

TEST 2 @ 110 points

You should use a pencil. SHOW AND JUSTIFY YOUR WORK OR YOU WILL NOT GET ANY CREDIT.

1. Decide whether the given ordered pair is a solution of the given equation:

(7, -2) $x=7, y=-2$

(1) $4x = 26 - y$
 $4(7) = 26 - (-2)$
 $28 = 28$ true
 $\Rightarrow (7, -2)$ satisfies the first equation

(2) $3x - 4y = 29$
 $3(7) - 4(-2) = 29$
 $21 + 8 = 29$
 $29 = 29$ true \Rightarrow
 $(7, -2)$ satisfies the second equation

Therefore, (7, -2) is a solution of the system.

2. Without graphing or solving the system of equations, find how many solutions each system has? How are the lines that represent the equations of each system? Justify your answers.

a) $\begin{cases} 2x + y = 6 & (1) \\ x - 3y = -4 & (2) \end{cases}$

(1) $2x + y = 6$
 $y = -2x + 6$
 $m = -2$

(2) $x - 3y = -4$
 $3y = x + 4$
 $y = \frac{1}{3}x + \frac{4}{3}$
 $m = \frac{1}{3}$

We have two distinct lines, not parallel, therefore they intersect in one point.
 The system has one solution.

c) $\begin{cases} 5x + 4y = 7 & (1) \\ 10x + 8y = 4 & (2) \end{cases}$

(1) $5x + 4y = 7$
 $4y = -5x + 7$
 $y = -\frac{5}{4}x + \frac{7}{4}$
 $\begin{cases} m_1 = -\frac{5}{4} \\ b_1 = \frac{7}{4} \end{cases}$

(2) $10x + 8y = 4$
 $8y = -10x + 4$
 $y = -\frac{10}{8}x + \frac{4}{8}$
 $y = -\frac{5}{4}x + \frac{1}{2}$
 $\begin{cases} m_2 = -\frac{5}{4} \\ b_2 = \frac{1}{2} \end{cases}$

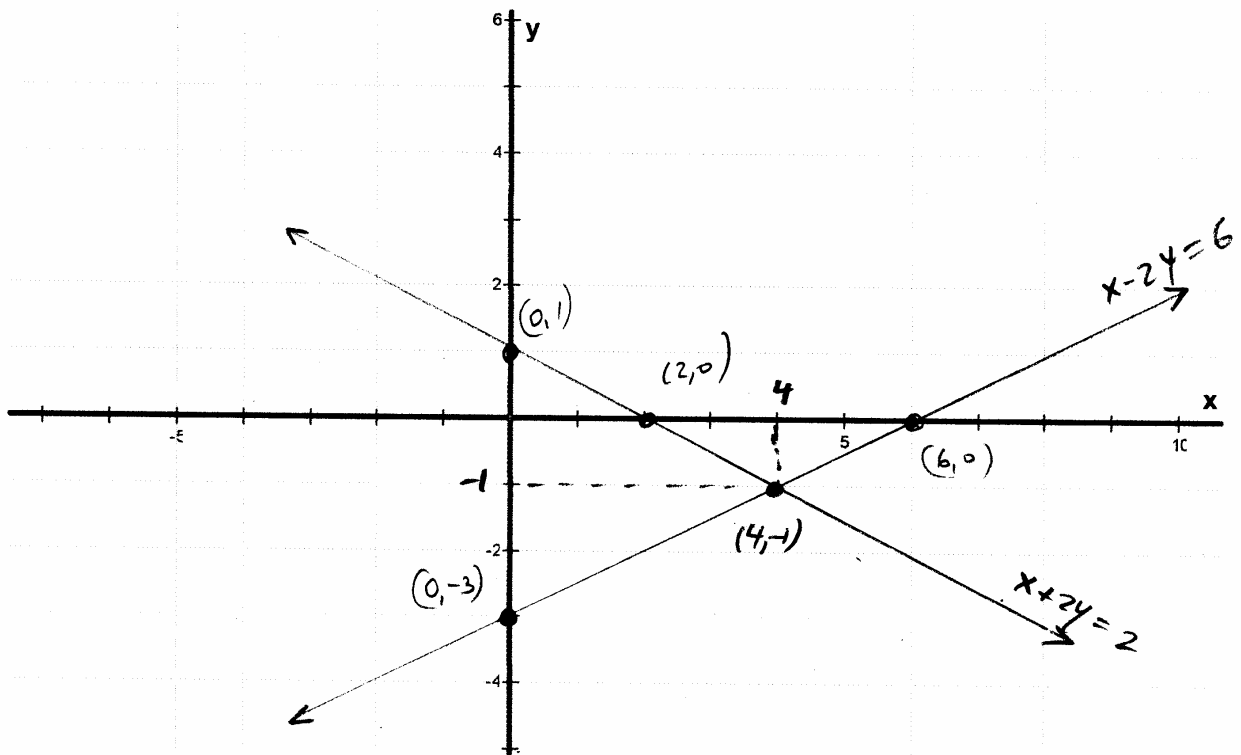
b) $\begin{cases} x = \frac{2}{3}y + 3 & (1) \\ 3x - 2y = 9 & (2) \end{cases}$

(1) $x = \frac{2}{3}y + 3$ $\cdot 3$
 $3x = 2y + 9$
 $3x - 2y = 9$

The equations of the system are equivalent. They represent the same line.
 Therefore, the system has an infinite number of solutions.

There are two distinct lines, parallel.
 Therefore, the system has no solutions.

3. Solve the following system of equations by graphing. Show clearly how you graph the lines. Label the points used. Highlight the solution of the system on the graph and give its coordinates.



$$\begin{cases} x - 2y = 6 \\ x + 2y = 2 \end{cases}$$

$$\boxed{x - 2y = 6}$$

x	y
0	-3
6	0

$$\boxed{x + 2y = 2}$$

x	y
0	1
2	0

The solution of the system is $(4, -1)$.

4. Solve the following system by **addition (elimination)**:

$$\begin{cases} \frac{3x}{4} + \frac{5y}{8} = \frac{10}{4} \\ \frac{x}{4} + \frac{y}{8} = 2 \end{cases} \quad \text{LCD} = 8$$

$$\begin{cases} \frac{x}{4} + \frac{y}{8} = 2 \\ \frac{3x}{4} + \frac{5y}{8} = \frac{10}{4} \end{cases} \quad \text{LCD} = 8$$

$$\begin{cases} 6x + 5y = 20 \\ 2x + y = 16 \end{cases} \quad | -3$$

$$\begin{cases} 6x + 5y = 20 \\ -6x - 3y = -48 \end{cases}$$

$$\begin{aligned} (+) \quad 2y &= -28 \\ y &= -14 \end{aligned}$$

$$2x + y = 16$$

$$2x - 14 = 16$$

$$2x = 16 + 14$$

$$2x = 30$$

$$x = 15$$

The solution is
(15, -14)

5. Solve the following system by **substitution**:

$$\begin{cases} -5A = 15B + 1 \\ A + 3B = -5 \end{cases} \Rightarrow A = -5 - 3B$$

$$-5A = 15B + 1$$

$$-5(-5 - 3B) = 15B + 1$$

$$25 + 15B = 15B + 1$$

$$25 = 1 \quad \text{Contradiction}$$

\Rightarrow

The system has no solutions

6. Solve the following system by the **method of your choice**:

$$\begin{cases} 6x = 9y \\ 2x - 3y = 0 \end{cases}$$

$$\begin{cases} 6x - 9y = 0 \\ 2x - 3y = 0 \end{cases} \quad | \div 3$$

$$\begin{cases} 2x - 3y = 0 \\ 2x - 3y = 0 \end{cases} \quad \text{same line}$$

\Rightarrow infinite number of solutions.

7. Translate the following problems into a system of two equations with two variables.
Define your variables. DO NOT SOLVE.

a) Mark invested \$22,000. Part of it was invested at 3% annual interest and the rest was invested at 5%. His interest income for the first year was \$760.
DO NOT SOLVE.

Let $x =$ amount \$ invested at 3%
interest rate
 $y =$ amount \$ invested at 5%
interest rate

$$\begin{cases} x + y = 22,000 \\ 0.03x + 0.05y = 760 \end{cases}$$

b) The width of a rectangle is half its length. The perimeter of the rectangle is 86 feet.
DO NOT SOLVE.

Let $w =$ width
 $l =$ length

$$\begin{cases} w = \frac{l}{2} \\ 2l + 2w = 86 \end{cases}$$

Solve the following problems. Define your variables.

8. A boat can travel 12 miles downstream in 1.5 hours. It takes the boat 6 hours to return. Find the speed of the boat in still water and the speed of the current.

	Distance	Rate	Time
downstream	12 mi	$x+y$	1.5 h
upstream	12 mi	$x-y$	6 h

Let $x =$ speed of boat in still water
 $y =$ speed of the current
We know that $\text{distance} = \text{rate} \cdot \text{time}$

$$\begin{cases} 12 = 1.5(x+y) \\ 12 = 6(x-y) \end{cases} \begin{array}{l} \div 1.5 \\ \div 6 \end{array}$$

$$\begin{cases} 8 = x+y \\ 2 = x-y \end{cases}$$

$$\textcircled{+} 10 = 2x = 0 \quad x = 5 \text{ mi/h}$$

$$x+y = 8 = 0 \quad 5+y = 8 \Rightarrow y = 3 \text{ mi/h}$$

The speed of the boat is 5 mi/h and the speed of the current is 3 mi/h.

9. A 90% antifreeze solution is to be mixed with a 75% solution to make 120 L of a 78% solution. How many liters of the 90% and 75% solutions will be used?

90%	75%	78%
x L	y L	120 L

Let $x = \#$ of liters of the 90% solution
 $y = \#$ of liters of the 75% solution

$$\begin{cases} x + y = 120 \\ 0.9x + 0.75y = 0.78(120) \end{cases}$$

$$\begin{cases} x + y = 120 \Rightarrow y = 120 - x \\ 0.9x + 0.75y = 93.6 \end{cases}$$

$$0.9x + 0.75(120 - x) = 93.6$$

$$0.9x + 90 - 0.75x = 93.6$$

$$0.15x = 93.6 - 90$$

$$0.15x = 3.6 \Rightarrow x = \frac{3.6}{0.15} = 24$$

$$x = 24$$

$$y = 120 - 24 = 96$$

There are 24 liters of the 90% solution and 96 liters of the 75% solution.

Extra Credit @ 5 points

A cup of rolled oats provides 310 calories. A cup of rolled wheat flakes provides 290 calories. A new breakfast cereal combines wheat and oats to provide 302 calories per cup. How much of each grain does 1 cup of the cereal include?

include?	calories/cup	# of cups
oats	310 cal/cup	x
wheat	290 cal/cup	y
cereal	302 cal/cup	1

Let $x = \#$ of cups of rolled oats
 $y = \#$ of cups of wheat flakes

$$\begin{cases} x + y = 1 \Rightarrow x = 1 - y \\ 310x + 290y = 302 \end{cases}$$

$$310(1 - y) + 290 = 302$$

$$310 - 310y + 290 = 302$$

$$310 - 20y = 302$$

$$310 - 302 = 20y$$

$$8 = 20y \Rightarrow y = \frac{8}{20} = \frac{2}{5}$$

$$y = \frac{2}{5}$$

$$x + y = 1 \Rightarrow x = \frac{3}{5}$$

One cup of cereal includes $\frac{3}{5}$ cup of rolled oats and $\frac{2}{5}$ cup of wheat flakes