Math 71 Summer 07 Name:

TEST 3 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

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1. Solve each equation in $\mathbb{C}$ (the set of complex numbers) by the indicated method.

   a) $2(x+3)^2 + 80 = 0$ by the square root property.

   b) $5y^2 = 2y - 7$ by completing the square.

   c) $t^2 + \frac{x}{2} = \frac{3}{2}$ by the quadratic formula.

   d) $h = -16t^2 + \frac{23}{3}t$ solve for t in terms of h.

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2. Solve the following equations. Give exact answers.

   a) $2x^4 - 3x^2 + 1 = 0$

   b) $\log_3(5x-1) - 1 = 0$

   c) $4^x = 11$

   d) $\log_8(x+5) - \log_8 2 = 1$

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3. Solve the following inequalities.

   a) $x^2 - 6x + 5 \le 0$

   b) $\dfrac{1}{x-3} < \dfrac{3}{x+2}$

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4. Let $f(x) = 5x - 2$ and $g(x) = \dfrac{1-x}{x+2}$. Answer the following questions:

   a) Find $(g \circ f)(x)$.

   b) $(f \circ g)(1)$

   c) Find $f^{-1}(x)$.

   d) Find $g^{-1}(x)$.

5. Simplify the following expressions.

a) $4\ln x + 7\ln y - 3\ln z$

b) $\frac{1}{2}\left(\log_5 x + \log_5 y\right) - 2\log_5\left(x+1\right)$

c) $\log_3 405 - \log_3 5 + \log 5 + \log 2$

d) $\log_{10}\left(\log_3\left(\log_5 125\right)\right)$

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6. For the equation given below, answer all the questions and graph the function (Be sure to label the axes and all points used). SHOW ALL WORK!

$$y = -10x^2 - 2x + 1$$

a) What type of curve is this?

b) What is the y-intercept?

c) What is the vertex

d) What are the x- intercept(s) (if any)?

e) What is the domain of the function?

f) What is the range of the function?

g) <u>Using the graph above</u>, solve the following inequality: $-10x^2 - 2x + 1 < 0$

h) What is the vertex form of the equation?

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7. Find the domain of each function:

a) $f\left(x\right) = -\frac{1}{2}x^2 + 3x - 57$

b) $g\left(x\right) = 7^x - 11$

c) $h\left(x\right) = \ln\left(7 - x\right)$

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8. Graph $f\left(x\right) = 4^x$ and $f^{-1}\left(x\right) = \log_4 x$ on the same coordinate system showing the symmetry about the bisector line $y = x$. Label the axes and all the points.

Choose THREE of the following problems.

1) The number of bacteria present in a culture after t hours is given by the formula $N = 500e^{0.59t}$.

a) How many bacteria will be there after ½ hour?

b) How long will it be before there are 500,000 bacteria?

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2) The owners of a small fruit orchard decide to produce gift baskets as a sideline. The cost per basket for producing *x* baskets is $C = 0.01x^2 - 2x + 120$. How many baskets should they produce in order to minimize the cost per basket? What will their total cost be at that production level?

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3) A model rocket launched with an upward velocity of 3.75 meters per second. The height of the rocket after *t* seconds if given by the formula: $h = -4.9t^2 + 3.75t + 12.25$.

a) How high is the rocket off the ground to start with?

b) How long does it take the rocket to hit the ground?

c) When does the rocket reach a height of 16 meters?

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4) You have 600 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

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5) In a laboratory experiment, researchers establish a colony of 100 bacteria and monitor its growth. The experiments discover that the colony triples in population every day.

a) Fill in Table 1. showing the population, $P(t)$, of bacteria *t* days later.

b) Find a function that gives the population of the colony at any time *t* in days.

c) Graph the function. Label the axes and the points used.

d) How many bacteria will be present after 13 days?

e) How long will it take before there are 37,000 bacteria?

t	**P(t)**
0	
10	
20	
30	
40	

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6) India is currently one of the world's fastest-growing countries. By 2040, the population of India will be larger than the population of China; by 2050, nearly one-third of the world's population will live in these two countries alone. The exponential function

$$f(x) = 574(1.026)^x$$

models the population of India, $f(x)$, in millions, x years after 1974.

a) What was India's population in 1974?

b) Find $f(27)$ and its meaning.

c) Find India's population, to the nearest million, in the year 2028 as predicted by this function.

d) How long will it be until the population reaches 1000 million people?

EXTRA CREDIT

#1 @ 3 points

Find the values of a and b such that the parabola $y = ax^2 + bx + 3$ contains the points $(-1,2)$ and $(1,6)$.

#2 @ 4 points

Find a formula for the cost of a sofa t years from now if it costs $1200 and the inflation rate is 8% annually.

#3 @ 1 point

Suppose a parabola has only one x-intercept. What can you say about the vertex of the parabola (where is it located)?

#4 @ 1 point

If you know the vertex of a parabola, how many more points do you need in order to find its equation?

#5 @ 2 points

What are the only x-values at which $y = ax^2 + bx + c$ can change signs?

#6 @ 3 points

The parabola $y = -x^2 + bx + c$ has x-intercepts at r_1 and r_2.

What are the solutions of the inequality $-x^2 + bx + c \leq 0$?

#7 @ 4 points

State whether each statement is TRUE or FALSE. Justify your answer.

a) $\log(a+b) = \log a + \log b$

b) $\log\left(\frac{a}{b}\right) \neq \frac{\log a}{\log b}$

c) $\log 5x^2 = 2\log 5x$

d) $\log(ab) \neq (\log a)(\log b)$

(3) $h = -4.9t^2 + 3.75t + 12.25$

t = time (in seconds)
h = height (in meters)

(a) initial height when $t=0$

$\boxed{h = 12.25 \text{ meters}}$

(b) $t = ?$ when $h = 0$

$-4.9t^2 + 3.75t + 12.25 = 0$

$4.9t^2 - 3.75t - 12.25 = 0$

$490t^2 - 375t - 1225 = 0 \quad / \div 5$

$98t^2 - 75t - 245 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-75) \pm \sqrt{(-75)^2 - 4(98)(-245)}}{2(98)}$$

$$= \frac{75 \pm \sqrt{101{,}665}}{196} \approx \frac{75 \pm 319}{196}$$

$t \approx 2$ sec OR $t \approx \cancel{1.2}$

It will hit the ground after about 2 seconds.

OR apply the quadratic formula in the beginning.

$-4.9t^2 + 3.75t + 12.25 = 0$

$$t = \frac{-3.75 \pm \sqrt{(3.75)^2 - 4(-4.9)(12.25)}}{2(-4.9)}$$

(c) $t = ?$ when $h = 16$

$-4.9t^2 + 3.75t + 12.25 = 16$

$4.9t^2 - 3.75t + 3.75 = 0$

$$t = \frac{3.75 \pm \sqrt{(3.75)^2 - 4(4.9)(3.75)}}{2(4.9)}$$

$$t = \frac{3.75 \pm \sqrt{-59}}{2(4.9)} \notin \mathbb{R}$$

The rocket will never reach a height of 16 meters.

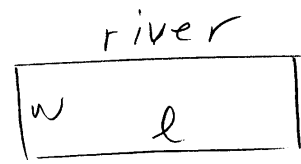
Note that max. height occurs at the vertex after $t_V = \frac{-3.75}{2(-4.9)} \approx 0.38$ seconds

and it is

$-4.9(0.38)^2 + 3.75(0.38) + 12.25$

max height ≈ 13 meters.

(4)



$\begin{cases} w = \text{width} \\ l = \text{length} \end{cases}$

600 ft fence $\Rightarrow$ $\underline{2w + l = 600}$

Area = maxim

A = area

$\underline{A = l \cdot w}$

$\begin{cases} 2w + l = 600 \Rightarrow l = 600 - 2w \\ A = l \cdot w \end{cases}$

$A = (600 - 2w)w$

$A = -2w^2 + 600w$

parabola opens down

$\Rightarrow$ maximum occurs at the vertex

$V(w_V, A_V)$

$w_v = \frac{-b}{2a} = \frac{-600}{2(-2)} = \boxed{150\,ft}$

the width that will maximize the area

Then $\ell = 600 - 2w$

$\ell = 600 - 300$

$\ell = 300$

Then $A_{max} = 150\,ft(300\,ft)$

$\boxed{A_{max} = 45{,}000\,ft^2}$

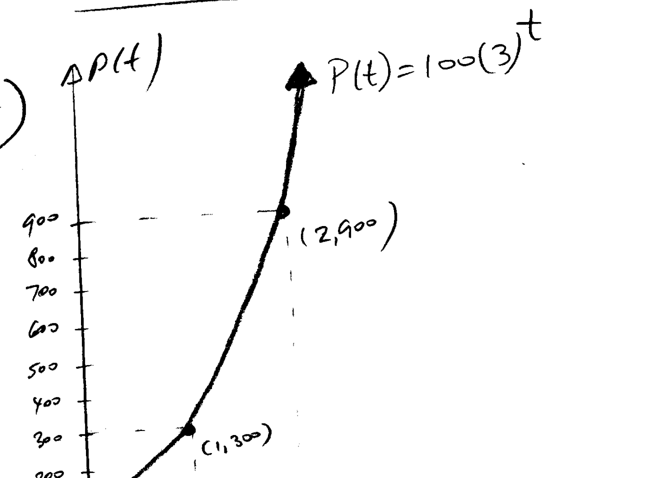
(5)

(a)

t	$P(t)$
0	100
1	$100(3) = 300$
2	$100(3)^2 = 900$
3	$100(3)^3 = 2700$
4	$100(3)^4$
t	$100(3)^t$

(b) $\boxed{P(t) = 100(3)^t}$

(c)



(d) If $t = 13$, then

$P(13) = 100(3)^{13} = \boxed{159{,}432{,}000}$ bacteria after 13 days

(e) $t = ?$ when $P(t) = 37{,}000$ bacteria

$37{,}0\cancel{00} = 1\cancel{00}(3)^t$

$370 = 3^t$

$\ln 370 = \ln 3^t$

$\ln 370 = t \ln 3$

$t = \frac{\ln 370}{\ln 3} \approx \boxed{5.4 \text{ days}}$

(6) $f(x) = 574(1.026)^x$

x = # years after 1974

$f(x)$ = population (in millions)

(a) when $x = 0$, $f(0) = 574(1.026)^0$
$= 574$ million people in 1974

(b) $f(27) = 574(1.026)^{27}$
≈ 1148 million people 27 years after 1974, that is in 2001

(c) 2028 is $x = 2028 - 1974$
$x = 54$

$f(54) = 574(1.026)^{54}$
≈ 2295 million people

(d) $t = ?$ when $f(x) = 1000$

$1000 = 574(1.026)^x$

$(1.026)^x = \frac{1000}{574}$

$\ln(1.026)^x = \ln\left(\frac{1000}{574}\right)$

$x \ln(1.026) = \ln \frac{1000}{574}$

$x = \frac{\ln \frac{1000}{574}}{\ln 1.026} \approx 22$ years after '74
in 1996

TEST 3 - SOLUTIONS

(1) (a) $2(x+3)^2 + 80 = 0$

$2(x+3)^2 = -80$

$(x+3)^2 = -40$

$\sqrt{(x+3)^2} = \sqrt{-40}$

$x+3 = \pm 2\sqrt{10}\,i$

$\boxed{x = -3 \pm 2\sqrt{10}\,i}$

(b) $5y^2 = 2y - 7$

$5y^2 - 2y = -7 \quad / \div 5$

$y^2 - \frac{2}{5}y = \frac{-7}{5} \quad / + \frac{1}{25}$

$\left(\frac{1}{2}\,\text{coef.}\,y\right)^2 = \left(\frac{1}{2}\cdot\frac{2}{5}\right)^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$

$y^2 - \frac{2}{5}y + \frac{1}{25} = \frac{-7}{5} + \frac{1}{25}$

$\left(y - \frac{1}{5}\right)^2 = \frac{-34}{25}$

$\sqrt{\left(y-\frac{1}{5}\right)^2} = \sqrt{\frac{-34}{25}}$

$y - \frac{1}{5} = \pm \frac{\sqrt{34}}{5}\,i$

$\boxed{y = \frac{1}{5} \pm \frac{\sqrt{34}}{5}\,i}$

(c) $t^2 + \frac{t}{2} = \frac{3}{2} \quad / \cdot 2$

$2t^2 + t - 3 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{cases} a = 2 \\ b = 1 \\ c = -3 \end{cases}$

$t = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-3)}}{2(2)} = \frac{-1 \pm \sqrt{1+24}}{4}$

$t = \frac{-1 \pm 5}{4} \quad \begin{cases} \frac{-1+5}{4} = 1 \\ \frac{-1-5}{4} = \frac{-6}{4} = -\frac{3}{2} \end{cases}$

$\boxed{t \in \{1, -\frac{3}{2}\}}$

(2) $2x^4 - 3x^2 + 1 = 0$

let $x^2 = t$

then $x^4 = t^2$

$2t^2 - 3t + 1 = 0$

$(2t-1)(t-1) = 0$

$2t - 1 = 0$ OR $t - 1 = 0$

$t = \frac{1}{2}$ $\qquad t = 1$

$x^2 = \frac{1}{2}$ $\qquad x^2 = 1$

$\sqrt{x^2} = \sqrt{\frac{1}{2}}$ $\qquad \sqrt{x^2} = \sqrt{1}$

$x = \pm \frac{1}{\sqrt{2}}$ $\qquad x = \pm 1$

$x = \pm \frac{\sqrt{2}}{2}$

$\boxed{x \in \{\pm \frac{\sqrt{2}}{2}, \pm 1\}}$

(b) $\log_3(5x-1) - 1 = 0$

condition: $5x - 1 > 0 \Rightarrow x > \frac{1}{5}$

$\log_3(5x-1) = 1$

$3^1 = 5x - 1 \Rightarrow 5x = 4 \Rightarrow \boxed{x = \frac{4}{5}}$

(c) $4^x = 11$

$\boxed{x = \log_4 11}$ or $\frac{\log 11}{\log 4}$

(1d) $h = -16t^2 + \frac{23}{3}t$

$16t^2 - \frac{23}{3}t + h = 0 \quad / \cdot 3$

$48t^2 - 23t + 3h = 0$

quadratic in t $\quad \begin{cases} a = 48 \\ b = -23 \\ c = 3h \end{cases}$

$t = \frac{23 \pm \sqrt{(23)^2 - 4(48)(3h)}}{2(48)}$

$\boxed{t = \frac{23 \pm \sqrt{529 - 576h}}{96}}$

(d) $\log_8(x+5) - \log_8 2 = 1$

Condition: $x+5>0$

$x>-5$

$\log_8 \frac{x+5}{2} = 1$

$8^1 = \frac{x+5}{2}$

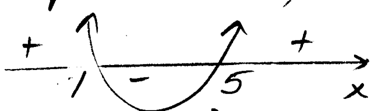
$x+5 = 16$

$\boxed{x=11}$

(3) (a) $x^2-6x+5 \le 0$

let $y = x^2-6x+5$

parabola opens upward



x-n: $x^2-6x+5=0$

$(x-1)(x-5)=0$

$x=1$ or $x=5$

So. $x^2-6x+5 \le 0$ iff $\boxed{x \in [1,5]}$

(b) $\frac{1}{x-3} < \frac{3}{x+2}$

$\frac{3}{x+2} - \frac{1}{x-3} > 0$

$LCD = (x+2)(x-3)$

$\frac{3(x-3)-(x+2)}{(x+2)(x-3)} > 0$

$\frac{3x-9-x-2}{(x+2)(x-3)} > 0$

$\frac{2x-11}{(x+2)(x-3)} > 0$

x	$-\infty$	-2	3	$\frac{11}{2}$	∞
$2x-11$	− − − − − − −		−	0 +	+
$x+2$	− − −	0 +	+ + +		+
$x-3$	−	−	− 0 +	+ +	+
$\frac{2x-11}{(x+2)(x-3)}$	−	+	−	+	

$\frac{1}{x-3} < \frac{3}{x+2}$ iff

$\frac{3}{x+2} - \frac{1}{x-3} > 0$ iff

$\boxed{x \in (-2,3) \cup (\frac{11}{2}, \infty)}$

(4) $f(x) = 5x-2$

$g(x) = \frac{1-x}{x+2}$

(a) $(g \circ f)(x) = g(f(x))$

$= g(5x-2)$

$= \frac{1-(5x-2)}{(5x-2)+2} = \frac{1-5x+2}{5x-2+2}$

$\boxed{(g \circ f)(x) = \frac{3-5x}{5x}}$

(b) $(f \circ g)(1) = f(g(1))$

$g(1) = \frac{1-1}{1+2} = \frac{0}{3} = 0$

So. $(f \circ g)(1) = f(g(1))$

$= f(0)$

$= 5(0)-2 = -2$

$\boxed{(f \circ g)(1) = -2}$

(c) $f(x) = 5x-2$

1st $y = 5x-2$

2nd solve for x

$5x = y+2$

$x = \frac{y+2}{5}$

3rd $x \longleftrightarrow y$

$y = \frac{x+2}{5}$

$$\boxed{f^{-1}(x) = \frac{x+2}{5}}$$

(d) $g(x) = \frac{1-x}{x+2}$

1st $y = \frac{1-x}{x+2}$

2nd solve for x

$y(x+2) = 1-x$

$yx+2y = 1-x$

$yx+x = 1-2y$

$x(y+1) = 1-2y$

$x = \frac{1-2y}{y+1}$

3rd $x \longleftrightarrow y$

$y = \frac{1-2x}{x+1}$

$$\boxed{g^{-1}(x) = \frac{1-2x}{x+1}}$$

(5)(a) $4\ln x + 7\ln y - 3\ln z =$

$= \ln x^4 + \ln y^7 - \ln z^3$

$= \ln x^4y^7 - \ln z^3$

$= \boxed{\ln\left(\frac{x^4y^7}{z^3}\right)}$

(b) $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1) =$

$= \frac{1}{2}\log_5(xy) - \log_5(x+1)^2$

$= \log_5(xy)^{\frac{1}{2}} - \log_5(x+1)^2$

$= \log_5\sqrt{xy} - \log_5(x+1)^2$

$= \boxed{\log_5 \frac{\sqrt{xy}}{(x+1)^2}}$

(c) $\log_3 405 - \log_3 5 + \log 5 + \log 2 =$

$= \log_3 \frac{405}{5} + \log 5\cdot 2$

$= \log_3 81 + \log 10$

$= 4+1 = \boxed{5}$

(d) $\log_{10}(\log_3(\log_5 125)) =$

$= \log_{10}(\log_3 3)$

$= \log_{10} 1$

$= \boxed{0}$

(6) $y = -10x^2 - 2x + 1$

(a) parabola opens downward $(a = -10 < 0)$

(b) y-n: let $x = 0$, $y = 1$

$\boxed{\text{y-n: } (0,1)}$

(c) $V(x_v, y_v)$

$$x_v = \frac{-b}{2a} = \frac{-(-2)}{2(-10)} = \frac{-1}{10}$$

$$y_v = -10\left(\frac{-1}{10}\right)^2 - 2\left(\frac{-1}{10}\right) + 1$$

$$= -10 \cdot \frac{1}{100} + \frac{2}{10} + 1$$

$$= \frac{-1}{10} + \frac{2}{10} + 1 = \frac{1}{10} + 1 = \frac{11}{10}$$

$$\boxed{V\left(\frac{-1}{10}, \frac{11}{10}\right)}$$

(d) x-n: let $y = 0$

$$-10x^2 - 2x + 1 = 0 \quad /(-1)$$

$$10x^2 + 2x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

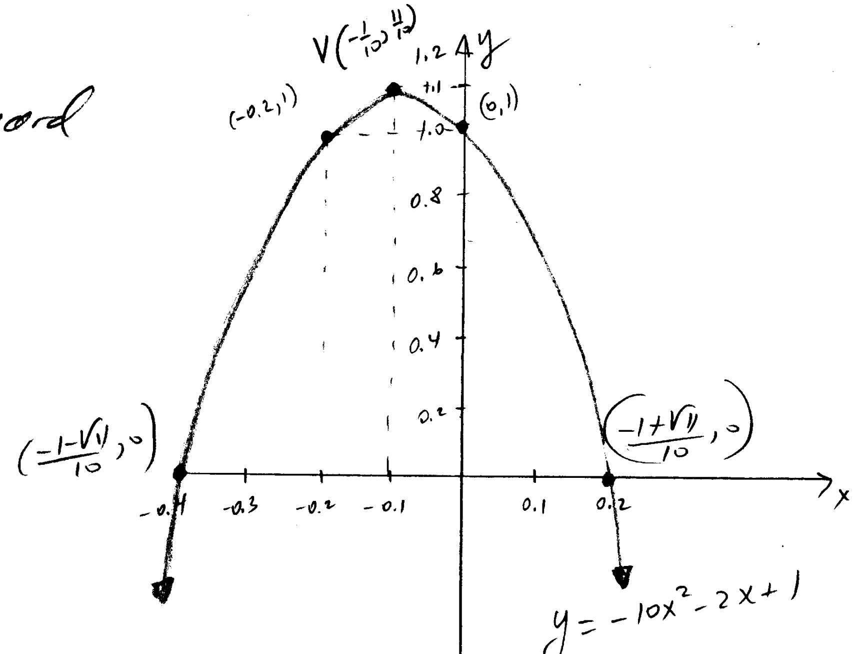
$$= \frac{-2 \pm \sqrt{2^2 - 4(10)(-1)}}{2(10)}$$

$$= \frac{-2 \pm \sqrt{4 + 40}}{20} = \frac{-2 \pm 2\sqrt{11}}{20}$$

$$= \frac{2(-1 \pm \sqrt{11})}{20} = \frac{-1 \pm \sqrt{11}}{10}$$

$$\boxed{\text{x-n: } \left(\frac{-1-\sqrt{11}}{10}, 0\right) \text{ and } \left(\frac{-1+\sqrt{11}}{10}, 0\right)}$$

x-n: $\approx (-0.4, 0)$ and $(0.2, 0)$



(e) Domain: $\boxed{x \in \mathbb{R}}$

(f) Range: $\boxed{y \in \left(-\infty, \frac{11}{10}\right]}$

(g) $-10x^2 - 2x + 1 < 0$ iff

$$\boxed{x \in \left(-\infty, \frac{-1-\sqrt{11}}{10}\right) \cup \left(\frac{-1+\sqrt{11}}{10}, \infty\right)}$$

(h) $y = a(x - x_v)^2 + y_v$

$$y = -10\left(x - \left(\frac{-1}{10}\right)\right)^2 + \frac{11}{10}$$

$$\boxed{y = -10\left(x + \frac{1}{10}\right)^2 + \frac{11}{10}}$$

(7) (a) $f(x) = \frac{-1}{2}x^2 + 3x - 57^{-5}$

Domain: $\boxed{x \in \mathbb{R}}$

(b) $g(x) = 7^{x} - 11$

Domain: $\boxed{x \in \mathbb{R}}$

(c) $h(x) = \ln(7-x)$

Condition: $7 - x > 0$

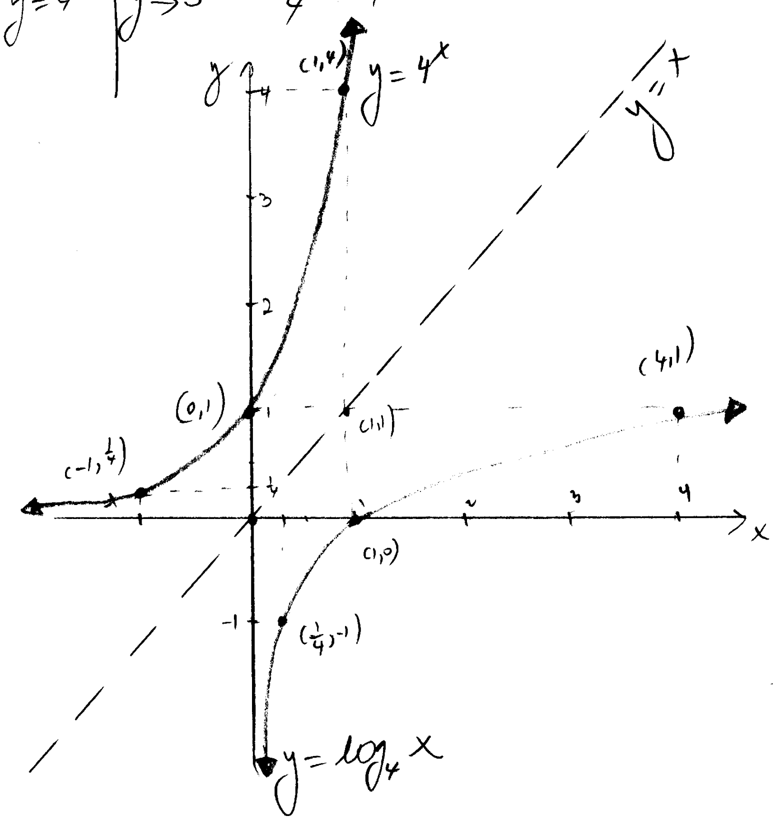
$7 > x$

$x < 7$

Domain: $\boxed{x \in (-\infty, 7)}$

(8) $f(x) = 4^x$

x	$-\infty$	-1	0	1	2	∞
$y=4^x$	$y \to 0$	$\frac{1}{4}$	1	4	16	∞



PART II

(1) $N = 500e^{0.59t}$

(a) $t = 0.5,\ N = ?$

$N = 500e^{0.59(0.5)} \approx \boxed{671 \text{ bacteria}}$ after $\frac{1}{2}$ hour

(b) $t = ?$ if $N = 500{,}000$

$500{,}000 = 500e^{0.59t}$

$1000 = e^{0.59t}$

$\ln 1000 = \ln e^{0.59t}$

$\ln 1000 = 0.59t$

$t = \frac{\ln 1000}{0.59} \approx \boxed{11.7 \text{ hours}}$

After aprox. 11.7 hours there will be 500,000 bacteria

(2) $C = 0.01x^2 - 2x + 120$

x = # baskets produced

C = cost per basket

The equation is a parabola that opens upward, therefore minimum occurs at the vertex $V(x_v, C_v)$

$x_v = \frac{-b}{2a} = \frac{-(-2)}{2(0.01)} = \boxed{100 \text{ baskets}}$

They should produce 100 baskets in order to minimize the cost per basket.

$C_v = 0.01(100)^2 - 2(100) + 120 =$

$C_v = 20\ \$/\text{basket}$

The total cost will be

$20\ \$/\text{basket} \cdot 100 \text{ baskets} = \boxed{2000\ \$}$

(3) $h=-4.9t^2+3.75t+12.25$

t = time (in seconds)
h = height (in meters)

(a) initial height when $t=0$

$\boxed{h = 12.25 \text{ meters}}$

(b) $t=?$ when $h=0$

$-4.9t^2+3.75t+12.25=0$

$4.9t^2-3.75t-12.25=0$

$490t^2-375t-1225=0 \quad \Big/ \div 5$

$98t^2-75t-245=0$

$t=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$t=\dfrac{-(-75)\pm\sqrt{(-75)^2-4(98)(-245)}}{2(98)}$

$=\dfrac{75\pm\sqrt{101,665}}{196}\approx\dfrac{75\pm319}{196}$

$t\approx 2$ sec OR $t\approx\cancel{-1.2}$

It will hit the ground after about 2 seconds.

OR apply the quadratic formula in the beginning.

$-4.9t^2+3.75t+12.25=0$

$t=\dfrac{-3.75\pm\sqrt{(3.75)^2-4(-4.9)(12.25)}}{2(-4.9)}$

(c) $t=?$ when $h=16$

$-4.9t^2+3.75t+12.25=16$

$4.9t^2-3.75t+3.75=0$

$t=\dfrac{3.75\pm\sqrt{(3.75)^2-4(4.9)(3.75)}}{2(4.9)}$

$t=\dfrac{3.75\pm\sqrt{-59}}{2(4.9)}\notin\mathbb{R}$

The rocket will never reach a height of 16 meters.

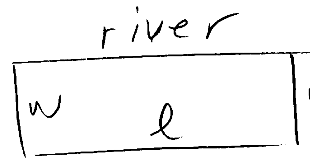
Note that max. height occurs at the vertex after $t_v=\dfrac{-3.75}{2(-4.9)}\approx 0.38$ seconds

and it is

$-4.9(0.38)^2+3.75(0.38)+12.25$

max height ≈ 13 meters.

(4)



$\begin{cases} w = \text{width} \\ l = \text{length}\end{cases}$

600 ft fence $\Rightarrow$ $\underline{2w+l=600}$

Area = maxim

A = area

$\underline{A=l\cdot w}$

$\begin{cases} 2w+l=600 \Rightarrow l=600-2w \\ A=l\cdot w \end{cases}$

$A=(600-2w)w$

$A=-2w^2+600w$

parabola opens down

$\Rightarrow$ maximum occurs at the vertex

$V(w_v, A_v)$

$w_v = \frac{-b}{2a} = \frac{-600}{2(-2)} = \boxed{150\ ft}$

the width that will maximize the area

Then $l = 600 - 2w$

$l = 600 - 300$

$l = 300$

Then $A_{max} = 150\ ft\ (300\ ft)$

$\boxed{A_{max} = 45{,}000\ ft^2}$

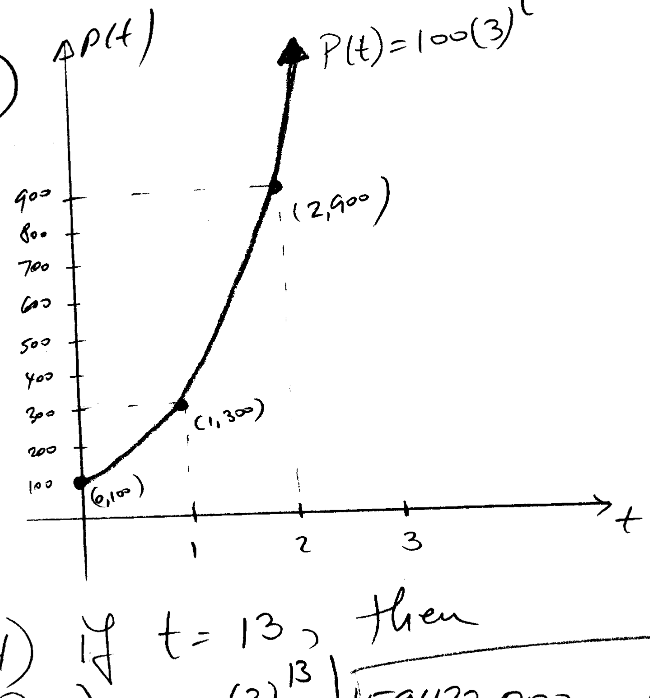
(5)

t	$P(t)$
0	100
1	$100(3) = 300$
2	$100(3)^2 = 900$
3	$100(3)^3 = 2700$
4	$100(3)^4$
t	$100(3)^t$

(a)

(b) $\boxed{P(t) = 100(3)^t}$

(c)



(d) If $t = 13$, then

$P(13) = 100(3)^{13} = \boxed{159{,}432{,}000}$ bacteria after 13 days

(e) $t = ?$ when $P(t) = 37{,}000$ bacteria

$37{,}0\cancel{00} = 1\cancel{00}(3)^t$

$370 = 3^t$

$\ln 370 = \ln 3^t$

$\ln 370 = t \ln 3$

$t = \frac{\ln 370}{\ln 3} \approx \boxed{5.4\ days}$

(6) $f(x) = 574(1.026)^x$

x = # years after 1974

$f(x)$ = population (in millions)

(a) when $x = 0$, $f(0) = 574(1.026)^0$

$= 574$ million people in 1974

(b) $f(27) = 574(1.026)^{27}$

≈ 1148 million people 27 years after 1974, that is in 2001

(c) 2028 is $x = 2028 - 1974$

$x = 54$

$f(54) = 574(1.026)^{54}$

≈ 2295 million people

(d) $t = ?$ when $f(x) = 1000$

$1000 = 574(1.026)^x$

$(1.026)^x = \frac{1000}{574}$

$\ln(1.026)^x = \ln\left(\frac{1000}{574}\right)$

$x \ln(1.026) = \ln \frac{1000}{574}$

$x = \frac{\ln \frac{1000}{574}}{\ln 1.026} \approx 22$ years after '74

in 1996