Math 71 Summer 07 Name:

TEST 1 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1.

a) State what it means that y is a function of x. Use function notation.
b) What is the domain of a function?
c) What is the range of a function?
d) Give an example of a function and state its domain.

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2. Let $f(x) = \dfrac{2x+1}{x+3}$.

a) Find $f(1)$.
b) Find $f(a+h)$
c) What is the domain of the given function ?

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3. Let $f(x) = \begin{cases} 3x^2, x \le 1 \\ 2x-1, x > 1 \end{cases}$.

a) Find $f(-1)$
b) Find $f(5)$.
c) What is the domain of the given function?

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4. Write an equation of a line passing through $(4,-7)$ and perpendicular to the line whose equation is $x - 2y - 3 = 0$.

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5. Graph the following line using the x- and y- intercepts. Then state the domain and range.

$$4x - 5y = 20$$

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6. Solve the following system:

$$\begin{cases} 2x+3y+7z=13 \\ 3x+2y-5z=-22 \\ 5x+7y-3z=-28 \end{cases}$$

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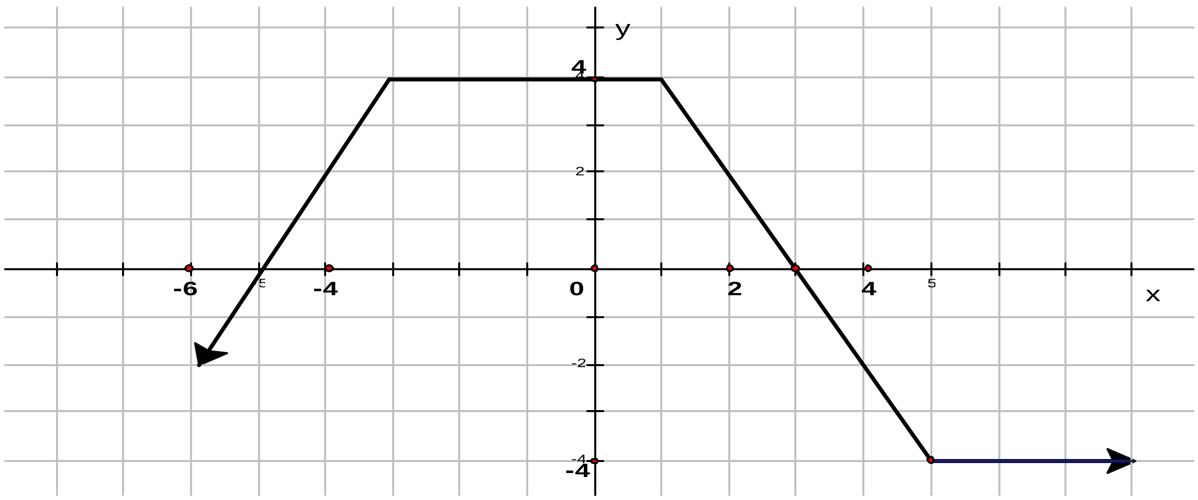
7. A graph is given. Answer the following:



a) Is y a function of x? Explain.

b) What is the domain of the function?

c) What is the range of the function?

d) If $y = f(x)$, find: $f(-4)$, $f(0)$, and $f(4)$.

e) Solve $f(x) = 0$

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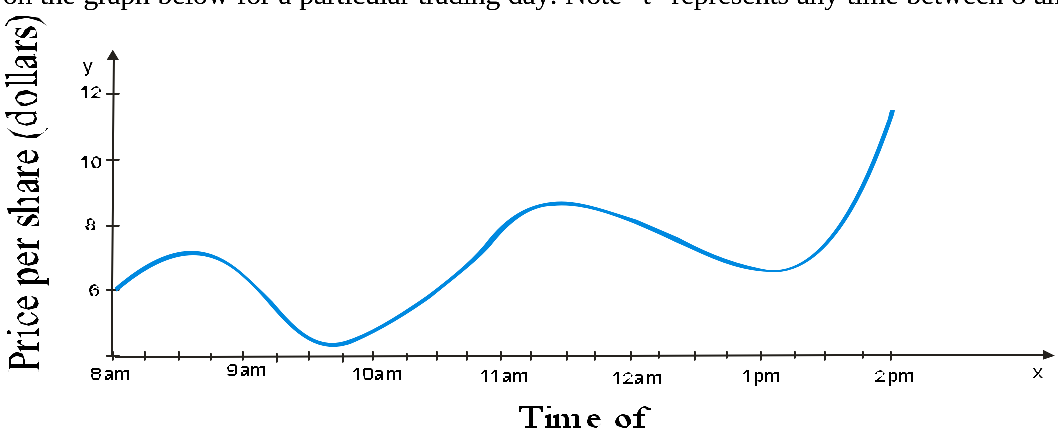
8. The value of a stock varies during the course of any trading day. The price per share "P" of a certain stock is shown on the graph below for a particular trading day. Note "t" represents any time between 8 am and 2 pm.



a) Is "t" (the time of the day) a function of "P" (the price per share)?

b) Is "P" a function of "t"? Explain using the <u>definition of function</u>.

Using the graph, estimate the answers to the following questions (Use the correct units).

c) What is the domain?

d) What is the range?

e) For what value(s) of "*t*" does P(t)=8 and what does it mean in practical terms?

f) What is P(2) and what does it mean in practical terms?

9. Solve the following equations :

a) $\left|2x+\frac{1}{3}\right|=\frac{4}{5}$

b) $\left|x+\frac{1}{2}\right|=|x-3|$

c) $|3x+14|+7=2$

e) $\frac{4}{5}(s+2)=\frac{1}{2}+\frac{5}{6}(s+3)$

f) $A=\frac{1}{2}h(a+b)$. Solve for a.

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10. Solve the following inequalities. Graph the solution set and write it in interval notation.

a) $|2x-1|+3\leq 11$

b) $4|3x-5|>12$

c) $\frac{2x+3}{3}+\frac{3x-4}{2}>\frac{x-2}{2}$

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11. Graph the solution set of the following system of inequalities. Show clearly how you graph the lines and what test points you're using. Clearly label the axes, the lines, and the points used.

$$\begin{cases} x\geq 0 \\ y\geq 0 \\ 2x+5y<10 \\ 3x+4y\leq 12 \end{cases}$$

12. **<u>Choose THREE</u>** of the following word problems. Show clearly what your variables represent. Show clearly the equation(s) you use to solve each problem. You may solve **the other two problems for extra credit.**

(A) A coin collection contains a mixture of 26 coins in nickels, dimes, and quarters. The coins have a total value of \$4.00. The number of quarters is 2 less than the number of nickels and dimes combined. Determine the number of nickels, dimes, and quarters in the collection

(B) The weight (in kilograms) of a pumpkin is measured as it grows over a particular month. After 2 days, the pumpkin weighed 3 kilograms while at 31 days, the pumpkin's weight was 9 kilograms.

a. Assuming the weight is growing at a linear rate, find a formula that gives the weight "W" (in kilograms) in terms of the number of days "D

b. What are the units of the slope and what does it mean in this problem.

(C) On the first of four exams, your grades are 70, 75, 87, and 92. There is still one more exam, and you are hoping to earn a B in the course. This will occur if the average of your five exam grades is greater than or equal to 80 and less than 90. What range of grades on the fifth exam will result in earning a B? Use interval notation to express this range.

(D) On your next vacation you will divide lodging between large resorts and small inns. Let x represent the number of nights spent in large resorts. Let y represent the number of nights spent in small inns.

a) Write a system of inequalities that model the following conditions:

You want to stay at least 5 nights.
At least one night should be spent at a large resort.
Large resorts average \$200 per night and small inns average \$100 per night.
Your budget permits no more than \$700 for lodging.

b) Graph the solution set of the system of inequalities in part (a).

(E) The function

$$W(x) = 0.07x + 4.1$$

models the number of women, $W(x)$, in millions, enrolled in U.S. colleges x years after 1984.
The function

$$M(x) = 0.01x + 3.9$$

models the number of men, $M(x)$, in millions, enrolled in U.S. colleges x years after 1984. Use these functions to answer the following questions:

a) Find and interpret $W(16)$.

b) Find and interpret $M(16)$

c) Find and interpret $W(20) - M(20)$.

SOLUTIONS

(1) a) $y = f(x)$

y is a function of x if for every input x there is only one output y.

b) Domain = the set of all values of x for which the function exists

Domain $= \{x \mid f(x) \in \mathbb{R}\}$

c) Range = the set of all values of y

Range $= \{y \mid y = f(x),\ x \in \text{Domain}\}$

d) $f(x) = 2x+1$

Domain $= \mathbb{R}$

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(2) $f(x) = \dfrac{2x+1}{x+3}$

a) $f(1) = \dfrac{2(1)+1}{1+3} = \boxed{\dfrac{3}{4}}$

b) $f(a+h) = \dfrac{2(a+h)+1}{(a+h)+3}$

$= \boxed{\dfrac{2a+2h+1}{a+h+3}}$

c) Condition: $x+3 \neq 0$

$x \neq -3$

$\boxed{x \in \mathbb{R} \setminus \{-3\}}$

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(3) $f(x) = \begin{cases} 3x^2, & x \le 1 \\ 2x-1, & x > 1 \end{cases}$

a) $f(-1) = 3(-1)^2 = \boxed{3}$

b) $f(5) = 2(5) - 1 = \boxed{9}$

c) $\boxed{x \in \mathbb{R}}$

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(4) $x - 2y - 3 = 0$

$2y = x - 3$

$y = \frac{1}{2}x - \frac{3}{2} \Rightarrow m = \frac{1}{2}$

$\begin{cases} m_\perp = -2 \\ (4,-7) \end{cases}$

$y - y_1 = m(x - x_1)$

$y - (-7) = -2(x-4)$

$\boxed{y + 7 = -2(x-4)}$

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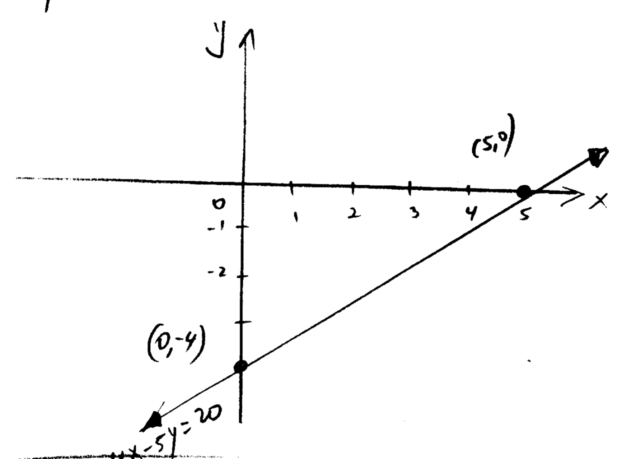
(5) $4x - 5y = 20$

x	y
0	-4
5	0

$-5y = 20 \Rightarrow y = -4$

$4x = 20 \Rightarrow x = 5$



$\boxed{\begin{array}{l} \text{Domain: } x \in \mathbb{R} \\ \text{Range: } y \in \mathbb{R} \end{array}}$

(6) $\begin{cases} 2x+3y+7z=13 & (1) \\ 3x+2y-5z=-22 & (2) \\ 5x+7y-3z=-28 & (3) \end{cases}$

Eliminate x:

(1) (2) $\left\{\begin{array}{l} 2x+3y+7z=13 \\ 3x+2y-5z=-22 \end{array}\right| \begin{array}{l} 3 \\ -2 \end{array}$

$\begin{cases} 6x+9y+21z=39 \\ -6x-4y+10z=44 \end{cases}$

(+) $5y+31z=83$ (4)

(1) (3) $\left\{\begin{array}{l} 2x+3y+7z=13 \\ 5x+7y-3z=-28 \end{array}\right| \begin{array}{l} 5 \\ -2 \end{array}$

$\begin{cases} 10x+15y+35z=65 \\ -10x-14y+6z=56 \end{cases}$

(+) $y+41z=121$ (5)

(4) (5) $\left\{\begin{array}{l} 5y+31z=83 \\ y+41z=121 \end{array}\right| \begin{array}{l} \\ -5 \end{array}$

$\begin{cases} 5y+31z=83 \\ -5y-205z=-605 \end{cases}$

$-174z=-522$

$z=\frac{-522}{-174}=3$ $z=3$

(5) $y+41z=121 \Rightarrow y=-2$

(1) $2x+3y+7z=13 \Rightarrow x=-1$

The solution is $\boxed{(-1,-2,3)}$

(7) a) y is a function of x because the graph passes the vertical line test.

b) $x \in \mathbb{R}$

c) $y \in (-\infty, 4]$

d) $f(-4)=2$

$f(0)=4$

$f(4)=-2$

e) $f(x)=0 \iff x=-5$ OR $x=3$

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(8) a) t is not a function of P because for a given P there could be more than one t value. For example, if $P=6$, $t=$ 8am, 9:10 am, 10:30am

b) P is a function of t because for every t there is only one P.

$P=f(t)$

c) $t \in$ [8am, 2pm]

d) $P \in$ [2\$, 12\$]

e) $P(t)=8$ when $t=$ 11am and $t=$ 12:15pm, $t=$ 1:30pm

The share is 8 dollars at 11am, 12:15pm, 1:30pm

f) $P(2)=12\$$ The share was \$12 at 2pm

(9) (a) $\left|2x+\frac{1}{3}\right|=\frac{4}{5}$

$2x+\frac{1}{3}=\frac{4}{5}$ OR $2x+\frac{1}{3}=-\frac{4}{5}$

$2x=\frac{^{3/}4}{5}-\frac{^{5/}1}{3}$ $\qquad$ $2x=\frac{^{3/}-4}{5}-\frac{^{5/}1}{3}$

$2x=\frac{12-5}{15}$ $\qquad$ $2x=\frac{-12-5}{15}$

$2x=\frac{7}{15}\ \Big/\cdot\frac{1}{2}$ $\qquad$ $2x=\frac{-17}{15}\ \Big/\cdot\frac{1}{2}$

$x=\frac{7}{30}$ $\qquad$ $x=\frac{-17}{30}$

$\boxed{x\in\left\{\frac{1}{30},\frac{-17}{30}\right\}}$

(b) $\left|x+\frac{1}{2}\right|=|x-3|$

$\cancel{x}+\frac{1}{2}=\cancel{x}-3$ OR $x+\frac{1}{2}=-(x-3)$

$\frac{1}{2}=-3$ $\qquad$ $x+\frac{1}{2}=-x+3$

NO SOLUTION $\qquad$ $x+x=3-\frac{1}{2}$

$2x=\frac{5}{2}\ \Big/\cdot\frac{1}{2}$

$x=\frac{5}{4}$

$\boxed{x\in\left\{\frac{5}{4}\right\}}$

(c) $|3x+14|+7=2$

$|3x+14|=2-7$

$|3x+14|=-5$

not possible

$\boxed{x\in\emptyset}$

(e) $\frac{^{6/}4}{5}(s+2)=\frac{^{15/}1}{2}+\frac{^{5/}5}{6}(s+3)$

LCD = 30

$24(s+2)=15+25(s+3)$

$24s+48=15+25s+75$

$24s+48=25s+90$

$48-90=25s-24s$

$\boxed{s=-42}$

(7) $A=\frac{1}{2}h(a+b)$ for a

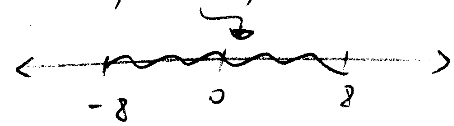
$2A=h(a+b)$

$a+b=\frac{2A}{h}\implies\boxed{a=\frac{2A}{h}-b}$

or $\boxed{a=\frac{2A-bh}{h}}$

(10) (a) $|2x-1|+3\le 11$

$|2x-1|\le 8$

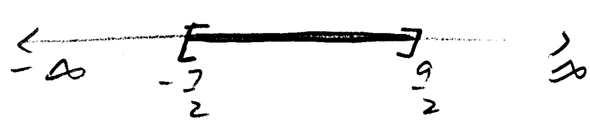


$-8\le 2x-1\le 8$

$+1\qquad +1\qquad +1$

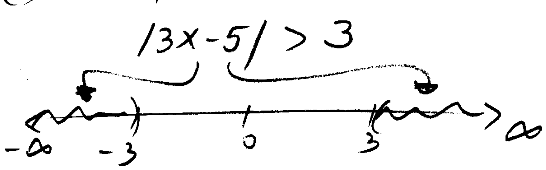
$-7\le 2x\le 9\ \Big/\div 2$

$\boxed{-\frac{7}{2}\le x\le\frac{9}{2}}$



(b) $4|3x-5| > 12$

$|3x-5| > 3$



$3x-5 < -3$	OR	$3x-5 > 3$
$3x < 2$		$3x > 8$
$x < \frac{2}{3}$		$x > \frac{8}{3}$

Therefor, $\boxed{x < \frac{2}{3} \text{ OR } x > \frac{8}{3}}$

$\boxed{x \in (-\infty, \frac{2}{3}) \cup (\frac{8}{3}, \infty)}$

(c) $\frac{2x+3}{3} + \frac{3x-4}{2} > \frac{x-2}{2}$

LCD = 6

$2(2x+3) + 3(3x-4) > 3(x-2)$

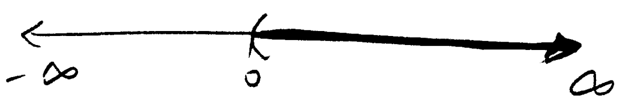
$4x + 6 + 9x - 12 > 3x - 6$

$13x - \cancel{6} > 3x - \cancel{6}$

$13x - 3x > 0$

$10x > 0$

$\boxed{x > 0}$



$\boxed{x \in (0, \infty)}$

(11) $\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x+5y < 10 \\ 3x+4y \leq 12 \end{cases}$ | quadrant I

$\boxed{2x+5y < 10}$

Boundary line: $2x+5y = 10$

x	y
0	2
5	0

Test point $(0,0) \notin$ line

$2(0) + 5(0) < 10$

$0 < 10$ true $\Rightarrow (0,0)$ = solution

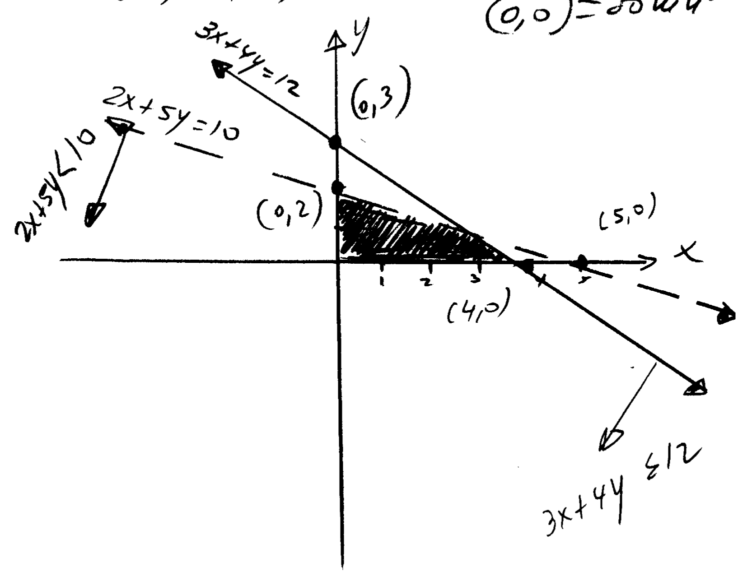
$\boxed{3x+4y \leq 12}$

Boundary line: $3x+4y = 12$

x	y
0	3
4	0

Test point: $(0,0) \notin$ line

$3(0) + 4(0) \leq 12$ true $\Rightarrow$ $(0,0)$ = soluti



(12) -5

(A) 26 coins / $4 total: nickels x, dimes y, quarters z

quarters is 2 less than the # of nickels and dimes combined

Solution

let x = # of nickels
y = # dimes
z = # quarters

$$\begin{cases} x+y+z=26 & (1)\ (26\text{ coins}) \\ 5x+10y+25z=400 & (2)\ (\$4=400\text{ cents}) \quad \div 5 \\ z=x+y-2 & (3) \end{cases}$$

$$\begin{cases} x+y+z=26 & (1) \\ x+2y+5z=80 & (2) \\ z=x+y-2 & (3) \end{cases}$$

$$\begin{cases} x+y+(x+y-2)=26 \\ x+2y+5(x+y-2)=80 \end{cases}$$

$$\begin{cases} 2x+2y=28 \\ 6x+7y=90 \end{cases} \Big| \cdot -3$$

$$\begin{cases} -6x-6y=-84 \\ 6x+7y=90 \end{cases}$$

$/\quad y=6$ dimes

$2x+2y=28 \Rightarrow 2x+12=28$
$x=8$ nickels

$z=x+y-2 \Rightarrow z=12$

There are 8 nickels, 6 dimes, 12 quarters

(B) a) D = # days (independent variable)
W = weight (in kg) (dependent variable)

D	W
2	3
31	9

We need a linear equation that gives W in terms of D (the equation of the line passing through (2,3) and (31,9))

$$\begin{cases} m=\frac{\Delta W}{\Delta D}=\frac{9-3}{31-2}=\frac{6}{29} \\ (2,3) \end{cases}$$

$$\boxed{W-3=\frac{6}{29}(D-2)}$$

b) $m=\frac{6}{29}$ kg/day

it shows the rate of growth of the pumpkin: $\frac{6}{29}$ kg/day

(C) I exam: 70
II 75
III 87
IV 92
V x

Let x = grade on the 5th exam

$80 \le$ average < 90

$$80 \le \frac{70+75+87+92+x}{5} < 90$$

$$80 \le \frac{324+x}{5} < 90 \quad \Big/ \cdot 5$$

$$400 \le 324 + x < 450$$

$-324 \qquad -324 \qquad -324$

$$76 \le x < 126$$

$$\boxed{x \in [76, 100]}$$

(D) large resorts - x nights \$200/night
small inns - y nights \$100/night

(a)
$$\begin{cases} x+y \ge 5 \\ x \ge 1 \\ 200x + 100y \le 700 \\ y \ge 0 \end{cases}$$

(b) $\boxed{x+y \ge 5}$ (1)

Boundary line: $x+y=5$

x	y
0	5
5	0

Test point: $(0,0) \notin$ line
$0+0 \ge 5$ false $\Rightarrow (0,0) \ne$ solution.

$\boxed{x \ge 1}$ (2)

Boundary line: $x=1$

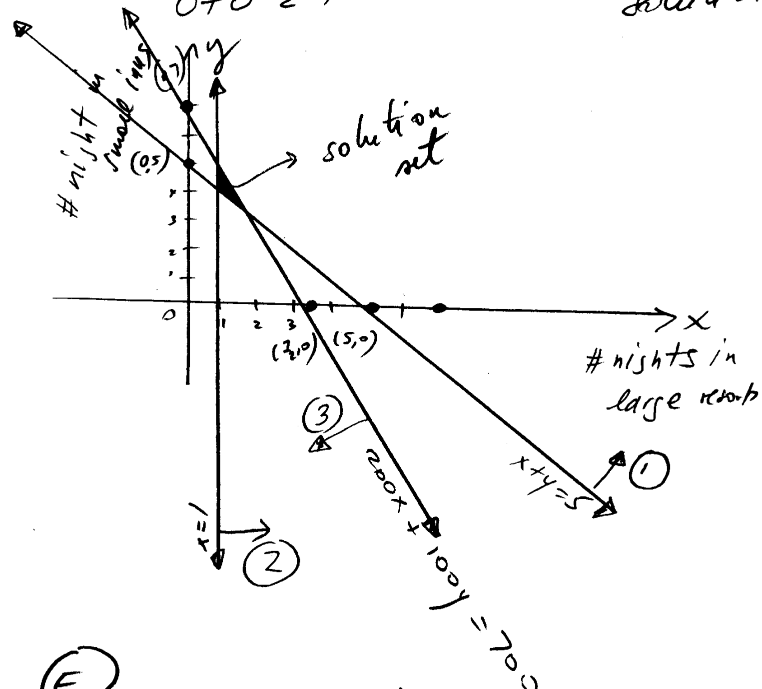
$\boxed{200x + 100y \le 700}$ (3) $\div 100$

$\boxed{2x + y \le 7}$

Boundary line: $2x+y=7$

x	y
0	7
$\frac{7}{2}$	0

Test point: $(0,0) \notin$ line
$0+0 \le 7$ true $\Rightarrow (0,0) =$ solution.



(E)
a) $W(16) = 0.07(16) + 4.1 = 5.22$
In 2000, there were 5.22 million women enrolled in U.S. colleges

b) $M(16) = 0.01(16) + 3.9 = 4.06$
In 2000, there were 4.06 million men enrolled in U.S colleges.

c) $W(20) - M(20) = (0.07(20)+4.1) - (0.01(20)+3.9) = 1.4$
In 2004, there were 1.4 million more women than men enrolled in U.S colleges.