Math 71 Summer 07 Name:

TEST 2 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Factor each expression completely:

a) $4y^3 + 12y^2 - 72y$

b) $3x^2 + 3y^2$

c) $a^9 + 1$

2. Do the following operations (simplify):

a) $5 + \dfrac{7}{x-2}$

b) $\dfrac{x+1}{x^2+x-2} - \dfrac{1}{x^2-3x+2} + \dfrac{2x}{x^2-4}$

c) $\left(\dfrac{2x^{-\frac{1}{2}}y}{x^{\frac{5}{2}}y^{-\frac{1}{3}}}\right)^{-\frac{2}{3}}$ and write the final answer using only positive exponents

d) $2\sqrt{75} + 4\sqrt{12} - \left(2\sqrt{2} - \sqrt{3}\right)\left(2\sqrt{2} + \sqrt{3}\right)$

e) $\dfrac{1+2i}{2-3i}$

f) $\dfrac{-14+\sqrt{-128}}{16}$

3. If $f(x) = x^2 - 2x + 5$, find $\dfrac{f(a+h) - f(a)}{h}$.

4. If $f(x) = \dfrac{4}{x-3}$ and $g(x) = \dfrac{10}{x^2+x-12}$, find all the values of a for which $f(a) = g(a) + 1$.

5. $f(x) = x^2 - 3x + 5$ Find the following:

a) $f(3i)$

b) $f\left(1-\sqrt{2}\right)$

6. Let $f(x)=\sqrt{x+3}$.

a) What is the domain of this function?

b) Sketch the graph of the function by plotting points. Label the axes and all the points used.

c) What is the range of this function.

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7. If $g(x)=\sqrt{x+9}-\sqrt{x-7}$, find x such that $g(x)=2$.

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8. If $f(x)=\dfrac{x+6}{x+3}$ and $g(x)=\dfrac{3}{x+3}$, find x such that $f(x)=g(x)+2$.

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9. Police use the function $f(x)=\sqrt{20x}$ to estimate the speed of a car, f(x), in miles per hour, based on the length, x, in feet, of its skid marks upon sudden braking on a dry asphalt road. A motorist is involved in an accident. A police officer measures the car's skid mark to be 45 feet long. Estimate the speed at which the motorist was traveling before braking. If the posted speed limit is 35 miles per hour and the motorist tells the officer she was not speeding, should the officer believe her? Explain.

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10. A gymnast dismounts the uneven parallel bars at a height of 8 feet with an initial upward velocity of 8 feet per second. The function $s(t)=-16t^2+8t+8$ describes the height of the gymnast's feet above ground, $s(t)$, in feet, t seconds after dismounting.

a) How long will it take the gymnast to reach the ground?

b) When will the gymnast be 8 feet above the ground?

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11. The rational function $P(x)=\dfrac{72{,}900}{100x^2+729}$ models the percentage of people in the U.S., $P(x)$, with x years of education who are unemployed. Answer the following:

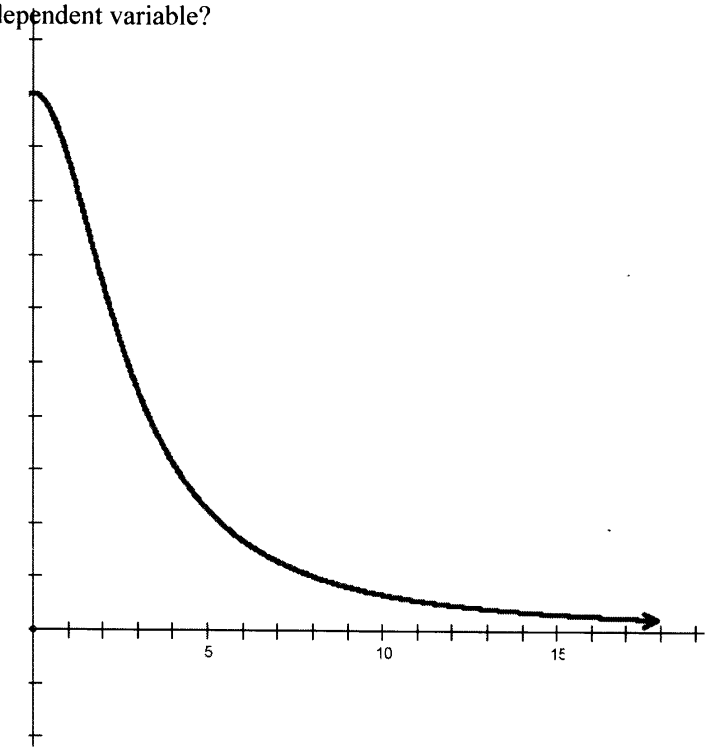
a) Which is the independent variable? Which is the dependent variable?
Label the axes of the graph accordingly.

b) What is the domain of the function?
What is the range?

c) Find and interpret $P(10)$.
Identify your solution as a point of the graph

d)
- What is the equation of the horizontal asymptote of the graph?
- Describe the end behavior of the graph.
Is there an education level that leads to guaranteed employment?
How is this indicated by the graph?

e) Find $P(0)$ and its meaning in the context of the problem.



TEST 2 - SOLUTIONS

① (a) $4y^3+12y^2-72y =$

$= 4y(y^2+3y-18)$

$= \boxed{4y(y+6)(y-3)}$

$P = c = -18 \begin{smallmatrix} +6 \\ -3 \end{smallmatrix}$
$S = b = 3$
$18 = 6\cdot 3$

(b) $3x^2+3y^2 = \boxed{3(x^2+y^2)}$

(c) $a^9+1 = (a^3)^3+1$

$= (a^3+1)((a^3)^2 - a^3 + 1^2)$

$= \boxed{(a+1)(a^2-a+1)(a^6-a^3+1)}$

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② (a) $5 + \dfrac{7}{x-2} = \dfrac{5}{1}^{\,x-2} + \dfrac{7}{x-2}$

$LCD = x-2$

$= \dfrac{5(x-2)+7}{x-2} = \boxed{\dfrac{5x-3}{x-2}}$

(b) $\dfrac{x+1}{x^2+x-2} - \dfrac{1}{x^2-3x+2} + \dfrac{2x}{x^2-4} =$

$= \dfrac{x+1}{(x+2)(x-1)}^{\,x-2} - \dfrac{1}{(x-1)(x-2)}^{\,x+2} + \dfrac{2x}{(x+2)(x-2)}^{\,x-1}$

$LCD = (x+2)(x-1)(x-2)$

$= \dfrac{(x-2)(x+1) - (x+2) + 2x(x-1)}{(x+2)(x-1)(x-2)}$

$= \dfrac{x^2 \cancel{+x} - 2x - 2 \cancel{-x} - 2 + 2x^2 - 2x}{(x+2)(x-1)(x-2)}$

$= \dfrac{3x^2-4x-4}{(x+2)(x-1)(x-2)} = \dfrac{(3x+2)\cancel{(x-2)}}{(x+2)(x-1)\cancel{(x-2)}}$

$= \boxed{\dfrac{3x+2}{(x+2)(x-1)}}$

① (c) $\left(\dfrac{2x^{-\frac{1}{2}}y}{x^{\frac{5}{2}}y^{-\frac{1}{3}}}\right)^{-\frac{2}{3}} = \left(2x^{-\frac{1}{2}-\frac{5}{2}}\,y^{1-(-\frac{1}{3})}\right)^{-\frac{2}{3}}$

$= \left(2x^{-3}y^{\frac{4}{3}}\right)^{-\frac{2}{3}}$

$= 2^{-\frac{2}{3}}\left(x^{-3}\right)^{-\frac{2}{3}}\left(y^{\frac{4}{3}}\right)^{-\frac{2}{3}}$

$= \dfrac{1}{2^{\frac{2}{3}}}\,x^2\,y^{-\frac{8}{9}} = \boxed{\dfrac{x^2}{2^{\frac{2}{3}}\,y^{\frac{8}{9}}}}$

(d) $2\sqrt{75} + 4\sqrt{12} - \underbrace{(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})}_{\text{difference of squares}} =$

$= 2\sqrt{25\cdot 3} + 4\sqrt{4\cdot 3} - \left((2\sqrt{2})^2 - (\sqrt{3})^2\right)$

$= 2\cdot 5\sqrt{3} + 4\cdot 2\sqrt{3} - (4\cdot 2 - 3)$

$= 10\sqrt{3} + 8\sqrt{3} - 5 = \boxed{18\sqrt{3} - 5}$

(e) $\dfrac{1+2i}{2-3i}^{\,2+3i} = \dfrac{(1+2i)(2+3i)}{(2-3i)(2+3i)}$

$= \dfrac{2+3i+4i+6i^2}{2^2-(3i)^2}$

$= \dfrac{2+7i-6}{4-9i^2} = \dfrac{-4+7i}{4+9}$

$= \dfrac{-4+7i}{13} = \boxed{-\dfrac{4}{13} + \dfrac{7}{13}i}$

(f) $\dfrac{-14+\sqrt{-128}}{16} = \dfrac{-14+\sqrt{128}\,i}{16}$

$= \dfrac{-14+\sqrt{64\cdot 2}\,i}{16} = \dfrac{-14+8\sqrt{2}\,i}{16}$

$= \dfrac{\cancel{2}(-7+4\sqrt{2}\,i)}{\cancel{16}\;8} = \boxed{\dfrac{-7+4\sqrt{2}\,i}{8}}$

3) $f(x) = x^2 - 2x + 5$

$$\frac{f(a+h)-f(a)}{h} =$$

$$= \frac{[(a+h)^2 - 2(a+h) + 5] - (a^2 - 2a + 5)}{h}$$

$$= \frac{\cancel{a^2} + 2ah + h^2 - \cancel{2a} - 2h + \cancel{5} - \cancel{a^2} + \cancel{2a} - \cancel{5}}{h}$$

$$= \frac{2ah + h^2 - 2h}{h} = \frac{\cancel{h}(2a+h-2)}{\cancel{h}}$$

$$= \boxed{2a + h - 2}$$

5) $f(x) = x^2 - 3x + 5$

a) $f(3i) = (3i)^2 - 3(3i) + 5$

$= 9i^2 - 9i + 5$

$= 9(-1) - 9i + 5$

$= \boxed{-4 - 9i}$

b) $f(1-\sqrt{2}) = (1-\sqrt{2})^2 - 3(1-\sqrt{2}) + 5$

$= 1 - 2\sqrt{2} + (\sqrt{2})^2 - 3 + 3\sqrt{2} + 5$

$= 1 - 2\sqrt{2} + 2 + 2 + 3\sqrt{2}$

$= \boxed{5 + \sqrt{2}}$

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4) $f(x) = \frac{4}{x-3}$, $g(x) = \frac{10}{x^2+x-12}$

$a = ?$ such that

$f(a) = g(a) + 1$

Solution

$$\frac{4}{a-3} = \frac{10}{a^2+a-12} + 1$$

$$\frac{\overset{a+4}{4}}{a-3} = \frac{10}{(a+4)(a-3)} + \frac{\overset{(a+4)(a-3)}{1}}{1}$$

Conditions: $\begin{cases} a \neq 3 \\ a \neq -4 \end{cases}$

$LCD = (a+4)(a-3)$

$4(a+4) = 10 + (a+4)(a-3)$

$4a + 16 = 10 + a^2 + a - 12$

$4a + 16 = a^2 + a - 2$

$a^2 - 3a - 18 = 0$

$(a-6)(a+3) = 0$

$a = 6$ or $a = -3$

$\boxed{a \in \{6, -3\}}$

6) $f(x) = \sqrt{x+3}$

a) Condition: $x + 3 \geq 0$

$x \geq -3$

$\boxed{x \in [-3, \infty)}$

b)

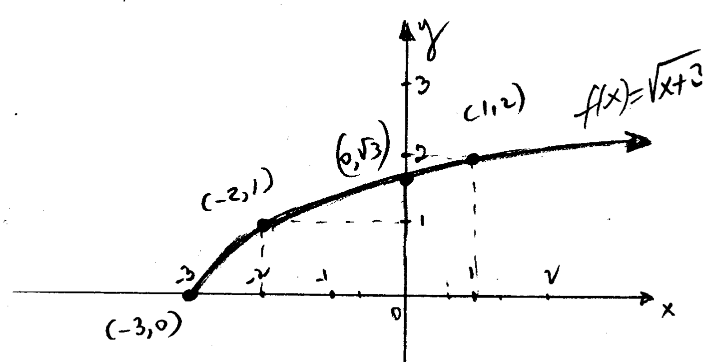
x	y
-3	0
-2	1
0	$\sqrt{3}$
1	2

$\sqrt{-3+3} = \sqrt{0} = 0$

$\sqrt{-2+3} = \sqrt{1} = 1$

$\sqrt{0+3} = \sqrt{3} \simeq 1.7$

$\sqrt{1+3} = \sqrt{4} = 2$



c) $\boxed{y \in [0, \infty)}$

(7) $g(x) = \sqrt{x+9} - \sqrt{x-7}$

$x = ?$ such that $g(x) = 2$

Solution

$\sqrt{x+9} - \sqrt{x-7} = 2$

$\sqrt{x+9} = 2 + \sqrt{x-7} \quad \Big|^2$

$\left(\sqrt{x+9}\right)^2 = \left(2 + \sqrt{x-7}\right)^2$

$\cancel{x} + 9 = 4 + 4\sqrt{x-7} + \cancel{x} - 7$

$9 = -3 + 4\sqrt{x-7}$

$12 = 4\sqrt{x-7} \quad \Big| \div 4$

$3 = \sqrt{x-7} \quad \Big|^2$

$3^2 = \left(\sqrt{x-7}\right)^2$

$9 = x - 7 \implies x = 16$

Check: $\sqrt{16+9} - \sqrt{16-7} \stackrel{?}{=} 2$

$\sqrt{25} - \sqrt{9} \stackrel{?}{=} 2$

$5 - 3 = 2$ true

$\boxed{x \in \{16\}}$

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(8) $f(x) = \dfrac{x+6}{x+3}$, $g(x) = \dfrac{3}{x+3}$

$x = ?$ such that $f(x) = g(x) + 2$

Solution

$\dfrac{x+6}{x+3} = \dfrac{3}{x+3} + 2$

Condition: $x \neq -3$

$\dfrac{x+6}{x+3} - \dfrac{3}{x+3} = 2$

$\dfrac{x+6-3}{x+3} = 2$

$\dfrac{x+3}{x+3} = 2$

$1 = 2$ false

$\boxed{x \in \emptyset}$

(9) $f(x) = \sqrt{20x}$

x = length of skid marks (ft)

$f(x)$ = speed of car (mi/h)

$x = 45$ ft

$f(45) = \sqrt{20(45)} = \sqrt{900} = 30$ mi/h

Yes, her speed was less than the posted speed limit

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(10) $s(t) = -16t^2 + 8t + 8$

t = time (sec)

$s(t)$ = height above ground (ft)

(a) $t = ?$ when $s(t) = 0$

$-16t^2 + 8t + 8 = 0$

$16t^2 - 8t - 8 = 0 \quad \Big| \div 8$

$2t^2 - t - 1 = 0$

$(2t+1)(t-1) = 0$

$t = -\frac{1}{2}$ not possible

$t = 1$ second

It will take the gymnast 1 second to reach the ground

(b) $t = ?$ when $s(t) = 8$

$\cancel{8} = -16t^2 + 8t + \cancel{8}$

$16t^2 - 8t = 0$

$8t(2t-1) = 0$

$t = 0$ seconds initial moment

$t = \frac{1}{2}$ seconds

The gymnast will be 8 ft above the ground at $t = 0$ seconds (initially) and again after 0.5 seconds.

(11) $P(x) = \dfrac{72,900}{100x^2 + 729}$

x = number of years of education
$P(x)$ = percentage of people unemployed

(a) x = # years of education is the independent variable
$P(x)$ = % of people unemployed is the dependent variable

(b) Domain: $x \geq 0$
Range: $0 < y \leq 100$ 100% is the maximum number of people

(c) $P(10) = \dfrac{72,900}{100(10)^2 + 729}$

$= \dfrac{72,900}{10000 + 729}$

$= \dfrac{72,900}{10,729} \approx 6.8\ \%$

About 6.8% of people in U.S. with 10 years of education are unemployed.

(d) $y = 0$ horizontal asymptote
As $x \to \infty$, $P(x) \to 0$
There is no education level that leads to guaranteed employment. The function values get closer to zero, but never actually reach 0.

(e) $P(0) = \dfrac{72,900}{729} = 100\ \%$.

100% of people (all) with no education (zero years of education) are unemployed.