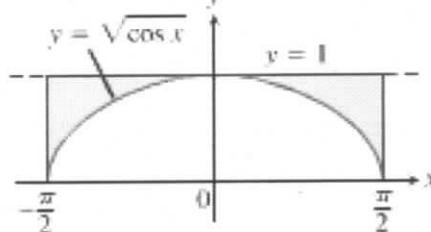


QUIZ #1 @ 100 points

Write neatly. Show all work. No work, no credit given. Write all responses on separate paper. Clearly label the exercises. No graphing calculator allowed. No phone allowed.

1. Using the disk method, find the volume of the solid generated by revolving the region bounded by $y = x - x^2$, $y = 0$ about the x -axis. Make a clear drawing of the region.

2. Using the washer method, find the volume of the solid generated by revolving the shaded region bounded above by the line $y = 1$, below by the curve $y = \sqrt{\cos x}$, on the right by the line $x = \frac{\pi}{2}$, and on the left by the line $x = -\frac{\pi}{2}$, about the x -axis.



3. Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = 3x$, $y = 0$, $x = 2$, about the line $y = 7$. Make a clear drawing of the region.

4. Find a curve through the point $(1,1)$ whose length integral is $L = \int_1^4 \sqrt{1 + \frac{1}{4x}} dx$.

5. Find the area of the surface generated by revolving the curve $y = \sqrt{x+1}$, with $1 \leq x \leq 5$, about the x -axis.

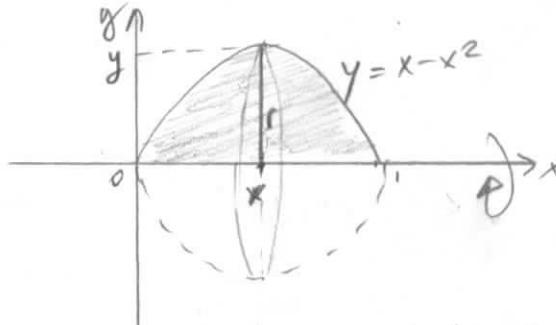
6. A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work done.

7. Show how to approximate the required work by a Riemann sum. Then express the work as an integral and evaluate it.
A vertical right-circular cylindrical tank measures 30 ft high and 20 ft in diameter. It is full of kerosene weighing 51.2 lb / ft^3 . How much work does it take to pump the kerosene to a level 2 ft above the top of the tank?
-
8. Calculate the fluid force on one side of a semicircular plate of radius 5 ft that rests vertically on its diameter at the bottom of a pool filled with water to a depth of 6 ft. The weight-density of water is 62.4 lb / ft^3 .

Quiz 1 - Solutions

$$(1) \begin{cases} y = x - x^2 \\ y = 0 \end{cases} \text{ about } x\text{-axis.}$$

$y = x - x^2$ - parabola opening downward with vertex at $x=0, y=1$



- use vertical planes, \perp to x -axis at x , with x from 0 to 1.

$$\Rightarrow V = \int_0^1 A(x) dx$$

where $A(x)$ = area of the cross-section at x

$$A(x) = \pi r^2 = \pi y^2 = \pi (x-x^2)^2$$

$$V = \int_0^1 \pi (x-x^2)^2 dx$$

$$= \pi \int_0^1 (x^2 - 2x^3 + x^4) dx$$

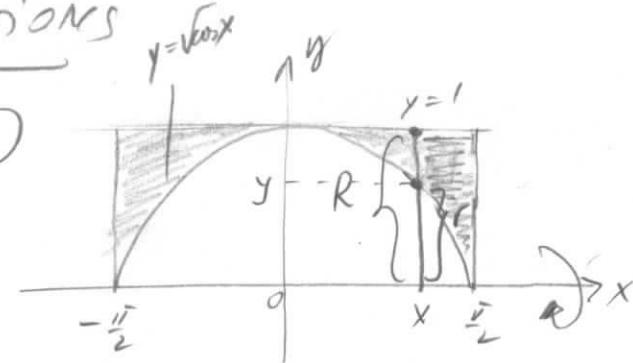
$$= \pi \left[\frac{x^3}{3} - 2 \frac{x^4}{4} + \frac{x^5}{5} \right]_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right)$$

$$= \pi \frac{10 - 15 + 6}{30} = \frac{\pi}{30}$$

$$V = \frac{\pi}{30} \text{ cubic units}$$

(2)



- use vertical planes, \perp x -axis at x , with x from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A(x) dx = 2 \int_0^{\frac{\pi}{2}} A(x) dx$$

where $A(x)$ = area of the cross-section (washer)

$$A(x) = \pi (R^2 - r^2)$$

$$\text{with } \begin{cases} R = 1 \\ r = y = \sqrt{x} \end{cases}$$

$$A(x) = \pi (1 - x)$$

Then,

$$V = 2 \int_0^{\frac{\pi}{2}} \pi (1 - x) dx$$

$$= 2\pi \left([x - \sin x] \right)_0^{\frac{\pi}{2}}$$

$$= 2\pi \left(\frac{\pi}{2} - 1 \right)$$

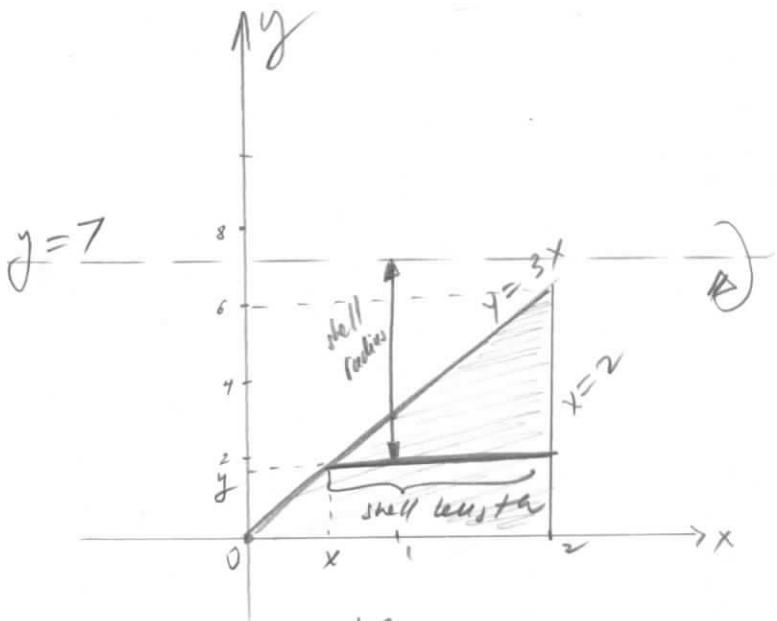
$$= \pi^2 - 2\pi$$

$$V = \pi^2 - 2\pi \text{ cubic units}$$

$$(3) \begin{cases} y = 3x \\ y = 0 \\ x = 2 \end{cases}$$

about the
line $y = 7$

-2-



We'll use shells, parallel to $y=7$
with y from $y=0$ to $y=3/2=6$

$$V = 2\pi \int_0^6 (\text{shell radius})(\text{height}) dy$$

$$\left\{ \begin{array}{l} \text{shell radius} = 7-y \\ \text{shell length} = 2-x = 2-\frac{y}{3} \end{array} \right.$$

$$\begin{aligned} V &= 2\pi \int_0^6 (7-y)(2-\frac{y}{3}) dy \\ &= 2\pi \int_0^6 (14 - \frac{13}{3}y + \frac{1}{3}y^2) dy \\ &= 2\pi \left[14y - \frac{13}{6}y^2 + \frac{1}{9}y^3 \right]_0^6 \\ &= 2\pi (84 - 78 + 24) = 60\pi \end{aligned}$$

$$V = 60\pi \text{ cubic units}$$

$$(4) L = \int_1^4 \sqrt{1 + \frac{1}{4x}} dx$$

$$\text{Also, } L = \int_1^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{1}{4x}$$

$$\text{then, } \frac{dy}{dx} = \pm \frac{1}{2\sqrt{x}}$$

$$\underline{\text{case I}} \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$y = \int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + C \quad \Rightarrow$$

$$(1,1) \in \text{curve} \Rightarrow x=1, y=1 \quad \Rightarrow$$

$$1 = \sqrt{1} + C, \text{ so } C = 0$$

$$\text{so } \left| \begin{array}{l} y = \sqrt{x} \text{ from} \\ (1,1) \text{ to } (4,2) \end{array} \right|$$

$$\underline{\text{case II}} \quad \frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$$

$$y = \int \frac{-1}{2\sqrt{x}} dx = -\sqrt{x} + C \quad \Rightarrow$$

$$(1,1) \in \text{curve} \Rightarrow x=1, y=1 \quad \Rightarrow$$

$$1 = -\sqrt{1} + C; \text{ so } C = 2$$

$$\text{so } \left| \begin{array}{l} y = -\sqrt{x} + 2 \text{ from} \\ (1,1) \text{ to } (4,0) \end{array} \right|$$

-3-

(5) $y = \sqrt{x+1}$, $1 \leq x \leq 5$
about $x\text{-axis}$

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$y = \sqrt{x+1}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1}{4(x+1)} = \frac{4x+5}{4(x+1)}$$

Then,

$$S = \int_1^5 2\pi \sqrt{x+1} \sqrt{\frac{4x+5}{4(x+1)}} dx$$

$$= \pi \int_1^5 \sqrt{4x+5} dx$$

$$\text{let } 4x+5 = u$$

$$4dx = du$$

$$dx = \frac{1}{4} du$$

$$\text{when } x=1, u=9 \\ x=5, u=25$$

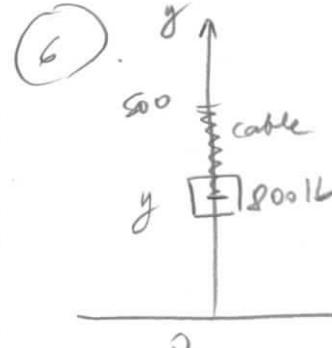
$$S = \frac{1}{4} \pi \int_9^{25} \sqrt{u} du$$

$$= \frac{1}{4} \pi \left[\frac{2u^{3/2}}{3} \right]_9^{25}$$

$$= \frac{5}{6} \left(25^{3/2} - 9^{3/2} \right)$$

$$> \frac{5}{6} (125 - 27) = \frac{98\pi}{6} = \frac{49\pi}{3}$$

$$S = \frac{49\pi}{3} \text{ square units}$$



$$W = \int_a^b F(y) dy$$

$F(y) = \text{weight (coal + cable)}$

$$F(y) = 800 + (500-y) \cdot 2$$

where $y = \text{number of ft lifted}$

Then,

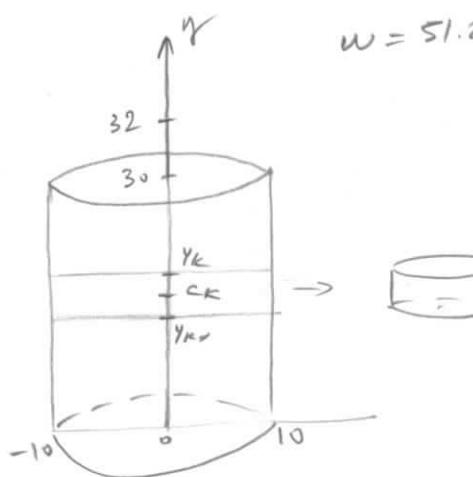
$$W = \int_0^{500} (1800 - 2y) dy$$

$$= 1800 \cdot 500 - 500^2$$

$$= 800 (1300) = 650,000 \text{ lb-ft}$$

$$W = 650,000 \text{ lb-ft} /$$

(7)



$$w = 51.2 \text{ lb/ft}^3$$

- partition $[0, 32]$: $y_0 = 0 < y_1 < \dots < y_{k-1} < y_k < 32$
(divide the fluid into n "layers")
- let $c_k \in [y_{k-1}, y_k]$

Then, $W \approx \sum W_k$

where W_k = work done to pump
the k^{th} layer up to 32 ft

let F_k = force needed to
pump the k^{th} layer up

F_k = weight of the layer
(constant)

$$F_k = (\bar{y} \cdot 10^2 \cdot \Delta y_k) 51.2 = 5120 \bar{y} \Delta y_k$$

$$\Rightarrow W_k \approx \left(\frac{1}{n} \sum_{j=1}^n 5120 \bar{y}_j \right) (32 - c_k) \Delta y_k$$

$$W = \lim_{n \rightarrow \infty} \sum_{k=1}^n 5120 \bar{y}_j (32 - c_k) \Delta y_k$$

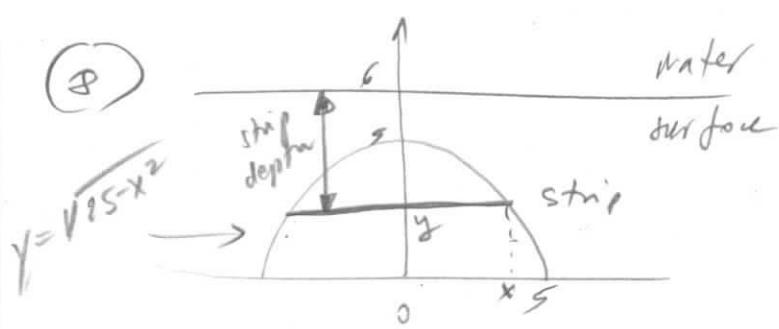
$$W = \int_0^{30} 5120 \bar{y} (32 - y) dy$$

$$= 5120 \bar{y} \left[32y - \frac{y^2}{2} \right]_0^{30}$$

$$= 5120 \bar{y} \cdot 510 = 2,611,200 \bar{y} \text{ ft-lb}$$

$$W \approx 2,611,200 \bar{y} \text{ ft-lb}$$

(8)



$$F = \int_a^b w \left(\frac{\text{strip depth}}{\text{strip height}} \right) \left(\frac{\text{strip length}}{\text{strip width}} \right) dy$$

$$w = \text{weight-density} = 62.4 \text{ lb/ft}^3$$

$$\text{strip depth} = 6 - y$$

$$\text{strip height} = 2x$$

where $x^2 + y^2 = 5^2$

$$x^2 = 25 - y^2$$

$$x = \sqrt{25 - y^2}$$

$$F = \int_0^5 62.4 (6 - y) \cdot 2 \sqrt{25 - y^2} dy$$

$$= 124.8 \int_0^5 (6 - y) \sqrt{25 - y^2} dy$$

$$= 124.8 \left[6 \int_0^5 \sqrt{25 - y^2} dy - \int_0^5 y \sqrt{25 - y^2} dy \right]$$

$$\text{let } j_1 = \int_0^5 \sqrt{25 - y^2} dy$$

= $\frac{1}{4}$ area of the circle
centered at (0, 0), with radius 5

$$= \frac{1}{4} \cdot \pi (5)^2 = \frac{25\pi}{4}$$

$$\text{let } j_2 = \int_0^5 y \sqrt{25 - y^2} dy$$

$$\text{let } \sqrt{25 - y^2} = u$$

$$\frac{-2y}{2\sqrt{25 - y^2}} dy = du$$

$$\therefore y dy = -\sqrt{25-y^2} du$$

$$y dy = -u du$$

when $y=0, u=5$

$y=5, u=0$

$$\begin{aligned} I_2 &= - \int_5^0 u \cdot u du = \int_0^5 u^2 du \\ &= \frac{u^3}{3} \Big|_0^5 = \frac{125}{3} \end{aligned}$$

Therefore,

$$\begin{aligned} F &= 124.8 \left[6 \cdot i, -I_2 \right] \\ &= 124.8 \left(6 \cdot \frac{25\pi}{4} - \frac{125}{3} \right) \\ &= 124.8 \left(\frac{75\pi}{2} - \frac{125}{3} \right) \approx 9502.715 \end{aligned}$$

$$\boxed{F \approx 9502.715}$$