Math 71B Summer 2006 Name:

# TEST 1 @ 140 points

Write in a neat and organized fashion. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. **No proof, no credit given!**

1. Let $f(x)=\sqrt{x-3}$

a) Find the domain of this function.

Condition: $x-3 \geq 0 \Rightarrow \boxed{x \geq 3}$

$\boxed{\text{Domain} = [3, \infty)}$

b) Find $f(0), f(5), f(4)$. If the function value is not a real number, so state.

$f(0) = \sqrt{0-3} = \sqrt{-3} \notin \mathbb{R}$

$f(5) = \sqrt{5-3} = \sqrt{2}$

$f(4) = \sqrt{4-3} = \sqrt{1} = 1$

2. Find the domain of $f(x)=\dfrac{\sqrt{x-2}}{\sqrt{4-x}}$

Conditions:

$$\begin{cases} (1)\ x-2 \geq 0 \\ \text{and} \\ (2)\ 4-x > 0 \end{cases} \iff \begin{cases} x \geq 2 \\ \text{and} \\ 4 > x \end{cases} \iff \boxed{x \in [2,4)}$$

$\boxed{\text{Domain} = [2,4)}$

3. Let $g(x)=x^2-2x-1$. Find $g\left(1-\sqrt{3}\right)$.

$g(1-\sqrt{3}) = (1-\sqrt{3})^2 - 2(1-\sqrt{3}) - 1$

$= 1 - \cancel{2\sqrt{3}} + (\sqrt{3})^2 - 2 + \cancel{2\sqrt{3}} - 1$

$= 1 + 3 - 3$

$= 1$

$\boxed{g(1-\sqrt{3}) = 1}$

4. Simplify the following expressions:

a) $\sqrt{(x-1)^2} = \boxed{|x-1|}$

b) $\left(x^{\frac{1}{2}}-3\right)\left(x^{\frac{1}{2}}+1\right) =$

$= \left(x^{\frac{1}{2}}\right)^2 + x^{\frac{1}{2}} - 3x^{\frac{1}{2}} - 3$

$= \boxed{x - 2x^{\frac{1}{2}} - 3}$

c) $\sqrt[3]{48(x-2)^3} = \sqrt[3]{2^4 \cdot 3(x-2)^3}$

$= \sqrt[3]{2^3 \cdot 2 \cdot 3(x-2)^3}$

$= \sqrt[3]{2^3}\ \sqrt[3]{(x-2)^3}\ \sqrt[3]{6}$

$= \boxed{2(x-2)\sqrt[3]{6}}$

d) $2\sqrt{75}+4\sqrt{12}-\left(2\sqrt{2}-\sqrt{3}\right)\left(2\sqrt{2}+\sqrt{3}\right) =$

$= 2\sqrt{25\cdot 3} + 4\sqrt{4\cdot 3} - \left(\left(2\sqrt{2}\right)^2 - \left(\sqrt{3}\right)^2\right)$

$= 2\cdot 5\sqrt{3} + 4\cdot 2\sqrt{3} - (4\cdot 2 - 3)$

$= 10\sqrt{3} + 8\sqrt{3} - 5$

$= \boxed{18\sqrt{3} - 5}$

e) $\left(a^{-1}+b^{-1}\right)^{-1} = \dfrac{1}{a^{-1}+b^{-1}}$

$= \dfrac{1}{\frac{1}{a}+\frac{1}{b}}$

$= \dfrac{1}{\frac{b+a}{ab}} = \boxed{\dfrac{ab}{a+b}}$

f) $3\sqrt{5}-\sqrt[3]{x}+5\sqrt{5}+2\sqrt[3]{x} =$

$= \boxed{8\sqrt{5} + \sqrt[3]{x}}$

g) $2\sqrt[3]{x^4y^2}+2x\sqrt[3]{xy^2} =$

$= 2\sqrt[3]{x^3xy^2} + 2x\sqrt[3]{xy^2}$

$= 2x\sqrt[3]{xy^2} + 2x\sqrt[3]{xy^2}$

$= \boxed{4x\sqrt[3]{xy^2}}$

h) $\left(\sqrt{2}-2\sqrt{7}\right)^2 =$

$= \left(\sqrt{2}\right)^2 - 2\cdot\sqrt{2}\left(2\sqrt{7}\right) + \left(2\sqrt{7}\right)^2$

$= 2 - 4\sqrt{14} + 4\cdot 7$

$= \boxed{30 - 4\sqrt{14}}$

i) $\dfrac{-6+\sqrt{-28}}{8} = \dfrac{-6+\sqrt{28}\,i}{8} = \dfrac{-6+\sqrt{7\cdot 4}\,i}{8}$

$= \dfrac{-6+2\sqrt{7}\,i}{8} = \dfrac{2(-3+\sqrt{7}\,i)}{8}$

$= \boxed{\dfrac{-3+\sqrt{7}\,i}{4} = -\dfrac{3}{4} + \dfrac{\sqrt{7}}{4}i}$

6. Do the following operations:

a) $(2-3i)-(5+i)+2i^2 =$

$= 2-3i-5-i+2(-1)$

$= -3-4i-2$

$= \boxed{-5-4i}$

b) $(5-3i)(1+2i) =$

$= 5+10i-3i-6i^2$

$= 5+7i-6(-1)$

$= 5+7i+6$

$= \boxed{11+7i}$

c) $\frac{1+i}{1+2i} = \frac{(1+i)(1-2i)}{(1+2i)(1-2i)}$

$= \frac{1-2i+i-2i^2}{1^2-(2i)^2} = \frac{1-i-2(-1)}{1-4i^2}$

$= \frac{1-i+2}{1-4(-1)} = \boxed{\frac{3-i}{5} = \frac{3}{5}-\frac{1}{5}i}$

d) $\frac{3}{2i} = \frac{3i}{2i \cdot i} = \frac{3i}{2i^2}$

$= \frac{3i}{2(-1)}$

$= \boxed{-\frac{3i}{2}}$

7. Solve the following equations:

a) $\sqrt{7-3x} = 2$ $|^2$

$\left(\sqrt{7-3x}\right)^2 = 2^2$

$7-3x = 4$

$7-4 = 3x$

$3 = 3x \Rightarrow x = 1$

check: $\sqrt{7-3(1)} \stackrel{?}{=} 2$

$\sqrt{4} = 2$ true

The solution set is $\{1\}$.

b) $\sqrt{2x+1} = x-7$ $/^2$

$\left(\sqrt{2x+1}\right)^2 = (x-7)^2$

$2x+1 = x^2-14x+49$

$x^2-16x+48 = 0$

$(x-12)(x-4) = 0$

$x = 12$ ✓ OR $x = 4$

check $x=12$

$\sqrt{2(12)+1} \stackrel{?}{=} 12-7$

$\sqrt{25} = 5$ true

check $x=4$

$\sqrt{2(4)+1} \stackrel{?}{=} 4-7$

$\sqrt{9} = -3$ false

Therefore, the solution set is $\{12\}$

c) $x-\sqrt{x-2}=4$

$x-4=\sqrt{x-2}$ $\Big/^2$

$(x-4)^2=(\sqrt{x-2})^2$

$x^2-8x+16=x-2$

$x^2-9x+18=0$

$(x-6)(x-3)=0$ $x=6$ ✓ OR $x=3$

check $x=6$

$6-\sqrt{6-2}\overset{?}{=}4$

$6-2=4$ true

check $x=3$

$3-\sqrt{3-2}\overset{?}{=}4$

$3-1=4$ false

The solution set is $\{6\}$.

d) $\sqrt{x-7}+\sqrt{x}=7$

$\sqrt{x-7}=7-\sqrt{x}$ $\Big/^2$

$(\sqrt{x-7})^2=(7-\sqrt{x})^2$

$x-7=49-14\sqrt{x}+(\sqrt{x})^2$

$\cancel{x}-7=49-14\sqrt{x}+\cancel{x}$

$14\sqrt{x}=49+7$

$14\sqrt{x}=56$

$\sqrt{x}=4$ $\Big/^2$

$(\sqrt{x})^2=4^2$

$x=16$

check:

$\sqrt{16-7}+\sqrt{16}\overset{?}{=}7$

$\sqrt{9}+4=7$

$3+4=7$ true

The solution set is $\{16\}$

8. Find the x-intercepts of the graph of the function

$$f(x)=\sqrt{x+16}-\sqrt{x}-2$$

$x=?$ $f(x)=0$

$\sqrt{x+16}-\sqrt{x}-2=0$

$\sqrt{x+16}=\sqrt{x}+2$

$(\sqrt{x+16})^2=(\sqrt{x}+2)^2$

$x+16=(\sqrt{x})^2+2(\sqrt{x})2+2^2$

$\cancel{x}+16=\cancel{x}+4\sqrt{x}+4$

$16-4=4\sqrt{x}$

$12=4\sqrt{x}$ $\Rightarrow$

$\sqrt{x}=3$

$(\sqrt{x})^2=3^2$

$x=9$

check: $\sqrt{9+16}-\sqrt{9}-2\overset{?}{=}0$

$5-3-2=0$ true

The x-intercept is $(9,0)$.