Review Test 2 – updated 05/17/2012

Chapter 10 (10.1 – 10.10)

Study all examples and exercises done in class, as well as all your homework problems.
Know the theory and exercises presented on all handouts.

Know how to prove the following theorems and properties:

Section 10.1

- Theorem 5 – Commonly occurring limits

Section 10.2

- Finding when a geometric series converges (page 564 textbook, handout 10.2 in class)

Section 10.3

- Finding the values for which a *p*-series is convergent (page 573 textbook, handout 10.3 in class)
- Property 1 (handout) / Corollary Theorem 6 (textbook)
- The Comparison Test

After studying sections 10.1 – 10.10, you should be able to answer the following:

- What is a convergent sequence?
- What is a convergent series?
- What is a bounded sequence?
- What is a monotonic sequence?
- What can you say about a bounded monotonic sequence?
- What is a geometric sequence? Under what circumstances is it convergent? What is its sum?
- What is a p-series? Under what circumstances is it convergent?
- State the following: The nth Test for Divergence, The Integral Test, The Comparison Test, The Limit Comparison Test, The Root Test, The Ratio Test, The Alternating Series Test.
- What is an absolutely convergent series? What can you say about such a series?
- If a series is convergent by the Integral Test, how do you estimate its sum?
- If a series is convergent by the Alternating Series Test, how do you estimate its sum?
- How do you estimate the error in a Taylor's Series?
- Write the general form of a power series. What is the radius of convergence of a power series? What is the interval of convergence of a power series?
- Suppose f(x) is the sum of a power series with radius of convergence R. How do you differentiate f? What is the radius of convergence of the series for f' ?How do you integrate f? What is the radius of convergence for the series for $\int f(x)\,dx$?
- Write an expression for the *n*-th degree Taylor polynomial of *f* centered at a.
- Write an expression for the Taylor series of *f* centered at a.
- Write an expression for the Maclaurin series of f.
- How do you show that f(x) is equal to the sum of its Taylor series?
- State Taylor's inequality.
- Write the Maclaurin series and the interval of convergence for each of the following functions: $\frac{1}{1-x}, \sin x, \cos x, e^x$ (known series).
- Write the binomial series for a function.

You should be able to solve the following types of exercises:

- Find the first terms of the binomial series for a given function; find the binomial series for a given function - 10.10 ex. 1 – 14
- Find the Maclaurin/Taylor series for a function and its radius of convergence using the direct method (definition of a Maclaurin/Taylor series) – 10.8 ex. 11 – 32
- Find the Maclaurin/Taylor series for a function and its radius of convergence using known series 10.9 – ex. 1 – 33; 10.8 – ex. 33 – 36
- Find error estimates – 10.10 examples 3, 4; ex 15 – 28; 10.9 ex 35-37; 10.6 ex 49-53 ; 10.3 ex. 49, 51;
- Find Taylor polynomials – 10.8 ex 1 – 10
- Identify a power series and find its radius and interval of convergence, find the values for which the series converges absolutely and conditionally – 10.7 ex 3 – 53
- Identify an alternating series and determine if the series converges or diverges – 10.6 ex 1 – 47
- Identify a series of positive terms and determine if the series converges or diverges - 10.5, 10.4, 10.3 ex. 3 – 44, 55, 56
- Identify a geometric series or a telescopic series and find its sum – 10.2
- Find limits of sequences – 10.1 ex 27 – 97
- Determine if a sequence is monotonic and bounded – 10.1 ex. 111 - 121