SOLUTIONS

Math 71B Summer 2006

Name:

TEST 2 @ 140 points

Write in a neat and organized fashion. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. **No proof, no credit given!**

1. Solve (in $\mathbb{C}$) by extracting roots:

$$4(x-1)^2+32=0$$

$$4(x-1)^2=-32$$

$$(x-1)^2=-8$$

$$\sqrt{(x-1)^2}=\sqrt{-8}$$

$$x-1=\pm 2\sqrt{2}\,i$$

$$\boxed{x=1\pm 2\sqrt{2}\,i}$$

2. Solve (in $\mathbb{C}$) by completing the square:

$$9x^2-6x+5=0$$

$$9x^2-6x=-5$$

$$x^2-\frac{6}{9}x=\frac{-5}{9}$$

$$x^2-\frac{2}{3}x=\frac{-5}{9}$$

$$\left(\frac{1}{2}\text{coef } x\right)^2=\left(\frac{1}{2}\cdot\frac{2}{3}\right)^2=\frac{1}{9}$$

$$x^2-\frac{2}{3}x+\frac{1}{9}=\frac{-5}{9}+\frac{1}{9}$$

$$\left(x-\frac{1}{3}\right)^2=\frac{-4}{9}$$

$$\sqrt{\left(x-\frac{1}{3}\right)^2}=\sqrt{-\frac{4}{9}}$$

$$x-\frac{1}{3}=\pm\frac{2}{3}i$$

$$\boxed{x=\frac{1}{3}\pm\frac{2}{3}i}$$

3. Solve (in $\mathbb{C}$) by the quadratic formula:

a) $2x^2-4x=-1$

$$2x^2-4x+1=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\qquad\begin{cases}a=2\\b=-4\\c=1\end{cases}$$

$$x_{1,2}=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(1)}}{2(2)}$$

$$=\frac{4\pm\sqrt{16-8}}{4}=\frac{4\pm\sqrt{8}}{4}$$

$$=\frac{4\pm2\sqrt{2}}{4}=\frac{2(2\pm\sqrt{2})}{4_2}$$

$$\boxed{x_{1,2}=\frac{2\pm\sqrt{2}}{2}}$$

b) $\overset{6}{\frac{x^2}{3}}+\overset{2}{\frac{x}{9}}-\overset{3}{\frac{1}{6}}=0$ LCD = 18

$$6x^2+2x-3=0\qquad\begin{cases}a=6\\b=2\\c=-3\end{cases}$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\pm\sqrt{4-4(6)(-3)}}{2(6)}$$

$$=\frac{-2\pm\sqrt{76}}{12}=\frac{-2\pm2\sqrt{19}}{12}$$

$$=\frac{2(-1\pm\sqrt{19})}{12}=\boxed{\frac{-1\pm\sqrt{19}}{6}=x_{1,2}}$$

4. Solve the following equations:

a) $x^{\frac{2}{3}} - 3 = 2x^{\frac{1}{3}}$

$x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 3 = 0$

Let $x^{1/3} = t$

Then $x^{2/3} = t^2$

$t^2 - 2t - 3 = 0$

$(t-3)(t+1) = 0 <$ $t = 3$ OR $t = -1$

if $t=3$

$x^{\frac{1}{3}} = 3 \;\Big|^{3}$

$x = 3^3$

$\boxed{x=27}$

if $t=-1$

$x^{1/3} = -1 \;\Big|^{3}$

$x = (-1)^3$

$\boxed{x=-1}$

Sol. set $\{27, -1\}$

b) $x^4 - 9x^2 + 20 = 0$

Let $x^2 = t$

Then $x^4 = t^2$

$t^2 - 9t + 20 = 0$

$(t-5)(t-4) = 0 <$ $t=5$ OR $t=4$

if $t=5$

$x^2 = 5$

$\sqrt{x^2} = \sqrt{5}$

$\boxed{x = \pm\sqrt{5}}$

if $t=4$

$x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

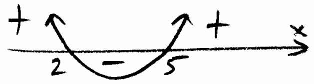
$\boxed{x = \pm 2}$

Solution set: $\{\pm\sqrt{5}, \pm 2\}$

5. Solve the following inequalities.

a) $x^2 - 7x + 10 \le 0$

$y = x^2 - 7x + 10$ parabola opens upward



x-n: $x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x = 5$, $x = 2$

Therefore, $x^2 - 7x + 10 \le 0$

if $\boxed{x \in [2, 5]}$

b) $\frac{x}{x+2} \le 2$

$\frac{x}{x+2} - 2 \le 0$

$\frac{x - 2(x+2)}{x+2} \le 0$

$\frac{x - 2x - 4}{x+2} \le 0$

$\frac{-x-4}{x+2} \le 0$

x	-∞	-4		-2	∞
-x-4	+ + +	0	−	−	− −
x+2	− −	−	−	0	+ +
$\frac{-x-4}{x+2}$	−	0	+	|	−

Therefore, $\frac{x}{x+2} \le 2 \Longleftrightarrow \frac{-x-4}{x+2} \le 0$

$\Longleftrightarrow \boxed{x \in (-\infty, -4] \cup (-2, \infty)}$

6. For the equation given below, answer all questions and graph. Show all work.

$$y = x^2 - 2x - 15$$

a) What type of curve is this? porabola opens upward $(a > 0)$

b) Find the coordinates of the vertex.

$x_v = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$ $\boxed{V(1,-16)}$

$y_v = 1^2 - 2(1) - 15 = -16$

c) Find the y-intercept.

$x = 0, \; y = -15$ $\boxed{0,-15}$

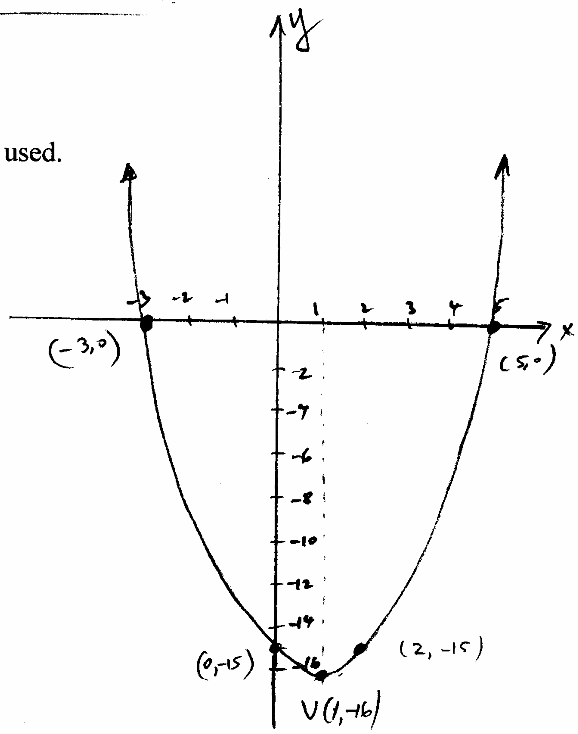
d) Find the x-intercepts (if any).

$y = 0, \; x^2 - 2x - 15 = 0$ $\boxed{(5,0) \text{ and } (-3,0)}$

$(x-5)(x+3) = 0$

$x = 5$ OR $x = -3$

e) Sketch the graph. Label the axes and all points used.



f) What is the domain of the function?

$\boxed{x \in \mathbb{R}}$

g) What is the range of the function?

$\boxed{y \geq -16}$

h) What is the vertex form of the equation?

$\boxed{y = (x-1)^2 - 16}$

i) Using just the graph above, solve the following :

i) $x^2 - 2x - 15 > 0$ $\iff$ $\boxed{x \in (-\infty,-3) \cup (5,\infty)}$

ii) $x^2 - 2x - 15 \leq 0$ $\iff$ $\boxed{x \in [-3, 5]}$

7. For the equation given below, answer all questions and graph. Show all work.

$$y=-2(x+1)^2+5$$

a) What type of curve is this?

parabola opens downward $(a<0)$

b) Find the coordinates of the vertex.

$\boxed{V(-1,5)}$ The equation is given in vertex form.

c) Find the y-intercept.

$x=0,\ y=-2(1)^2+5$ $\boxed{(0,3)}$

$y=3$

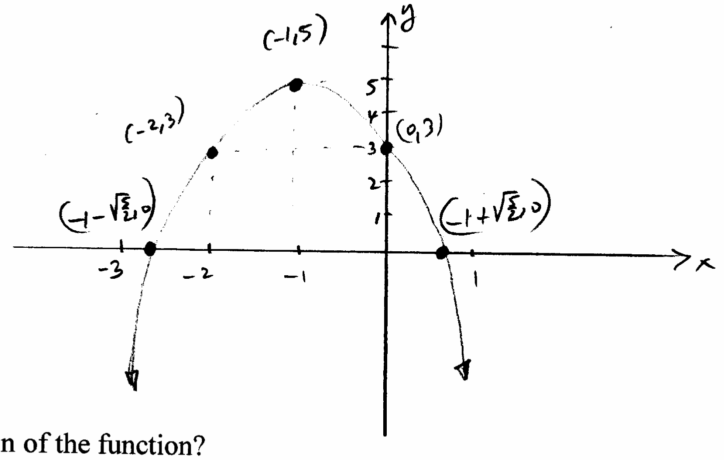
d) Find the x-intercepts (if any).

$y=0,\ -2(x+1)^2+5=0 \rightarrow \sqrt{(x+1)^2}=\sqrt{\frac{5}{2}}$ $x_1 \approx 0.6$

$2(x+1)^2=5$ $x+1=\pm\sqrt{\frac{5}{2}}$

$(x+1)^2=\frac{5}{2}$ $\boxed{x=-1\pm\sqrt{\frac{5}{2}}}$ $x_2 \approx -2.6$

e) Sketch the graph. Label the axes and all points used.



f) What is the domain of the function?

$x \in \mathbb{R}$

g) What is the range of the function?

$y \le 5$

h) What is the ~~vertex~~ form of the equation?
standard

$y=-2(x+1)^2+5$

$y=-2(x^2+2x+1)+5$

$\boxed{y=-2x^2-4x+3}$

i) Using just the graph above, solve the following:

i) $-2(x+1)^2+5\ge 0$ $\Leftrightarrow$ $\boxed{x\in\left[-1-\sqrt{\frac{5}{2}},\ -1+\sqrt{\frac{5}{2}}\right]}$

ii) $-2(x+1)^2+5<0$ $\Leftrightarrow$ $\boxed{x\in\left(-\infty,\ -1-\sqrt{5/2}\right)\cup\left(-1+\sqrt{5/2},\ \infty\right)}$

Choose TWO of the following problems.

1. When Maria serves in volleyball, the ball leaves her hand with an upward velocity of 20 feet per second. The height "h" of the volleyball above the ground after "t" seconds is given by: $h = -16t^2 + 20t + 5$.

a) If nobody hits the ball, how long will it take the ball to hit the ground?

b) If nobody hits the ball, how long will it take the ball to reach its initial height again?

2. You have 600 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

3. The owners of a small fruit orchard decide to produce gift baskets as a sideline. The cost per basket for producing x baskets is $C = 0.01x^2 - 2x + 120$.
How many baskets should they produce in order to minimize the cost per basket? What will their total cost be at that production level?

① $h = -16t^2 + 20t + 5$

t = time

h = height of the ball

a) t = ? when h = 0

$-16t^2 + 20t + 5 = 0$

$16t^2 - 20t - 5 = 0$

$t_{1,2} = \frac{20 \pm \sqrt{20^2 - 4(16)(-5)}}{2(16)}$

$= \frac{20 \pm \sqrt{400 + 320}}{32}$

$t_1 = \frac{20 + \sqrt{720}}{32} \approx 1.46$ sec.

$t_2 = \frac{20 - \sqrt{80}}{32} < 0$

not possible

Therefore, it takes the ball about 1.46 seconds until it hits the ground

b) t = ? h = initial height

First, find the initial height (when t = 0)

t = 0, h = 5 = initial height

t = ? h = 5

$5 = -16t^2 + 20t + 5$

$16t^2 - 20t = 0$

$4t(4t - 5) = 0$

t = 0 (initial moment)

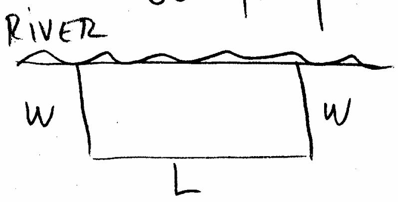
OR

$t = 5/4$

So, the ball reaches 5 ft again at $\frac{5}{4}$ seconds

② Given

600 ft fence



A = area

Let w = width

L = length

w = ? , l = ? for maximum area

$$\begin{cases} 2w + L = 600 \Rightarrow L = 600 - 2w \\ A = w \cdot L \end{cases}$$

$$A = w(600 - 2w)$$

$$A = -2w^2 + 600w$$

parabola opens downward => maximum occurs at the vertex

$$V(w_v, A_v)$$

$$w_v = \frac{-b}{2a} = \frac{-600}{2(-2)} = 150$$

if $w = 150$, then $L = 600 - 2w$

$$L = 600 - 2(150)$$

$$L = 300$$

Therefore, w = 150 ft and L = 300 ft maximize the area.

The maximum area is (150 ft)(300 ft) =

= 45,000 ft^2

③ $C = 0.01x^2 - 2x + 120$

x = the number of gift baskets produced
C = cost per basket

$C = 0.01x^2 - 2x + 120$
parabola opens upward =>
the minimum occurs at the vertex

$V(x_v, C_v)$

$$x_v = \frac{-b}{2a} = \frac{-(-2)}{2(0.01)} = \frac{1}{0.01} = 100$$

So, they should produce 100 baskets in order to minimize the cost per basket

Total cost = (# baskets)(cost per basket)

$$= 100 \cdot C_v$$
$$= 100\left(0.01(100)^2 - 2(100) + 120\right)$$
$$= 2000 \$$$