Math 71 Spring 09 Name:

TEST 3 @ 120 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

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1. Solve each equation in $\mathbb{C}$ (the set of complex numbers) by the indicated method.

a) $4(x-3)^2+50=0$ by the square root property.

b) $3y^2-4y+1=0$ by completing the square.

c) $\frac{t^2}{5}-\frac{t}{3}=\frac{2}{3}$ by the quadratic formula.

d) $2x^2+xy+y^2=3$ solve for y in terms of x.

e) $2x^4-3x^2+1=0$ using substitution.

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2. Solve the following inequalities.

a) $x^2-6x+5\le 0$

b) $\frac{1}{x-5}<\frac{3}{2-x}$

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3. Let $f(x)=3x-1$ and $g(x)=\frac{2-x}{x+1}$. Answer the following questions:

a) Find $(g\circ f)(x)$.

b) $(f\circ g)(1)$

c) Find $f^{-1}(x)$.

d) Find $g^{-1}(x)$.

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4. Simplify the following expressions.

a) $3\ln x-5\ln y+2\ln z$

b) $\frac{1}{3}(\log_5 x-\log_5 y)+3\log_5(x+2)$

c) $\log_3 405-\log_3 5+\log 5+\log 2$

d) $\log_{10}(\log_3(\log_5 125))$

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5. a) Graph $f(x)=3^x$ by plotting at least 3 points . Find its domain, range, and asymptote. Label the axes and all the points

b) Graph $g(x)=\log_2 x$ by plotting at least 3 points . Find its domain, range, and asymptote. Label the axes and all the points

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6. For the equation given below, answer all the questions and graph the function (Be sure to label the axes and all points used). SHOW ALL WORK!

$$y = -2x^2 + 3x + 2$$

a) What type of curve is this?
b) What is the y-intercept?
c) What is the vertex
d) What are the x- intercept(s) (if any)?
e) What is the domain of the function?
f) What is the range of the function?
g) <u>Using the graph above</u>, solve the following inequality: $-2x^2 + 3x + 2 > 0$
h) What is the vertex form of the equation?

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7. State whether each statement is TRUE or FALSE. Justify your answer.

a) $\log(a+b) = \log a \cdot \log b$

b) $\log\left(\frac{a}{b}\right) = \frac{\log a}{\log b}$

c) $\log 3x^5 = 5\log 3x$

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8) The owners of a small fruit orchard decide to produce gift baskets as a sideline. The cost per basket for producing x baskets is $C = 0.01x^2 - 2x + 120$. How many baskets should they produce in order to minimize the cost per basket? What will their total cost be at that production level?

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9) India is currently one of the world's fastest-growing countries. By 2040, the population of India will be larger than the population of China; by 2050, nearly one-third of the world's population will live in these two countries alone. The exponential function

$$f(x) = 573(1.027)^x$$

models the population of India, $f(x)$, in millions, x years after 1974.

a) What was India's population in 1974?
b) Find $f(25)$ and its meaning.
c) Find India's population, to the nearest million, in the year 2025 as predicted by this function.

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10) Hortense is investing $2600 in an account where interest is calculated according to the formula $A = P(1+r)^t$ where P is the original principal, r is the interest rate and t is the time measured in years. If Hortense wants her money to grow to double in two years, what interest rate must the account have? (Approximate the answer to the nearest hundredth of a percent)

MATH 71

TEST 3 – SOLUTIONS

(1) (a) $4(x-3)^2+50=0$

$4(x-3)^2=-50$

$(x-3)^2=\frac{-50}{4}$ $\quad /\sqrt{\ }$

$\sqrt{(x-3)^2}=\sqrt{\frac{-50}{4}}$

$x-3=\pm\frac{\sqrt{-50}}{\sqrt{4}}$

$\boxed{x=3\pm\frac{5i\sqrt{2}}{2}}$

(b) $3y^2-4y+1=0 \quad /\div 3$

1st leading coefficient = 1

$y^2-\frac{4}{3}y+\frac{1}{3}=0$

2nd isolate constant

$y^2-\frac{4}{3}y=\frac{-1}{3}$

3rd find missing term

$\left(\frac{1}{2}\text{coef. }y\right)^2=\left(\frac{1}{2}\cdot\frac{-4}{3}\right)^2=\left(\frac{2}{3}\right)^2=\frac{4}{9}$

$y^2-\frac{4}{3}y+\frac{4}{9}=\frac{-1}{3}+\frac{4}{9}$

$\left(y-\frac{2}{3}\right)^2=\frac{-3+4}{9}$

$\left(y-\frac{2}{3}\right)^2=\frac{1}{9}$ $\quad /\sqrt{\ }$

$\sqrt{\left(y-\frac{2}{3}\right)^2}=\sqrt{\frac{1}{9}}$

$y-\frac{2}{3}=\pm\frac{1}{3}$

$y=\frac{2}{3}\pm\frac{1}{3}$ $\quad y=\frac{2}{3}+\frac{1}{3}=1$ OR $y=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$

$\boxed{y\in\left\{1,\frac{1}{3}\right\}}$

(c) $\frac{t^2}{5}-\frac{t}{3}=\frac{2}{3}$

$\frac{t^2}{5}-\frac{t}{3}-\frac{2}{3}=0$

LCD = 15

$3t^2-5t-10=0$

$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=3,\ b=-5,\ c=-10$

$t=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-10)}}{2(3)}$

$t=\frac{5\pm\sqrt{25+120}}{6}$

$\boxed{t=\frac{5\pm\sqrt{145}}{6}}$

(d) $2x^2+xy+y^2=3$

solve for y

$y^2+xy+2x^2-3=0$

$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=1,\ b=x,\ c=2x^2-3$

$y=\frac{-x\pm\sqrt{x^2-4(1)(2x^2-3)}}{2(1)}$

$y=\frac{-x\pm\sqrt{x^2-8x^2+12}}{2}$

$\boxed{y=\frac{-x\pm\sqrt{12-7x^2}}{2}}$

(e) $2x^4 - 3x^2 + 1 = 0$

let $x^2 = t$

$2t^2 - 3t + 1 = 0$

$(2t - 1)(t - 1) = 0$

$t = \frac{1}{2}$ or $t = 1$

$x^2 = \frac{1}{2}$ $\qquad$ $x^2 = 1$

$\sqrt{x^2} = \sqrt{\frac{1}{2}}$ $\qquad$ $\sqrt{x^2} = \sqrt{1}$

$x = \pm \frac{1}{\sqrt{2}}$ $\qquad$ $x = \pm 1$

$x = \pm \frac{\sqrt{2}}{2}$

$\boxed{x \in \{\pm \frac{\sqrt{2}}{2}, \pm 1\}}$

(2) (a) $x^2 - 6x + 5 \leq 0$

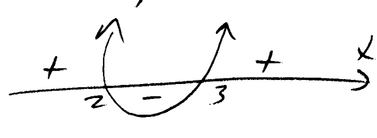
let $y = x^2 - 6x + 5$ parabola that opens upward

x-I:

$x^2 - 6x + 5 = 0$

$(x-2)(x-3) = 0$

$x = 2$ or $x = 3$



so $x^2 - 6x + 5 \leq 0$ when

$\boxed{x \in [2, 3]}$

(b) $\frac{1}{x-5} < \frac{3}{2-x}$

$\frac{1}{x-5} - \frac{3}{2-x} < 0$

LCD $= (x-5)(2-x)$

$\frac{2 - x - 3(x-5)}{(x-5)(2-x)} < 0$

$\frac{2 - x - 3x + 15}{(x-5)(2-x)} < 0$

$\frac{17 - 4x}{(x-5)(2-x)} < 0$

We will study the sign of each factor

x	$-\infty$		2		$\frac{17}{4}$		5		∞
$17-4x$	+	+	+	+	0	−	−	−	−
$x-5$	−	−	−	−	−	−	0	+	+
$2-x$	+	+	0	−	−	−	−	−	−
$\frac{17-4x}{(x-5)(2-x)}$		−	|	+	0	−	|	+	

so $\frac{1}{x-5} < \frac{3}{2-x}$ when

$\boxed{x \in (-\infty, 2) \cup (\frac{17}{4}, 5)}$

(3) $f(x) = 3x - 1$

$g(x) = \dfrac{2-x}{x+1}$

(a) $(g \circ f)(x) = g(f(x))$

$= g(3x-1)$

$= \dfrac{2-(3x-1)}{(3x-1)+1}$

$= \dfrac{2-3x+1}{3x}$

$= \dfrac{3-3x}{3x} = \dfrac{\cancel{3}(1-x)}{\cancel{3}x}$

so $\boxed{(g \circ f)(x) = \dfrac{1-x}{x}}$

(b) $(f \circ g)(1) = f(g(1))$

but $g(1) = \dfrac{2-1}{1+1} = \dfrac{1}{2}$

so $(f \circ g)(1) = f(g(1))$

$= f\left(\frac{1}{2}\right)$

$= 3 \cdot \frac{1}{2} - 1$

$= \frac{1}{2}$

so $\boxed{(f \circ g)(1) = \frac{1}{2}}$

(c) $f(x) = 3x - 1$

1st $y = 3x - 1$

2nd solve for x

$y + 1 = 3x \Rightarrow x = \dfrac{y+1}{3}$

3rd $x \leftrightarrow y$

$y = \dfrac{x+1}{3}$

so $\boxed{f^{-1}(x) = \dfrac{x+1}{3}}$

(d) $g(x) = \dfrac{2-x}{x+1}$

1st $y = \dfrac{2-x}{x+1}$

2nd solve for x

$y(x+1) = 2 - x$

$yx + y = 2 - x$

$yx + x = 2 - y$

$x(y+1) = 2 - y$

$x = \dfrac{2-y}{y+1}$

3rd $x \rightarrow y$

$y = \dfrac{2-x}{x+1}$

so $\boxed{g^{-1}(x) = \dfrac{2-x}{x+1}}$

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(4) (a) $3\ln x - 5\ln y + 2\ln z =$

$= \ln x^3 - \ln y^5 + \ln z^2$

$= \ln \dfrac{x^3}{y^5} + \ln z^2$

$= \boxed{\ln\left(\dfrac{x^3 z^2}{y^5}\right)}$

(b) $\frac{1}{3}(\log_5 x - \log_5 y) + 3\log_5(x+2) =$

$= \frac{1}{3}\left(\log_5 \frac{x}{y}\right) + \log_5 (x+2)^3$

$= \log_5 \left(\frac{x}{y}\right)^{\frac{1}{3}} + \log_5 (x+2)^3$

$= \log_5 \left(\frac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}} \cdot (x+2)^3\right)$

$= \boxed{\log_5 \left(\frac{\sqrt[3]{x}\,(x+2)^3}{\sqrt[3]{y}}\right)}$

(c) $\log_3 405 - \log_3 5 + \log 5 + \log 2$

$= \log_3 \left(\frac{405}{5}\right) + \log (5)\cdot 2$

$= \log_3 81 + \log 10$

$= 4 + 1 = \boxed{5}$

(d) $\log_{10}(\log_3(\log_5 125)) =$

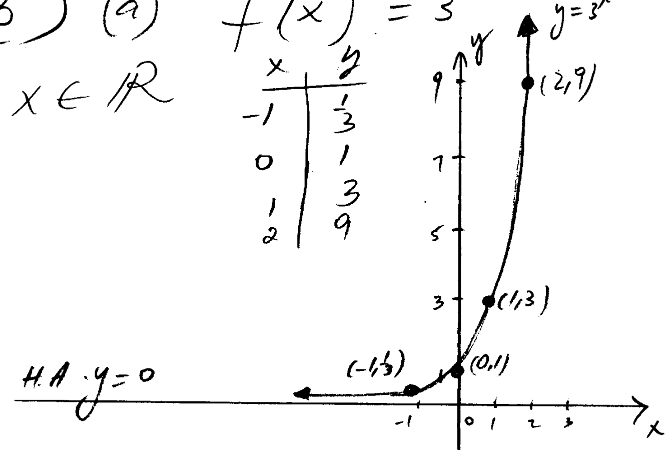
$= \log_{10}(\log_3 3)$

$= \log_{10} 1 = \boxed{0}$

(5) (a) $f(x) = 3^x$

$x \in \mathbb{R}$

x	y
-1	$\frac{1}{3}$
0	1
1	3
2	9



Domain: $x \in \mathbb{R}$

Range $y \in (0,\infty)$

H.A. $y=0$

(b) $g(x) = \log_2 x$

$x > 0$

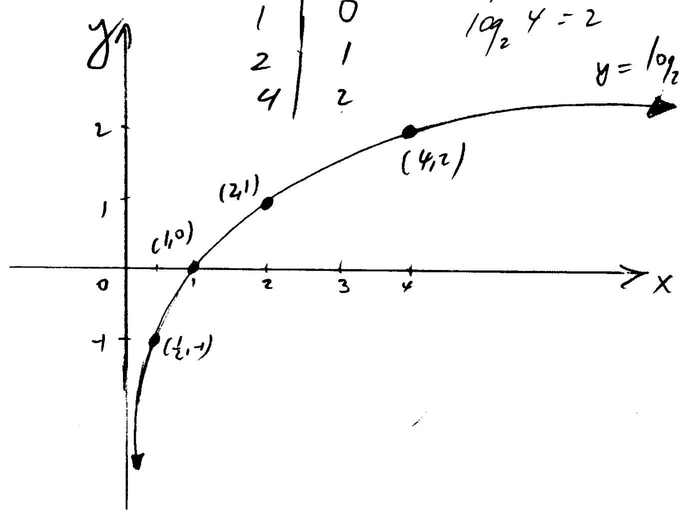
x	y
$\frac{1}{2}$	-1
1	0
2	1
4	2

$\log_2 \frac{1}{2} = -1$

$\log_2 1 = 0$

$\log_2 2 = 1$

$\log_2 4 = 2$



V.A. x=0

Domain $x \in (0,\infty)$

Range $y \in \mathbb{R}$

V.A. $x=0$

(6) $y = -2x^2 + 3x + 2$

(a) parabola that opens downward $(a = -2 < 0)$

(b) y-n: let $x = 0$, then $y = 2$

$\boxed{(0, 2)}$

(c) Vertex

$x_v = \frac{-b}{2a} = \frac{-3}{2(-2)} = \frac{3}{4}$

$y_v = -2 \cdot \left(\frac{3}{4}\right)^2 + 3 \cdot \frac{3}{4} + 2$

$y_v = -\frac{9}{8} + \frac{9}{4} + 2$

$= +\frac{9}{8} + 2 = \frac{25}{8}$

$\boxed{V\left(\frac{3}{4}, \frac{25}{8}\right)}$

(d) x-n: let $y = 0$, then

$-2x^2 + 3x + 2 = 0 \quad | (-)$

$2x^2 - 3x - 2 = 0$

$(2x + 1)(x - 2) = 0$

$2x + 1 = 0$ OR $x - 2 = 0$

$x = \frac{-1}{2}$ $\quad x = 2$

$\boxed{\text{x-n: } \left(-\tfrac{1}{2}, 0\right) \text{ and } (2, 0)}$

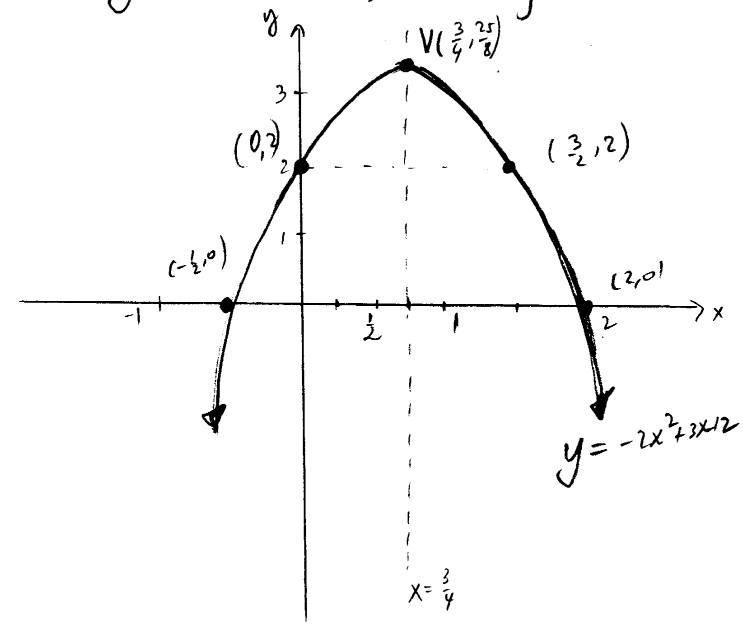
(e) Domain: $x \in \mathbb{R}$

(f) Range: $y \in \left(-\infty, \frac{25}{8}\right]$

(g) $-2x^2 + 3x + 2 > 0$ when $x \in \left(-\frac{1}{2}, 2\right)$

(h) $y = a(x - x_v)^2 + y_v$

$y = -2\left(x - \frac{3}{4}\right)^2 + \frac{25}{8}$



(7) (a) False

$\log(a+b) \neq \log a \log b$

For example,

$\log(10+10) = \log 20$

while

$\log 10 \cdot \log 10 = 1$

and $\log 20 \neq 1$

(b) False

$\log\left(\frac{a}{b}\right) = \log a - \log b$

(c) False

$\log 3x^5 = \log 3 + 5 \log x$

(8) $C = 0.01x^2 - 2x + 120$
represents a parabola
opening upward
Therefore, the minimum
occurs at the vertex
$V(x_v, C_v)$

$x_v = \frac{-b}{2a} = \frac{-(-2)}{2(0.01)} = $ 100 baskets

$C_v = 0.01(100)^2 - 2(100) + 120$
$C_v = 100 - 200 + 120 = 20$ \$/basket

They should produce
100 baskets in order to
minimize the cost per
basket

The total cost at that
production level will be
$C = 100$ baskets $\cdot$ (20 \$/basket)
$C_{total} = 200$ \$

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(9) $f(x) = 573(1.027)^x$
x = years after 1974
$f(x)$ = population (in millions)

a) 1974 : $x = 0$
$f(0) = 573$ million people
(population in 1974)

b) $f(25) = 573(1.027)^{25}$
$f(25) = 1115.36$ million people

25 years from 1974 – in 1999 –
India's population was
1115.36 million people.

(c) 2025:
$2025 - 1974 = 51$, so $x = 51$
$f(51) = 573(1.027)^{51}$
$f(51) = 2229.7$ million
In 2025 the population
will be about 2230 million
people.

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(10) $A = P(1+r)^t$
$P = 2600$ \$
$A = 2(2600) = 5200$ \$
$t = 2$
$r = ?$

$5200 = 2600(1+r)^2$
$(1+r)^2 = \frac{5200}{2600}$
$(1+r)^2 = 2 \quad /\sqrt{\ }$

$\begin{cases} 1+r = \sqrt{2} & \text{or} \\ 1+r = -\sqrt{2} \end{cases}$

$r = -1 + \sqrt{2}$ or
$r = -1 - \sqrt{2}$ (not possible)
(r must be positive)

so $r \simeq 0.4142$
The interest should
be 41.42 %