

TEST 1 @ 110 points

You should use a pencil. SHOW AND JUSTIFY YOUR WORK OR YOU WILL NOT GET ANY CREDIT.

1. Answer each question:

a) What is the standard form of a linear equation in two variables?

$$ax + by = c, \quad a, b, c \in \mathbb{R}, \quad a \text{ and } b \text{ not both zero}$$

c) What is the slope-intercept form of a line?

$$y = mx + b, \quad m = \text{slope}, \quad (0, b) = y\text{-int}$$

d) What is the point-slope form of a line?

$$y - y_1 = m(x - x_1), \quad m = \text{slope}, \quad (x_1, y_1) \in \text{line}$$

e) When are two lines parallel?

$$\text{They have same slope } m_1 = m_2$$

f) When are two lines perpendicular?

$$\text{when } m_1 \cdot m_2 = -1$$

g) What is the definition of the slope?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$$

2. Let $3x - y = -2$ be a linear equation in two variables.

a) Complete each ordered pair so that it is a solution of the given equation:

i) $(?, -1)$

$$\begin{aligned} y = -1 &\Rightarrow 3x - (-1) = -2 \\ 3x + 1 &= -2 \\ 3x &= -2 - 1 \\ 3x &= -3 \\ x &= -1 \end{aligned}$$

$$\boxed{(-1, -1)}$$

ii) $(2, ?)$

$$\begin{aligned} x = 2 &\Rightarrow 3(2) - y = -2 \\ 6 - y &= -2 \\ 6 + 2 &= y \\ y &= 8 \end{aligned}$$

$$\boxed{(2, 8)}$$

c) What is the slope of the line?

$$3x - y = -2$$

$$3x + 2 = y$$

$$\boxed{m = 3}$$

OR

$$(-1, -1) \text{ and } (2, 8)$$

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} = \frac{8 - (-1)}{2 - (-1)} \\ &= \frac{9}{3} = 3 \end{aligned}$$

d) Is the ordered pair $(1, 1)$ a solution of the equation?

Justify your answer.

$$x = 1, \quad y = 1$$

$$3(1) - 1 \stackrel{?}{=} -2$$

$$3 - 1 = -2$$

$$2 = -2 \quad \text{false}$$

$\Rightarrow (1, 1)$ is not a solution

3 Graph the following lines. Show all work. Label all points, lines, and axes.

a) $2x = -y$

b) $2 - y = 0$

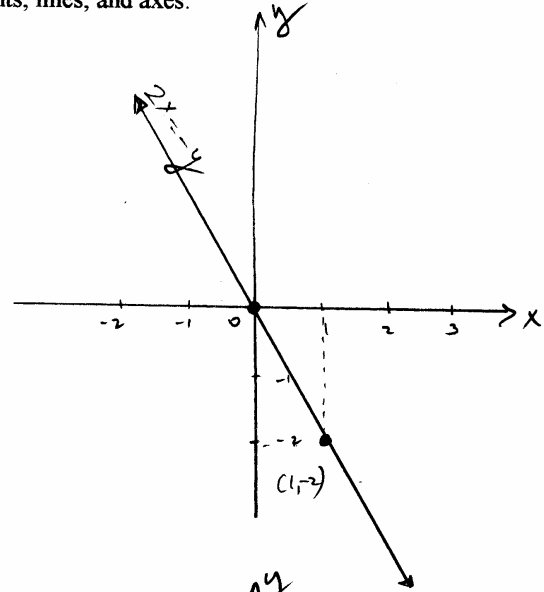
c) $2x = 5$

d) $3x - 5y = 15$

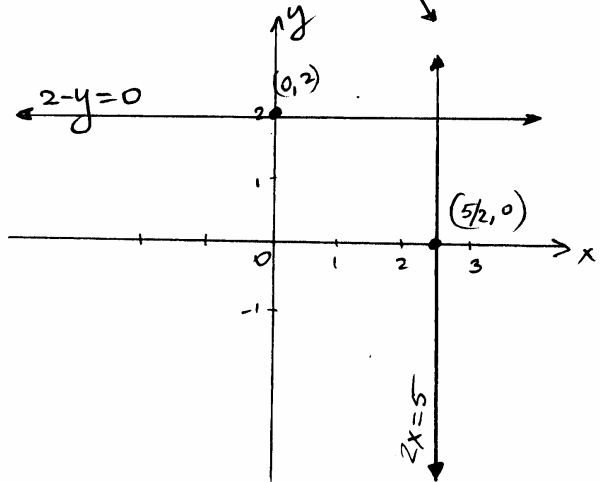
(a) $2x = -y$

x	y
0	0
1	-2

$2(1) = -y$
 $2 = -y$
 $y = -2$



(b) $2 - y = 0$
 $y = 2$ horizontal line

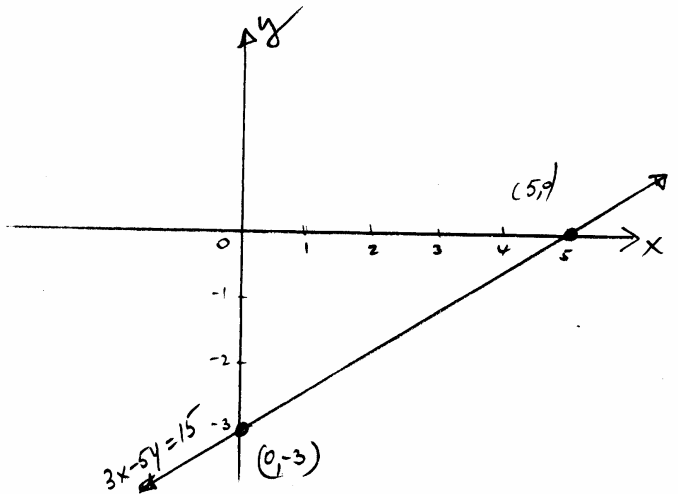


(c) $2x = 5$
 $x = \frac{5}{2}$ vertical line.

(d) $3x - 5y = 15$

x	y
0	-3
5	0

 $x = 0 \Rightarrow -5y = 15$
 $y = -3$
 $y = 0 \Rightarrow 3x = 15$
 $x = 5$



4. a) Write an equation for the line that passes through the given point and has the given slope: $(-2, 3)$, $m = \frac{4}{5}$.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{4}{5}(x - (-2))$$

$$y - 3 = \frac{4}{5}(x + 2)$$

- b) put your equation into slope-intercept form;

$$y - 3 = \frac{4}{5}(x + 2)$$

$$y - 3 = \frac{4}{5}x + \frac{8}{5}$$

$$y = \frac{4}{5}x + \frac{8}{5} + 3$$

$$y = \frac{4}{5}x + \frac{23}{5}$$

- c) put your equation in standard form with integer coefficients.

$$y = \frac{4}{5}x + \frac{23}{5} \quad | \cdot 5$$

$$5y = 4x + 23$$

$$-4x + 5y = 23$$

5. Find an equation of the line that passes through the points $(2, -1)$ and $(-3, 1)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{1 - (-1)}{-3 - 2} = \frac{1 + 1}{-5} = -\frac{2}{5}$$

$$m = -\frac{2}{5} \quad \left| \quad y - y_1 = m(x - x_1) \right.$$

$$(2, -1) \quad \left| \quad y - (-1) = -\frac{2}{5}(x - 2) \right.$$

$$y + 1 = -\frac{2}{5}(x - 2)$$

6. Let $2x + \frac{1}{5}y = \frac{1}{3}$ be the equation of a line.

- a) What is the slope of a line perpendicular to the given line?

First, we find the slope of the given line

$$\frac{1}{5}y = -2x + \frac{1}{3} \quad | \cdot 5$$

$$y = -10x + \frac{5}{3} \Rightarrow m = -10$$

Therefore, a line perpendicular to the given line has slope

$$m_{\perp} = \frac{1}{10}$$

- b) Find an equation of the line that passes through the point $(2, 5)$ and is perpendicular to the given line?

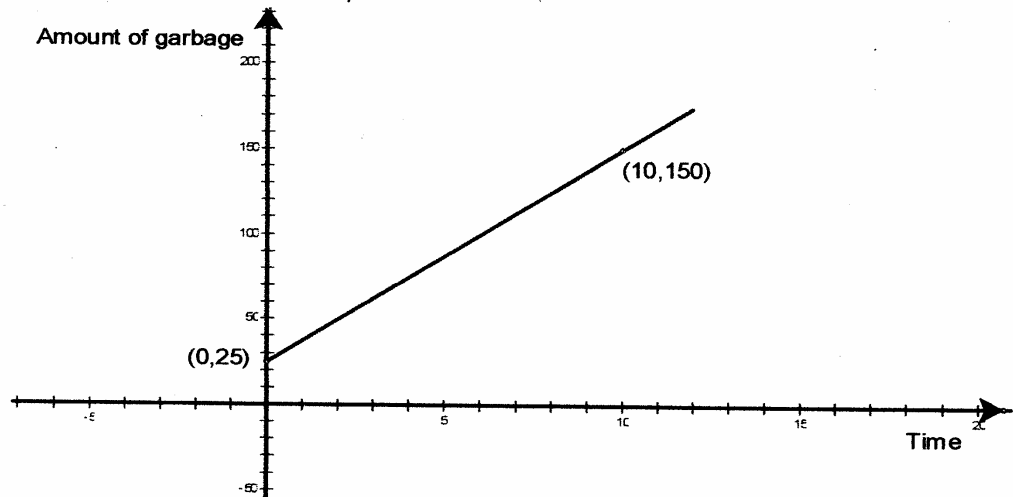
$$m = \frac{1}{10}$$

$$(2, 5)$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{10}(x - 2)$$

7.



The figure above shows the amount of garbage, G (in tons) that has been deposited at a dump t years after new regulations go into effect.

- a) What is the significance of the G -intercept $(0, 25)$ in the context of this problem?
(0, 25) shows the amount of garbage that had been deposited at the time when new regulations went into effect
- b) What is the significance of the point $(10, 150)$ in the context of this problem?
(10, 150) shows the amount of garbage that had been deposited 10 years after new regulations went into effect
- c) Compute the slope (including units) and explain what the slope measures in the context of the problem.

$$m = \frac{\Delta G}{\Delta t} = \frac{150 - 25}{10 - 0} = \frac{125}{10} = 12.5 \text{ t/year}$$

It shows the rate at which the garbage is deposited per year.

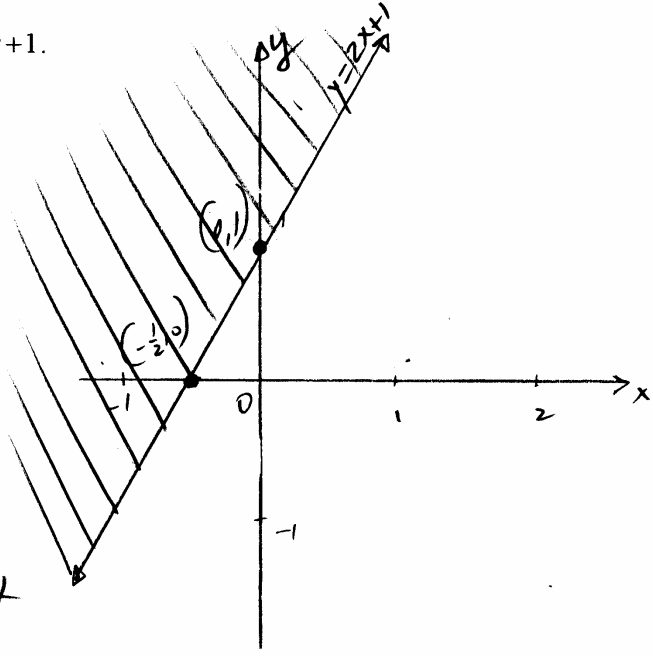
8. Graph the following linear inequality in two variables: $y \geq 2x + 1$. Show all work. Label the points, line and axes used.

$$y \geq 2x + 1$$

Boundary line: $y = 2x + 1$

x	y
0	1
$-\frac{1}{2}$	0

If $x=0, y=1$
 If $y=0, 2x+1=0$
 $x = -\frac{1}{2}$



Test point not on the line: $(0, 0)$

$$0 \geq 2(0) + 1$$

$$0 \geq 1 \text{ false} \Rightarrow (0, 0) \text{ not a solution}$$

Extra Credit @ 6 points

As dry air moves upward, it expands and cools. If the ground temperature is 20°C , then for each kilometer increase in altitude the temperature will go down by 10°C .

a) Write an equation for the temperature "T" (in $^{\circ}\text{C}$) in terms of the altitude "H" (in kilometers)

two variables $\left\{ \begin{array}{l} H = \text{the altitude (independent variable)} \\ T = \text{the temperature (the dependent variable)} \end{array} \right.$

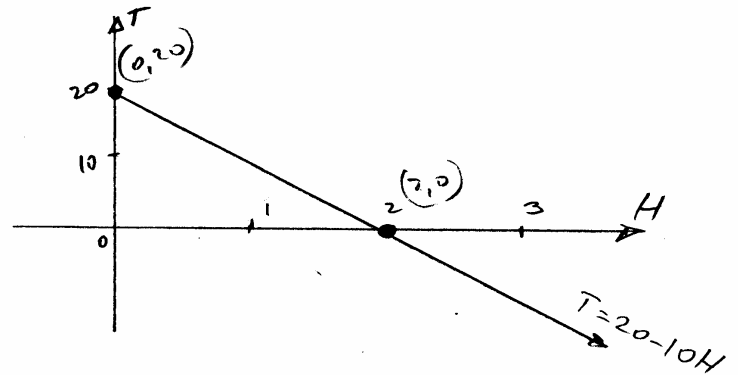
$$| T = 20 - 10H |$$

b) Graph the equation above (Find the intercepts and be sure to label your axis)

$$T = 20 - 10H$$

H	T
0	20
2	0

$\begin{cases} T=0, \\ 20-10H=0 \\ 20=10H \\ H=2 \end{cases}$



c) Explain what the intercepts represent in terms of the temperature and the altitude.

$(0, 20)$ T-intercept
It shows the temperature at ground level.
There are 20°C at ground level.

$(2, 0)$ H-intercept
It shows the altitude at which the temperature is 0.
At 2 kilometers altitude, the temperature is 0°C .