Math 71 – Summer 2007 Name:

QUIZ #3 @ 30 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. If $f(x)=2x^2-x+1$, find (and simplify): $f(a+h)-f(a)$.

2. Factor each polynomial completely:

a) $15a^3-25a^2+10a$

b) $6x^2+19x+15$

c) t^9+1

d) $x^2-8xy+64y^2$

e) $2x^{n+2}-5x^{n+1}+3x^n$

3. Factor by introducing an appropriate substitution:

a) $2x^6+11x^3+15$

b) $9x^{2n}+x^n-8$

4. Solve each equation by factoring:

a) $2x^2-8x=90$

b) $(x-3)(x+8)=-30$

c) $3x^4-48x^2=0$

d) $6(5x+2)\left(2x-\frac{31}{5}\right)(x^2-8x)=0$

5. The function $f(x)=-\frac{1}{4}x^2+3x+17$ models the number of people, f(x), in millions, receiving food stamps x years after 1990.

a) In which year did 25 million people receive food stamps?

b) How many people received food stamps in 1996?

6. James Bond stands on top of a 240-foot building and throws a film canister upward to a fellow agent in a helicopter 16 feet above the building. The height of the film above the ground t seconds later is given by the formula $h(t)=-16t^2+32t+240$ where h is in feet.

a) Calculate $h(0)$. What is its meaning in this context?

b) How long will it take the film canister to reach the agent in the helicopter?

c) How long will it take to hit the ground?

SOLUTIONS

(1) $f(x) = 2x^2 - x + 1$

$f(a+h) - f(a) =$

$= \left(2(a+h)^2 - (a+h) + 1\right) - \left(2a^2 - a + 1\right)$

$= 2(a^2 + 2ah + h^2) - a - h + 1 - 2a^2 + a - 1$

$= 2a^2 + 4ah + 2h^2 - h - 2a^2$

$= \boxed{4ah + 2h^2 - h}$

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(2) (a) $15a^3 - 25a^2 + 10a =$

$= 5a(3a^2 - 5a + 2)$

$= \boxed{5a(3a-2)(a-1)}$

(b) $6x^2 + 19x + 15 = \%$

product $= ac = 6(15) = 90 \begin{cases} +9 \\ +10 \end{cases}$

sum $= b = 19$

$90 = 9 \cdot 10$

$\% = 6x^2 + 9x + 10x + 15$

$= 3x(2x+3) + 5(2x+3)$

$= \boxed{(2x+3)(3x+5)}$

(c) $t^9 + 1 = (t^3)^3 + 1$

$= (t^3+1)((t^3)^2 - t^3 + 1^2)$

$\neq \boxed{(t+1)(t^2-t+1)(t^6-t^3+1)}$

(d) $x^2 - 8xy + 64y^2 = \boxed{\text{prime}}$

$= (x \quad y)(x \quad y)$

product $= c = 64 \begin{cases} - \\ - \end{cases}$

sum $= b = -8$

$64 = 8 \cdot 8$, $4 \cdot 12$, $2 \cdot 32$

not factorable

Also, $b^2 - 4ac < 0$ (not perfect square)

(e) $2x^{n+2} - 5x^{n+1} + 3x^n =$

$= x^n(2x^2 - 5x + 3)$

$= \boxed{x^n(2x-3)(x-1)}$

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(3) (a) $2x^6 + 11x^3 + 15$

let $x^3 = t$

then $(x^3)^2 = t^2$, $x^6 = t^2$

$2x^6 + 11x^3 + 15 = 2t^2 + 11t + 15$

product $= ac = 30 \begin{cases} +6 \\ +5 \end{cases}$

sum $= b = 11$

$30 = 6 \cdot 5$

$= 2t^2 + 6t + 5t + 15$

$= 2t(t+3) + 5(t+3)$

$= (t+3)(2t+5)$

$\neq \boxed{(x^3+3)(2x^3+5)}$

(b) $9x^{2n} + x^n - 8$

let $x^n = t$

then $(x^n)^2 = t^2$, $x^{2n} = t^2$

$9x^{2n} + x^n - 8 = 9t^2 + t - 8$

product $= ac = 9(-8) \begin{cases} +9 \\ -8 \end{cases}$

sum $= b = 1$

$= 9t^2 + 9t - 8t - 8$

$= 9t(t+1) - 8(t+1)$

$= (t+1)(9t-8)$

$\neq \boxed{(x^n+1)(9x^n-8)}$

(4) (a) $2x^2 - 8x = 90$

$2x^2 - 8x - 90 = 0$ | $\div 2$

$x^2 - 4x - 45 = 0$

product $= c = -45$ $<$ -9, $+5$

sum $= b = -4$

$45 = 9 \cdot 5$

$(x+5)(x-9) = 0 \Rightarrow$

$x+5=0$ OR $x-9=0$

$x = -5$ $\quad x = 9$

$\boxed{x \in \{-5, 9\}}$

(b) $(x-3)(x+8) = -30$

$x^2 + 8x - 3x - 24 = -30$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x+2=0$ OR $x+3=0$

$x = -2$ $\quad x = -3$

$\boxed{x \in \{-2, -3\}}$

(c) $3x^4 - 48x^2 = 0$

$3x^2(x^2 - 16) = 0$

$3x^2(x-4)(x+4) = 0$

$x^2 = 0$ OR $x-4=0$ OR $x+4=0$

$x = 0$ $\quad x = 4$ $\quad x = -4$

$\boxed{x \in \{0, 4, -4\}}$

(d) $6(5x+2)\left(2x - \frac{31}{5}\right)(x^2 - 8x) = 0$

$6(5x+2)\left(2x - \frac{31}{5}\right) \times (x-8) = 0$

$5x+2=0 \Rightarrow x = -\frac{2}{5}$

OR

$2x - \frac{31}{5} = 0 \Rightarrow 2x = \frac{31}{5}, \; x = \frac{31}{10}$

OR

$x = 0$

OR

$x - 8 = 0 \Rightarrow x = 8$

$\boxed{x \in \left\{-\frac{2}{5}, \frac{31}{10}, 0, 8\right\}}$

(5) $f(x) = -\frac{1}{4}x^2 + 3x + 17$

x = number of years after 1990

$f(x)$ = number of people (in millions)

(a) $x = ?$ if $f(x) = 25$

$25 = -\frac{1}{4}x^2 + 3x + 17 = 0$

$\frac{1}{4}x^2 - 3x - 17 + 25 = 0$

$\frac{1}{4}x^2 - 3x + 8 = 0$ | $\cdot 4$

$x^2 - 12x + 32 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{12 \pm \sqrt{144 - 4(1)(32)}}{2(1)}$$

$$= \frac{12 \pm \sqrt{16}}{2} = \frac{12 \pm 4}{2}$$

$x = 8$ OR $x = 4$

in 1994 and 1998, 25 million people received food stamps

OR $x^2 - 12x + 32 = 0$

$(x-8)(x-4) = 0$

$x = 8$ OR $x = 4$

(b) $f(x) = ?$ if $x = 6$

$f(6) = \frac{-1}{4}(6)^2 + 3(6) + 17$

$= \frac{-36}{4} + 18 + 17$

$= -9 + 18 + 17 =$ 26 million

In 1996, 26 million people received food stamps.

(c) $h(t) = -16t^2 + 32t + 240$

t = time (in seconds)

$h(t)$ = height (in feet)

(a) $h(0) = 240$ ft

It represents the initial height; the film canister was initially on the building (240 ft high)

(b) $t = ?$ if $h = 240 + 16$

$h = 256$

$256 = -16t^2 + 32t + 240$

$16t^2 - 32t - 240 + 256 = 0$

$16t^2 - 32t + 16 = 0$ $/\div 16$

$t^2 - 2t + 1 = 0$

$(t-1)^2 = 0 \Rightarrow t = 1$ second

It will take the film canister 1 second to reach the agent in the helicopter

(c) $t = ?$ if $h = 0$

$-16t^2 + 32t + 240 = 0$

$16t^2 - 32t - 240 = 0$ $/\div 16$

$t^2 - 2t - 15 = 0$

$(t-5)(t+3) = 0$

$t = 5$ seconds or $t = -3$

It will take the canister 5 seconds to hit the ground.