Math 71 – Summer 2007 Name:

QUIZ #6 @ 30 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Solve the following inequalities. Write the answer using interval notation.

a) $x^2+x-6>0$

b) $9x^2+3x-2\le 0$

c) $\dfrac{2x+1}{5-x}\ge 0$

d) $\dfrac{x-2}{x+2}\le 2$

2. Answer the following questions:

a) Is y a function of x ? Why?

b) Find $f(1)$, $f(2)$, and $f(3)$.

c) Let $y=f(x)$. Is f one-to-one? Why?

d) Does f have an inverse? Why?

e) Find $f^{-1}(1)$, $f^{-1}(2)$, and $f^{-1}(3)$.

x	y
1	2
3	1
2	5
4	7
5	3

3. Let $f(x)=2x+3$, $g(x)=\dfrac{x+1}{1-x}$.

a) Find $(f\circ g)(x)$.

b) Find $f^{-1}(x)$.

c) Find $g^{-1}(x)$.

4. a) Graph $f(x)=5^x$ by plotting points. Label the axes and all points used.

b) Is the function one-to-one? Why?

c) What is the domain of the function? What is the range ?

d) What kind of asymptote does the graph have? What is the equation of the asymptote?

5. In a laboratory experiment, researchers establish a colony of 100 bacteria and monitor its growth. The experiments discover that the colony doubles in population every three days.

a) Write a function that gives the population of the colony , $P(t)$,at any time t in days.

b) Graph the function. Label the axes and the points used.

c) Find the number of bacteria present after 15 days.

6. John invests \$10,000 for 6 years at an interest rate of 5.5%. Find the amount in the account after 5 years if:

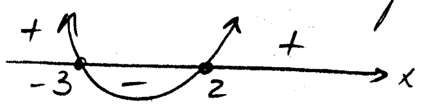
a) the interest is compounded monthly ($A=Pe^{rt}$ $A=P\left(1+\dfrac{r}{n}\right)^{nt}$).

b) the interest is compounded continuously.

QUIZ # 6 - SOLUTIONS

(1) (a) $x^2+x-6>0$

$y=x^2+x-6$ parabola opens upward



x-int: $x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3, \quad x=2$

Therefore, $x^2+x-6>0$ iff $\boxed{x\in(-\infty,-3)\cup(2,\infty)}$

OR

$x^2+x-6>0$

$(x+3)(x-2)>0$

x	$-\infty$	-3		2	∞
$x+3$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	0	$+$
$(x+3)(x-2)$	$+$	0	$-$	0	$+$

$x^2+x-6>0$ iff

$x\in(-\infty,-3)\cup(2,\infty)$

(b) $9x^2+3x-2\le 0$

$y=9x^2+3x-2$ parabola opens up

x-int: $9x^2+3x-2=0$

$$x=\frac{-3\pm\sqrt{9+72}}{18}=\frac{-3\pm 9}{18}$$

$$x=\frac{-12}{18}=-\frac{2}{3}$$

$$x=\frac{+6}{18}=\frac{+1}{3}$$

$9x^2+3x-2\le 0$ iff $\boxed{x\in\left[-\frac{2}{3},+\frac{1}{3}\right]}$

OR

$9x^2+3x-2\le 0$

$(3x+2)(3x-1)\le 0$

x	$-\infty$	$-\frac{2}{3}$		$\frac{1}{3}$	∞
$3x+2$	$-$	0	$+$	$+$	$+$
$3x-1$	$-$	$-$	$-$	0	$+$
$(3x+2)(3x-1)$	$+$	0	$-$	0	$+$

$9x^2+3x-2\le 0$ iff $x\in\left[-\frac{2}{3},\frac{1}{3}\right]$

(c) $\dfrac{2x+1}{5-x}\ge 0$

x	$-\infty$	$-\frac{1}{2}$		5	∞
$2x+1$	$-$	0	$+$	$+$	$+$
$5-x$	$+$	$+$	$+$	0	$-$
$\frac{2x+1}{5-x}$	$-$	0	$+$	$\vert$	$-$

$\dfrac{2x+1}{5-x}\ge 0$ iff $\boxed{x\in\left[-\frac{1}{2},5\right)}$

(d) $\dfrac{x-2}{x+2}\le 2$

$$\frac{x-2}{x+2}-2\le 0$$

$$\frac{x-2-2(x+2)}{x+2}\le 0$$

$$\frac{x-2-2x-4}{x+2}\le 0$$

$$\frac{-x-6}{x+2}\le 0$$

x	$-\infty$	-6		-2	∞
$-x-6$	$+$	0	$-$	$-$	$-$
$x+2$	$-$	$-$	$-$	0	$+$
$\frac{-x-6}{x+2}$	$-$	0	$+$	$\vert$	$-$

$\dfrac{x-2}{x+2}\le 2$ iff $\dfrac{-x-6}{x+2}\le 0$ iff $x\in(-\infty,-6]\cup[-2,\infty)$

(2) (a) y is a function of x because for every input x there is only one output y

(b) $f(1) = 2$
$f(2) = 5$
$f(3) = 1$

(c) $f(x) = y$
f is a one-to-one function because every input has a different output
(no two input values have the same output)

(d) f has an inverse because it is one-to-one.

(e) $f^{-1}(1) = 3$ because $f(3) = 1$
$f^{-1}(2) = 1$ because $f(1) = 2$
$f^{-1}(3) = 5$ because $f(5) = 3$

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(3) $f(x) = 2x+3$
$g(x) = \dfrac{x+1}{1-x}$

(a) $(f \circ g)(x) = f(g(x))$
$= f\left(\dfrac{x+1}{1-x}\right)$

$= 2 \cdot \dfrac{x+1}{1-x} + 3$

$= \dfrac{2(x+1) + 3(1-x)}{1-x}$

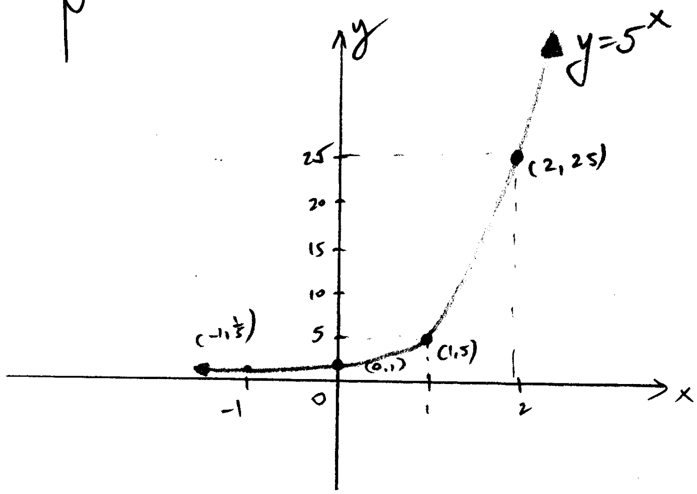
$= \dfrac{2x+2+3-3x}{1-x} = \boxed{\dfrac{5-x}{1-x}}$

(b) $f(x) = 2x+3$
1st $y = 2x+3$
2nd (solve the equation for x)
$2x = y-3$
$x = \dfrac{y-3}{2}$
3rd $x \leftrightarrow y$
$y = \dfrac{x-3}{2}$
$\boxed{f^{-1}(x) = \dfrac{x-3}{2}}$

(c) $g(x) = \dfrac{x+1}{1-x}$
1st $y = \dfrac{x+1}{1-x}$
2nd solve the eq. for x
$y(1-x) = x+1$
$y - yx = x+1$
$y - 1 = x + yx$
$y - 1 = x(1+y)$
$x = \dfrac{y-1}{y+1}$
3rd $x \leftrightarrow y$
$y = \dfrac{x-1}{x+1}$
$\boxed{g^{-1}(x) = \dfrac{x-1}{x+1}}$

(4) (a) $f(x) = 5^x$

x	$-\infty$	-2	-1	0	1	2	∞
$y=5^x$	$y \to 0$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	∞



(b) f is one-to-one because it passes the horizontal line test

(c) Domain: $x \in \mathbb{R}$
Range: $y \in (0, \infty)$

(d) Horizontal asymptote
$y = 0$

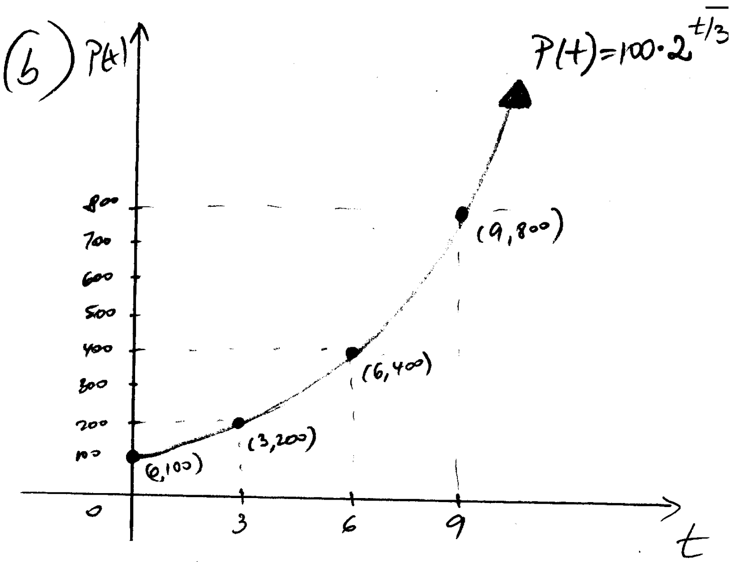
(5) t = # days
$P(t)$ = # bacteria

(a)

t	$P(t)$
0	100
3	$100(2) = 200$
6	$100(2)^2 = 400$
9	$100(2)^3 = 800$
12	$100(2)^4$

in general, $\boxed{P(t) = 100(2)^{\frac{t}{3}}}$

(b)



(c) $P(15) = 100 \cdot 2^{\frac{15}{3}}$
$= 100 \cdot 2^5$
$= 3200$ bacteria

(6) (a) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = 10{,}000$
$r = 0.055$
$n = 12$
$t = 5$

$A = 10{,}000\left(1 + \frac{0.055}{12}\right)^{12 \cdot 5}$

$\boxed{A = 13{,}157 \text{ dollars}}$

(b) $A = Pe^{rt}$

$P = 10{,}000$
$r = 0.055$
$t = 5$

$A = 10{,}000\, e^{0.055(5)}$

$\boxed{A = 13{,}165.3 \text{ dollars}}$