Review Test #2

Chapter 5 (5.3 – 5.8), Chapter 6
Chapter 7 (7.1 – 7.3, 7.5 – 7.7), 8.1

To prepare for the test, you should study all exercises and examples done in class, plus:

- Homework assigned from the listed sections
- Handout Section 7.3 – Trigonometric Equations

Trigonometry

You should know the following:

- how to graph the basic functions sin, cos, tan, cot, sec, csc
- domain, range, period, amplitude (when defined) and vertical asymptotes (when applicable) for the basic functions
- how to graph transformations of trigonometric functions (vertical translations, vertical stretching and compression, horizontal stretching and compression, horizontal shifting
- how to graph the inverse sine, inverse cosine, and inverse tangent functions
- domain and range for the inverse functions (all six inverse functions)
- evaluate all inverse functions
- compose trigonometric functions and their inverses
- solve trigonometric equations
- IMPORTANT FORMULAS
  - $\tan x = \dfrac{\sin x}{\cos x}$
  - $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$
  - $\sec x = \dfrac{1}{\cos x}$
  - $\csc x = \dfrac{1}{\sin x}$
  - $\sin^2 x + \cos^2 x = 1$
  - sine and cosine functions have period $2p$

    $\sin(x + 2kp) = \sin x$

    $\cos(x + 2kp) = \cos x$
  - tangent function has period p

    $\tan(x + kp) = \tan x$
  - $\cos(a+b) = \cos a \cos b - \sin a \sin b$
  - $\cos(a-b) = \cos a \cos b + \sin a \sin b$
  - $\sin(a+b) = \sin a \cos b + \sin b \cos a$
  - $\sin(a-b) = \sin a \cos b - \sin b \cos a$
  - $\cos 2a = \cos^2 a - \sin^2 a$

OTHER FORMULAS

$\sin(x + p) = -\sin x$

$\cos(x + p) = -\cos x$

$\cos(-x) = \cos x$ cosine is an even function

$\sin(-x) = -\sin x$ sine is an odd function

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

$\tan 2a = \dfrac{2\tan a}{1 - \tan^2 a}$

$\cos a = \pm\sqrt{\dfrac{1 + \cos 2a}{2}}$

- $\cos 2a = 2\cos^2 a - 1$
- $\cos 2a = 1 - 2\sin^2 a$ $\qquad\qquad \sin a = \pm\sqrt{\dfrac{1-\cos 2a}{2}}$
- $\sin 2a = 2\sin a \cos a$

More practice Chapter 5 - Exponential and Logarithmic Functions and Equations
Note: Important Homework exercises
5.4 - # 39 – 50, 73 – 88, 89 – 112, 115 – 118
5.5 - # 37 – 56, 87 – 95, 104
5.6 - All Homework exercises are important.

1. Solve the following equations:

a) $10^{x+3} = 5e^{7-x}$ b) $2e^{3x} = 4e^{5x}$ c) $2x - 1 = e^{\ln x^2}$ d) $5^x = 3^{2x-1}$ e) $5e^{0.3x} = 8$

f) $2\log(x-1) = \dfrac{5}{2}\log x^5 - \log\sqrt{x}$ g) $\log_2 x + \log_3 x = 1$ h) $x^{\log x + 2} = 1000$ i) $\log_4(3x-2) + 1 = 0$

2. Graph each function. Label at least 2 points and the asymptote for each graph.

a) $f(x) = e^x$ b) $g(x) = 3^{-x}$ c) $h(x) = \ln(x+1)$ d) $F(x) = \log_2 x$

3. Find the domain of each function.

a) $f(x) = \sqrt{\ln x}$ b) $g(x) = \log\left(\dfrac{1}{x+1}\right)$ c) $h(x) = \ln\left(\dfrac{x+1}{1-x}\right)$

4. Write the expression as a sum and/or difference of logarithms. Express powers as factors

$$\ln\left[\frac{x^2 - x - 2}{3(x+4)^2}\right]^{\frac{1}{3}}$$

5. If $f(x) = \log_a x$, show that $\dfrac{f(x+h) - f(x)}{h} = \log_a\left(1 + \dfrac{h}{x}\right)^{\frac{1}{h}}$, $h \neq 0$

6. Express y as a function of x. The constant C is a positive number.

$\ln y = \ln x + \ln(x+1) + \ln C$

More practice Chapters 6 , 7 and 8.1 – Trigonometry

Note: Important Homework exercises
6.3 - #19 – 76, 95 – 112
6.4 - # 33 – 52, 75 – 78
6.5 - # 17 – 28, 41 – 44
7.1 - # 15 – 58
7.2 – # 57 - 66
7.3 – # 13 – 82
7.5 - # 105, 106
7.6 - # 43 – 61 , 69, 70, 71 - 79
7.7 - # 7 – 15

7. Find a formula for $\sin(x + y + z)$ in terms of sine and cosine of x, y and z.

8. Prove that tangent is an odd function.

9. Graph the following functions on graphing paper. In each case, identify the amplitude (when defined) and the period and label the axes accurately. Clearly label the problems. Explain in words what and how you are graphing.

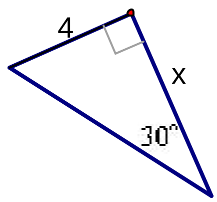
a) $y = 1 + \sin x$ from $-2p$ to $4p$ b) $y = 4\sin\frac{1}{3}x$ over one period

c) $y = -2\cos x$ over one period d) $y = \tan 2x$ over one period e) $y = 2\sin\left(x - \frac{p}{3}\right)$

10. Find all real numbers x that satisfy each equation. Justify your answers.

a) $\cos x = 0$ b) $\sin x = 0$ c) $\tan x = 0$ d) $\cot x = 0$

11. Find the side labeled x.



12. Sketch a right triangle that has one acute angle q, and find the other five trigonometric ratios of q

$\sin q = \frac{3}{7}$

13. Find the exact value of each expression.

a) $\sin\frac{p}{6} + \cos\frac{5p}{3}$ b) $\tan\frac{3p}{4}\left(\sin\frac{2p}{3} - \sec\frac{5p}{4}\right)$ c) $\sin\left(-\frac{3p}{4}\right)$

d) $\tan\left(\frac{13p}{6}\right)$ e) $\cos\left(\frac{5p}{6}\right)$

14. Use the unit circle to find all the values of q between 0 and $2p$ for which

a) $\sin q = \frac{1}{2}$ b) $\cos q = -\frac{\sqrt{2}}{2}$

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15. Graph $f(x) = \sin x$ and $f^{-1}(x) = \sin^{-1}(x)$ on the same coordinate system, showing the relation between the two graphs (symmetry about the line $y = x$). Answer the following questions:

a) What is the domain and range of $f(x) = \sin x$?

b) What is the domain and range of $f^{-1}(x) = \sin^{-1}(x)$?

16. Evaluate the following. Give exact answers whenever possible.

a) $\sin^{-1}\left(\frac{1}{2}\right)$ b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ c) $\tan^{-1}(-1)$ d) $\cos\left(\sin^{-1}\frac{3}{5}\right)$

e) $\sin^{-1}\left(\sin\frac{5p}{8}\right)$ f) $\cos^{-1}\left(\cos\frac{2p}{7}\right)$ g) $\tan\left(\tan^{-1}100.23\right)$ h) $\cos\frac{p}{12}$

17.

a) Write $\cos^2 x$ in terms of $\cos 2x$.

b) Write $\sin^2 \frac{x}{3}$ in terms of a trig function of power 1.

c) Write $\cos^4 x$ and $\sin^4 x$ in terms of cosine and/or sine functions with power 1.

d) Write $\tan^2 t$ in terms of $\sec^2 t$.

e) Write $\cot^2 y$ in terms of $\cos^2 y$.

f) Write tant in terms of cost

18. Find the average rate of change of each function from 0 to $\frac{p}{2}$.

a) $f(x) = \sin x$ b) $g(x) = \cos(2x)$

19. Find the average rate of change of f from 0 to $\frac{p}{6}$

$f(x) = \tan x$

20. Show that the difference quotient for $f(x) = \sin x$ is given by

$$\frac{f(x+h) - f(x)}{h} = \cos x \cdot \frac{\sin h}{h} - \sin x \cdot \frac{1-\cos h}{h}, \quad h \neq 0$$

21. Show that the difference quotient for $g(x) = \cos x$ is given by

$$\frac{g(x+h) - g(x)}{h} = -\sin x \cdot \frac{\sin h}{h} - \cos x \cdot \frac{1-\cos h}{h}, \quad h \neq 0$$

22. Prove the following formulas:

a) $\ln|\sin q| = \frac{1}{2}\left(\ln|1-\cos(2q)| - \ln 2\right)$ b) $\ln|\cos x| = \frac{1}{2}\left(\ln|1+\cos(2x)| - \ln 2\right)$

23. Simplify:

a) $\cos^4 x - \sin^4 x$ b) $\frac{\cot a - \tan a}{\cot a + \tan a}$ c) $\frac{1}{2}\sec t \csc t$ d) $4\sin u \cos u\left(1 - 2\sin^2 u\right)$

e) $\sin^2 x \cos^2 x$

24. Express each product as a sum containing only sines or only cosines.

a) $\sin(4t)\sin(2t)$ b) $\sin(4x)\cos(6x)$ c) $\cos(3q)\cos(4q)$

25. Write each trigonometric expression as an algebraic expression in x. What are the restrictions on x?

a) $\sin\left(\cos^{-1} x\right)$ b) $\cos\left(\sin^{-1} x\right)$ c) $\sin\left(\tan^{-1} x\right)$ d) $\cos\left(\tan^{-1} x\right)$

e) $\sec\left(\tan^{-1} x\right)$ f) $\tan\left(\sec^{-1} x\right)$ g) $\sin\left(\sec^{-1} x\right)$ h) $\cos\left(\sec^{-1} x\right)$ i) $\tan\left(\sin^{-1} x\right)$

j) $\tan\left(\cos^{-1} x\right)$ k) $\sin\left(\cot^{-1} x\right)$ l) $\cos\left(\cot^{-1} x\right)$ m) $\cos\left(\csc^{-1} x\right)$ n) $\cos\left(\sec^{-1} x\right)$