Math 71 – Summer 2007 Name:

QUIZ #4 @ 30 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Solve or simplify, whichever is appropriate:

a) $\dfrac{x^3-27}{4x^2-4x}\cdot\dfrac{4x}{x-3}$

b) $\dfrac{3x^2-7x-6}{3x^2-13x-10}\div\dfrac{2x^2-x-1}{4x^2-18x-10}$

c) $\dfrac{x+6}{x+3}=\dfrac{3}{x+3}+2$

d) $\dfrac{x+2}{x^2-x}-\dfrac{6}{x^2-1}=0$

2. Find all values of a for which $(f+g)(a)=h(a)$ knowing that

$$f(x)=\frac{5}{x-4},\quad g(x)=\frac{3}{x-3},\quad \text{and } h(x)=\frac{x^2-20}{x^2-7x+12}.$$

3. Divide the following using long division for polynomials:

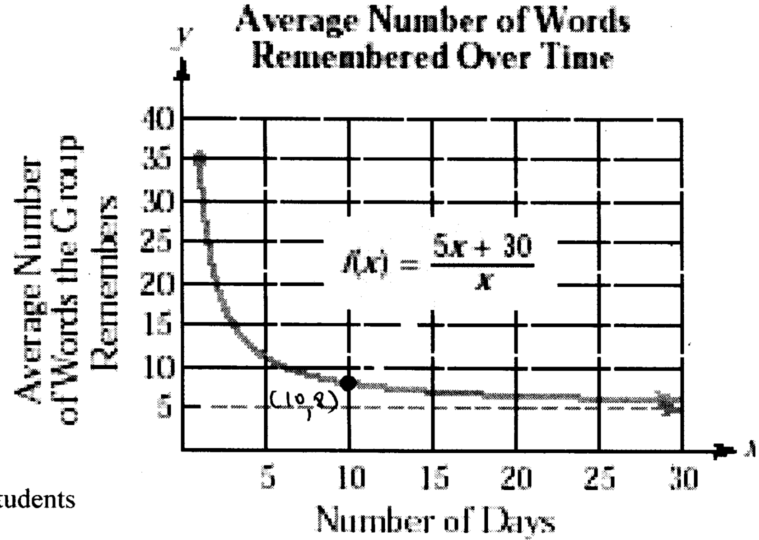
a) $\dfrac{9x^3-3x^2-3x+4}{3x+2}$

b) $\dfrac{15x^4+3x^3+4x^2+4}{3x^2-1}$

4. In an experiment about memory, students in a language class are asked to memorize 40 vocabulary words in Latin. After studying the words for one day, students are tested each day thereafter to see how many words they remember. The class average is found. The function

$$f(x) = \frac{5x+30}{x}$$

models the average number of Latin words remembered by the students, $f(x)$, after x days.
The graph of the rational function is shown.



a) After how many days do the students remember 8 words? Identify your solution as a point of the graph.

b) What is the horizontal asymptote of the graph? Describe what it means about the average number of Latin words remembered by the students over an extended period of time.

c) According to the graph, between which two days do students forget the most? Explain your choice.

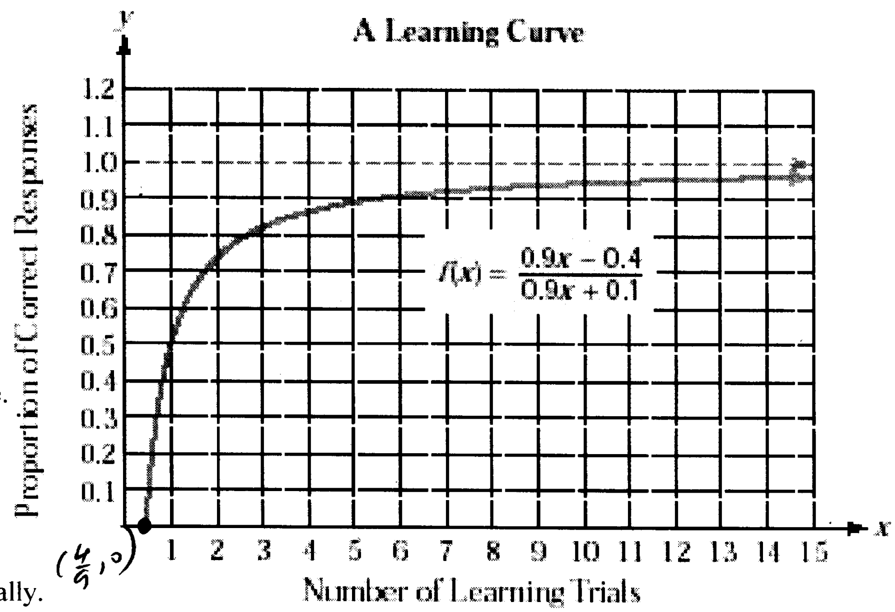
EXTRA CREDIT @ 6 points

Rational functions can be used to model learning. Many of these functions model the proportion of correct responses as a function of the number of trials of a particular task. One such model, called a learning curve, is

$$f(x) = \frac{0.9x-0.4}{0.9x+0.1}$$

where $f(x)$ is the proportion of correct responses after x trials.

The graph of the function is shown.
Answer the following question:



a) Which one is the independent variable and which one is the dependent variable in the given situation? Explain your choice.

b) Using the graph of the function, find the domain and the range.

c) Solve the equation $f(x) = 0$ algebraically.
Then identify the point on the graph. What is the meaning of $f(x) = 0$ in the context of the learning curve?

d) How many learning trials are necessary for 0.05 of the responses to be correct? Identify your solution as a point on the graph.

e) Describe the trend shown by the graph in terms of learning new tasks. What happens initially and what happens as time increases?

f) What is the horizontal asymptote of the graph? Write its equation and its meaning in the given context.

SOLUTIONS

① (a) algebraic expression (we simplify it).

$$\frac{x^3-27}{4x^2-4x}\cdot\frac{4x}{x-3}=$$

$$\frac{x^3-3^3}{4x(x-1)}\cdot\frac{4x}{x-3}=$$

$$\frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{4x}(x-1)}\cdot\frac{\cancel{4x}}{\cancel{x-3}}=$$

$$\boxed{\frac{x^2+3x+9}{x-1}}$$

(b) algebraic expression (we simplify it)

$$\frac{3x^2-7x-6}{3x^2-13x-10}\div\frac{2x^2-x-1}{4x^2-18x-10}=$$

$$=\frac{3x^2-7x-6}{3x^2-13x-10}\cdot\frac{2(2x^2-9x-5)}{2x^2-x-1}=\%$$

- $3x^2-7x-6=3x^2-9x+2x-6$

  $\begin{cases}\text{product}=-18 \lessdot \begin{matrix}+2\\-9\end{matrix}\\ \text{sum}=-7\end{cases}$

  $=3x(x-3)+2(x-3)$

  $=(x-3)(3x+2)$

- $3x^2-13x-10=3x^2-15x+2x-10$

  $\begin{cases}\text{product}=-30 \lessdot \begin{matrix}+2\\-15\end{matrix}\\ \text{sum}=-13\end{cases}$

  $=3x(x-5)+2(x-5)$

  $=(x-5)(3x+2)$

$$\% = \frac{(x-3)\cancel{(3x+2)}}{\cancel{(x-5)}\cancel{(3x+2)}}\cdot\frac{2\cancel{(2x+1)}\cancel{(x-5)}}{\cancel{(2x+1)}(x-1)}$$

$$=\boxed{\frac{2(x-3)}{x-1}}$$

(c) equation (we solve it)

$$\frac{x+6}{x+3}=\frac{3}{x+3}+2$$

Condition: $x+3\neq 0$

$x\neq -3$

$$\frac{x+6}{x+3}-\frac{3}{x+3}=2$$

$$\frac{x+6-3}{x+3}=2$$

$$\frac{x+3}{x+3}=2$$

$1=2$ Contradiction

No solutions $\boxed{x\in\emptyset}$

(d) equation (we solve it)

$$\frac{x+2}{x^2-x}-\frac{6}{x^2-1}=0$$

$$\frac{x+2}{x\cancel{(x-1)}}=\frac{6}{(x+1)\cancel{(x-1)}}\quad \Big|\cdot\underset{\neq 0}{(x-1)}$$

Conditions $\begin{cases}x\neq 0\\ x\neq 1\\ x\neq -1\end{cases}$

$$\frac{x+2}{x}=\frac{6}{x+1}$$

cross-product property

$(x+2)(x+1) = 6x$

$x^2+3x+2 = 6x$

$x^2-3x+2 = 0$

$(x-1)(x-2) = 0$

$x-1=0$ OR $x-2\neq 0$

$\cancel{x=1}$ OR $x = 2$

not possible

$\boxed{x \in \{2\}}$

(2) $f(x) = \dfrac{5}{x-4}$, $g(x) = \dfrac{3}{x-3}$

$h(x) = \dfrac{x^2-20}{x^2-7x+12}$

$(f+g)(a) = h(a)$

$f(a)+g(a) = h(a)$

$$\frac{5}{a-4} + \frac{3}{a-3} = \frac{a^2-20}{a^2-7a+12}$$

$$\overset{a-3}{\frac{5}{a-4}} + \overset{a-4}{\frac{3}{a-3}} = \frac{a^2-20}{(a-4)(a-3)}$$

Conditions: $\begin{cases} a-4\neq 0 \\ a-3\neq 0 \end{cases}$ $\quad a\neq 4$, $a\neq 3$

$LCD = (a-4)(a-3)$

$5(a-3)+3(a-4) = a^2-20$

$5a-15+3a-12 = a^2-20$

$8a-27 = a^2-20$

$a^2-8a-20+27=0$

$a^2-8a+7=0$

$(a-1)(a-7) = 0$

$a=1$ OR $a=7$

$\boxed{a \in \{1,7\}}$

(3)

(a) $\dfrac{9x^3-3x^2-3x+4}{3x+2} = \boxed{3x^2-3x+1+\dfrac{2}{3x+}}$

$$\begin{array}{r|l} & 3x^2-3x+1 \\ 3x+2 & 9x^3-3x^2-3x+4 \\ & -9x^3-6x^2 \\ \hline & -9x^2-3x+4 \\ & +9x^2+6x \\ \hline & 3x+4 \\ & -3x-2 \\ \hline & 2 \;\; R \end{array}$$

(b) $\dfrac{15x^4+3x^3+4x^2+4}{3x^2-1} = \boxed{5x^2+x+3+\dfrac{x+}{3x^2-}}$

$$\begin{array}{r|l} & 5x^2+x+3 \\ 3x^2-1 & 15x^4+3x^3+4x^2+0x+4 \\ & -15x^4+\quad 5x^2 \\ \hline & 3x^3+9x^2+0x+4 \\ & -3x^3 \quad +x \\ \hline & 9x^2+x+4 \\ & -9x^2 \quad +3 \\ \hline & x+7 \end{array}$$

(4) $x = \#$ days

$f(x) = \#$ words remembered

$$f(x) = \frac{5x+30}{x}$$

(a) $x = ?$ if $f(x) = 8$

$$\frac{5x+30}{x} = 8$$

$5x + 30 = 8x$

$30 = 3x \Rightarrow \boxed{x = 10 \text{ days}}$

They remember 8 words after 10 days.

The point on the graph is $\boxed{(10, 8)}$

(b) $\boxed{y = 5}$ horizontal asymptote

On average, the students remembered 5 words over an extended period of time.

(c) Between the first and second days because that's where the graph decreases most rapidly.

EXTRA CREDIT

$$f(x) = \frac{0.9x - 0.4}{0.9x + 0.1} \quad (1)$$

$x = \#$ trials

$f(x) =$ proportion of correct responses

a) $x =$ independent variable (# of trials).

- It is the input in equation (1)
- It is on the horizontal axis of the ~~sys~~ rectangular coordinate system
- the function "models the proportion of correct responses as a function of the number of trials"

$f(x) =$ dependent variable (proportion of correct responses)

- On the vertical axis
- the function "models the proportion of correct responses as a function of # trials"
- it is the output in eq. (1)

b) $x \geq 0.5$ (approximately)

~~$y \geq$~~ $0 \leq y < 1$

c) $f(x) = 0$

$$\frac{0.9x - 0.4}{0.9x + 0.1} = 0 \quad \text{iff}$$

$0.9x - 0.4 = 0$

$0.9x = 0.4$

$$x = \frac{0.4}{0.9} = \frac{4}{9}$$

$f(x) = 0$ iff $x = \frac{4}{9}$

The point on the graph is $(\frac{4}{9}, 0)$, the x-intercept

If $f(x) = 0$, there are no correct responses

d) $x = ?$ if $f(x) = 0.5$

$$\frac{0.9x - 0.4}{0.9x + 0.1} = 0.5$$

$0.9x - 0.4 = 0.5(0.9x + 0.1)$

$0.9x - 0.4 = 0.45x + 0.05$

$0.9x - 0.45x = 0.05 + 0.4$

$0.45x = 0.45 \Rightarrow x = 1$ learning trial

The point on the graph is $(1, 0.5)$

One learning trial is necessary for 0.5 of the responses to be correct.

e) As the number of learning trials increases, the proportion of correct responses increases; initially the proportion of correct responses increases rapidly, then slows down as time increases.

f) $y = 1$ Additional practice has little effect when performance is near peak efficiency.

The graph is leveling off.