Review Test 1

Chapter 6

- Finding volumes of solids of revolution (disk method, washer method, shell method) (6.1, 6.2)
- Finding lengths of curves (6.3)
- Finding surface areas of solids of revolution (6.4)
- Work done by a variable force along a line (6.5)
- Pumping liquids from containers (6.5)
- Fluid forces against a vertical plate (6.5)
- Know the following proofs:
  - Prove the formula for the arc length (6.3)
  - Prove the formula for the area of a surface generated by revolving a curve about the *x*-axis (6.4)

Chapter 7

- Definition of hyperbolic functions (7.3)
- Proofs for the identities for hyperbolic functions (table 7.4 in section 7.3) and proofs of formulas for the derivatives of hyperbolic functions (table 7.5 in section 7.3)

Chapter 8

- Integration by parts (8.1)
- Trigonometric integrals (8.2)
- Trigonometric substitutions (8.3)
- Integration of rational functions (8.4)
- Numerical integration (8.6)
  - Know how to approximate a definite integral using the trapezoidal rule
  - Know how to estimate the number of subintervals needed to approximate an integral within a given error
  - Know how to prove the formula used to approximate an integral using parabolas (from Simpson's rule)
- Improper integrals (8.7)

To prepare for the test, study all examples and exercises **done in class**, all exercises from the **handouts** used in class, as well as all **homework** examples and exercises.

<u>Volumes – more problems</u>

1. The disk $x^2 + y^2 \le a^2$ is revolved about the line $x = b$ $((b > a)$ to generate a solid shaped like a doughnut and called a *torus*. Find its volume.

2. A bowl has a shape that can be generated by revolving the graph of $y = \frac{x^2}{2}$ between $y = 0$ and $y = 5$ about the y-axis.

   a) Find the volume of the bowl.
   b) If we fill the bowl with water at a constant rate of 3 cubic units per second, how fast will the water level in the bowl be rising when the water is 4 units deep?

3. a) A hemispherical bowl of radius a contains water to a depth *h*. Find the volume of water in the bowl.
   b) Water runs into a sunken concrete hemispherical bowl of radius 5 m at the rate of $0.2m^3/\sec$. How fast is the water level in the bowl rising when the water is 4 m deep?

4. The arch $y = \sin x$, $0 \le x \le p$, is revolved about the line $y = c$, $0 \le c \le 1$, to generate a solid .

a) Find the value of c that minimizes the volume of the solid. What is the minimum volume?
b) What value of c in $[0,1]$ maximizes the volume of the solid?

5. Let S be the solid obtained by rotating about the y-axis the region bounded by $y = x(x-1)^2$ and $y = 0$. Find the volume of the solid S.

6. A Bundt cake , well known for having a ringed shape, is formed by revolving around the y-axis the region bounded by the graph of $y = \sin(x^2 - 1)$ and the x-axis over the interval $1 \le x \le \sqrt{1+p}$. Find the volume of the cake.

- You may find more volume, length, and surface area problems in the following list: 61 in section 6.1, 46 and 48 in section 6.2, 18, 21, and 32 in section 6.4, as well as problems 7 – 12, 14 – 24 on page 414.

Pumping liquids from containers and fluid forces – more problems

5. The graph of $y = x^2$ on $0 \le x \le 2$ is revolved about the y-axis to form a tank that is filled with salt water (weighing approximately 73 lbs/ft^3). How much work does it take to pump all of the water to the top of the tank?

6. a) A reservoir shaped like a right-circular cone, point down, 20 ft across the top and 8 ft deep, is full of water. How much work does it take to pump the water to a level 6 ft above the top?

b) If the reservoir is filled to a depth of 5 ft, and the water is to be pumped to the same level as the top, how much work does it take?

7. We model pumping from spherical containers the way we do from other containers, with the axis of integration along the vertical axis of the sphere. Find how much work it takes to empty a full hemispherical water reservoir of radius 5 m by pumping the water to a height of 4 m above the top of the reservoir. Water weighs 9800 N/m^3.

8. A large tank is designed with ends in the shape of the region between the curves $y = \frac{x^2}{2}$ and $y = 12$, measured in feet. Find the hydrostatic force on one end of the tank if it is filled to a depth of 8 ft with gasoline. (Assume the gasoline's density is 42.0 lb/ft^3.)

- You may find **more work and fluid forces** problems in the following list: problems 25, 27, 29, 30, 31a, 32, 39, 40, 41 on page 414

Integrals – more problems

7. Obtain a formula that expresses $\int \sin^n x dx$ in terms of an integral of a lower power of x (a reduction formula).

8. Find a reduction formula for $I_n = \int \frac{1}{(x^2 + a^2)^n} dx$, $n \in \mathbb{N}, n \ge 2$.

9. Find the following:

a) $\int \sec^{-1} x dx$ b) $\int \sin q \cos q \cos 3q \, dq$ c) $\int \frac{\sin^3 x}{\cos^4 x} dx$

d) $\int \frac{\sqrt{y^2 - 25}}{y^3} dy$ e) $\int_{\ln(3/4)}^{\ln(4/3)} \frac{e^t dt}{\left(1+e^{2t}\right)^{3/2}}$ f) $\int \frac{e^{4t} + 2e^{2t} - e^t}{e^{2t} + 1} dt$

g) $\int \frac{1}{\left(x^{1/3} - 1\right)\sqrt{x}} dx$ h) $\int \frac{1}{x^6\left(x^5 + 4\right)} dx$ i) $\int \frac{x^2 + 1}{x\sqrt{x^4 + 1}} dx, x > 0$

10. Solve the initial value problem for y as a function of x.

$$\sqrt{x^2 - 9}\frac{dy}{dx} = 1, \quad x > 3, \quad y(5) = \ln 3$$

11. Solve the initial value problem for x as a function of t.

$$\left(3t^4 + 4t^2 + 1\right)\frac{dx}{dt} = 2\sqrt{3}\,, \; x(1) = -\frac{p\sqrt{3}}{4}$$

<u>Improper Integrals – more problems</u>

1. If $\int_{-\infty}^{\infty} f(x) dx$ is convergent and a and b are real numbers, show that

$$\int_{-\infty}^{a} f(x) dx + \int_{a}^{\infty} f(x) dx = \int_{-\infty}^{b} f(x) dx + \int_{b}^{\infty} f(x) dx.$$

2. The average speed of molecules in an ideal gas is

$$v = \frac{4}{\sqrt{p}}\left(\frac{M}{2RT}\right)^{3/2} \int_0^{\infty} v^3 e^{-Mv^2/(2RT)} dv$$

where M is the molecular weight of gas, R is the gas constant, T is the gas temperature, and v is the molecular speed. Show that $v = \sqrt{\frac{8RT}{pM}}$.

3. Show that $\int_0^{\infty} x^2 e^{-x^2} dx = \frac{1}{2}\int_0^{\infty} e^{-x^2} dx$.

4. Find the value of the constant C for which the integral

$$\int_0^{\infty}\left(\frac{1}{\sqrt{x^2+4}} - \frac{C}{x+2}\right)dx$$

converges. Evaluate the integral for this value of C.