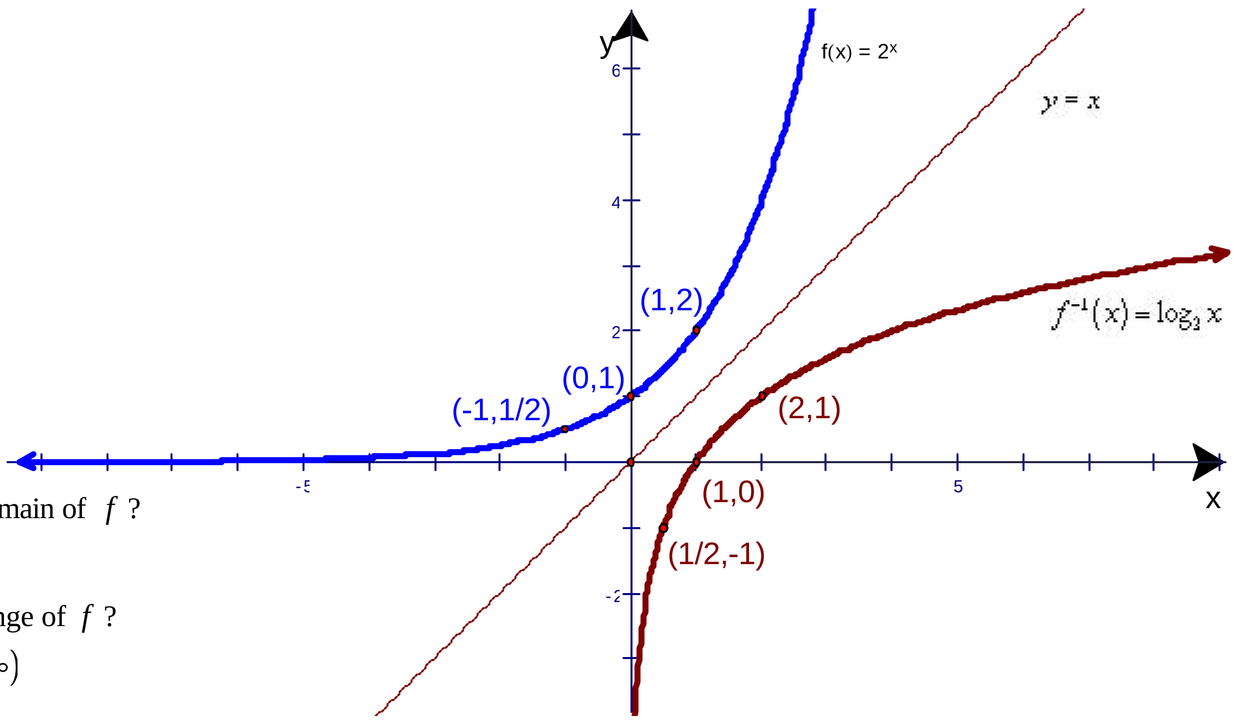
Math 71B Summer 2006 Name:

TEST 3 @ 140 points

Write in a neat and organized fashion. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. **No proof, no credit given!**

1. Let $f(x) = 2^x$. Answer the following questions:

a) Graph the function. Label the axes and all the points used.



b) What is the domain of f ?

$x \in \mathbb{R}$

c) What is the range of f ?

$y \in (0, \infty)$

d) What kind of asymptote does the graph have?
What is the equation of the asymptote?

The graph has a horizontal asymptote . Its equation is $y = 0$ (the x-axis).

e) What are the x- and y- intercepts (if any)?

no x- intercept; y-intercept: $(0,1)$

f) What is $f^{-1}(x)$?

$f^{-1}(x) = \log_2 x$

g) Explain how you can obtain the graph of f^{-1} from the graph of f . Graph f^{-1} on the same coordinate system as f.

The graph of $f^{-1}(x) = \log_2 x$ is obtained by reflecting the graph of $f(x) = 2^x$ about the line $y = x$.

h) What is the domain of $f^{-1}(x)$?

$x \in (0, \infty)$

i) What is the range of $f^{-1}(x)$?

$y \in \mathbb{R}$

2. Let $f(x) = 2x - 3$ and $g(x) = \dfrac{x+3}{2}$. Answer the following questions:

a) Find $(g \circ f)(x)$.

$$(g \circ f)(x) = g(f(x))$$
$$= g(2x-3)$$
$$= \frac{(2x-3)+3}{2} = \frac{2x}{2}$$
$$= x$$

b) $(f \circ g)(2)$

$$(f \circ g)(2) = f(g(2))$$
$$= f\left(\frac{5}{2}\right)$$
$$= 2\left(\frac{5}{2}\right) - 3$$
$$= 2$$

c) Find $f^{-1}(x)$.

Step1: $y = 2x - 3$

Step 2: Solve the equation for x

$$y + 3 = 2x$$
$$x = \frac{y+3}{2}$$

Step 3: $y = \dfrac{x+3}{2}$

$$f^{-1}(x) = \frac{x+3}{2}$$

d) Find $g^{-1}(x)$.

Step 1: $y = \dfrac{x+3}{2}$

Step 2: Solve the equation for x

$$2y = x + 3$$
$$x = 2y - 3$$

Step 3: $y = 2x - 3$

$$g^{-1}(x) = 2x - 3$$

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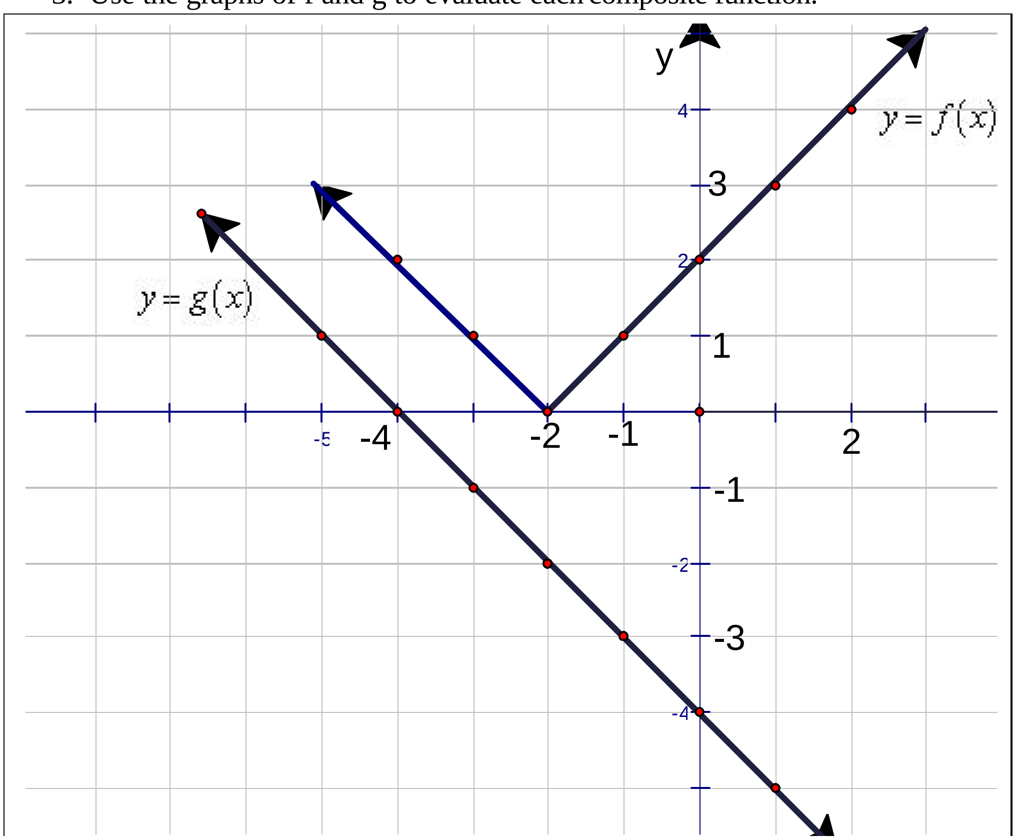
3. Use the graphs of f and g to evaluate each composite function.



a) $(f \circ g)(-1)$

$$(f \circ g)(-1) = f(g(-1))$$
$$= f(-3)$$
$$= 1$$

b) $(g \circ f)(0)$

$$(g \circ f)(0) = g(f(0))$$
$$= g(2)$$
$$= -6$$

4. Find the domain of each logarithmic function.

a) $f(x) = \log_5(3-4x)$

Condition: $3-4x > 0$

$3 > 4x$

$x < \frac{3}{4}$

Therefore $x \in \left(-\infty, \frac{3}{4}\right)$.

b) $g(x) = \log(x-1)^2$

Condition: $(x-1)^2 > 0$

$x-1 \neq 0$

$x \neq 1$

Therefore $x \in (-\infty, 1) \cup (1, \infty)$.

5. Simplify the following expressions.

a) $4\ln x + 7\ln y - 3\ln z =$

$= \ln x^4 + \ln y^7 - \ln z^3$

$= \ln\left(x^4 y^7\right) - \ln z^3$

$= \ln \frac{x^4 y^7}{z^3}$

b) $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1) =$

$= \frac{1}{2}\log_5 x + \frac{1}{2}\log_5 y - \log_5(x+1)^2$

$= \log_5 \sqrt{x} + \log_5 \sqrt{y} - \log_5(x+1)^2$

$= \log_5 \frac{\sqrt{xy}}{(x+1)^2}$

c) $\log_3 405 - \log_3 5 + \log 5 + \log 2 =$

$= \log_3 \frac{405}{5} + \log(5 \cdot 2)$

$= \log_3 81 + \log 10$

$= 4 + 1 = 5$

d) $\log_4(\log_2 16) =$

$= \log_4 4$

$= 1$

6. Solve the following equations.

a) $\log_3(x-1) = 2$

Condition: $x-1 > 0$, so $x > 1$

$3^2 = x - 1$

$x = 10$

b) $\log_5 x = 4\log_5 2 - \log_5 8$

Condition: $x > 0$

$\log_5 x = \log_5 2^4 - \log_5 8$

$\log_5 x = \log_5 \frac{16}{8}$

$\log_5 x = \log_5 2$

$x = 2$

c) $3^{1-x}=\dfrac{1}{27}$

$$3^{1-x}=\frac{1}{3^3}$$
$$3^{1-x}=3^{-3}$$
$$1-x=-3$$
$$x=4$$

d) $\log_3(x-5)+\log_3(x+3)=2$

Conditions: $x-5>0$ and $x+3>0$

Therefore, $x>5$

$$\log_3(x-5)(x+3)=2$$
$$3^2=(x-5)(x+3)$$
$$9=x^2-2x-15$$
$$x^2-2x-24=0$$
$$(x-6)(x+4)=0$$

$x=6$ or ~~$x=-4$~~

e) $e^{1-2x}=20$

$$\ln\left(e^{1-2x}\right)=\ln 20$$
$$1-2x=\ln 20$$
$$2x=1-\ln 20$$
$$x=\frac{1-\ln 20}{2}$$

f) $5^x=17$

$$\log_5 17=x$$
$$x=\log_5 17$$

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7. Using the formulas for compound interest $A=P\left(1+\dfrac{r}{n}\right)^{nt}$ or $A=Pe^{rt}$, solve the following problem.

Find the accumulated value of an investment of $10,000 for 5 years at an interest rate of 5.25% if the money is:

a) compounded monthly

$$A=P\left(1+\frac{r}{n}\right)^{nt}$$

$P=10,000$

$r=0.0525$, $n=12$, $t=5$

$$A=10,000\left(1+\frac{0.0525}{12}\right)^{12\cdot 5}$$

$A=\$12994.32$

b) compounded continuously.

$A=Pe^{rt}$

$P=10,000$

$r=0.0525$, $t=5$

$$A=10,000e^{0.05255}$$

$A=10,000e^{0.05255}$, so $A=\$13001.77$.

c) If $10,000 are deposited in an investment account that pays an interest rate of 5% compounded continuously, how many years is it going to take for the accumulated value to be $15,000?

$$A=Pe^{rt}$$
$$15,000=10,000e^{0.05t}$$
$$1.5=e^{0.05t}$$
$$\ln(1.5)=0.05t$$
$$t=\frac{\ln(1.5)}{0.05}\approx 8.1 \text{ years}$$

Therefore, it will take 8.1 years for the 10,000 to grow to15,000 at a rate of 5% compounded continuously.