Math 71 Spring 09 Name:

# TEST 2 @ 120 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Factor each expression completely:

a) $x^5 - x^3 + 27x^2 - 27$

b) $216x - x^4$

c) $10a^2 + 19a + 6$

2. Do the following operations (simplify):

a) $\dfrac{1}{x^2 + x - 2} - \dfrac{3}{2x^2 + 3x - 2} + \dfrac{2}{2x^2 - 3x + 1}$

b) $\dfrac{\frac{1}{x} + \frac{y}{x^2}}{\frac{1}{y} + \frac{x}{y^2}}$

c) $\left(4 - \dfrac{3}{x+2}\right)\left(1 + \dfrac{5}{x-1}\right)$

d) $\left(\dfrac{x^{\frac{-5}{4}} y^{\frac{1}{3}}}{x^{-\frac{3}{4}}}\right)^{-6}$ and write the final answer using only positive exponents

e) $2\sqrt{45} - 3\sqrt{20} - \left(\sqrt{5} - 3\right)\left(\sqrt{5} + 3\right)$

f) $\dfrac{2 - 5i}{1 + 2i}$

g) $\dfrac{-14 + \sqrt{-128}}{16}$

h) $\left(\sqrt{x} + \sqrt{x - 3}\right)^2$

3. If $f(x)=x^2+3x-2$, find $\dfrac{f(a+h)-f(a)}{h}$.

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4. If $f(x)=\dfrac{5}{x-4}$ , $g(x)=\dfrac{3}{x-3}$, and $h(x)=\dfrac{x^2-20}{x^2-7x+12}$ find all the values of $a$ for which $(f+g)(a)=h(a)$.

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5. $f(x)=-x^2+2x+3$ Find the following and simplify:

a) $f(i)$

b) $f\left(3-\sqrt{2}\right)$

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6. Let $f(x)=\sqrt{x+2}$.

a) What is the domain of this function?

b) Sketch the graph of the function by plotting points. Label the axes and all the points used.

c) What is the range of this function.

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7. Solve the following equations:

a) $x^3-4x^2-x+4=0$

b) $(5x+4)(x-1)=2$

c) $\sqrt{x+16}-\sqrt{x}-2=0$

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8. Find the perimeter and area of a rectangle whose width is $3\sqrt{28}$ feet and whose length is $\sqrt{63}$ feet. Simplify.

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9. The height , *h*, of a baseball *t* seconds after being hit is given by $h=-16t^2+64t+4$. When will the baseball reach a height of 64?

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10. Do the following division using long division and relate dividend, division, quotient, and remainder.

$$\frac{3x^5-x^3+4x^2-12x-8}{x^2-2}$$

11. The Food Stamp Program is America's first line of defense against hunger for millions of families. Over half of all participants are children; one out of six is a low-income older adult. The next questions involve the number of participants in the program from 1990 through 2002.

The function $f(x) = -\frac{1}{4}x^2 + 3x + 17$ models the number of people, $f(x)$, in millions, receiving food stamps $x$ years after 1990.

a) In which year did 26 million people receive food stamps?
b) How many people received food stamps in 1995?

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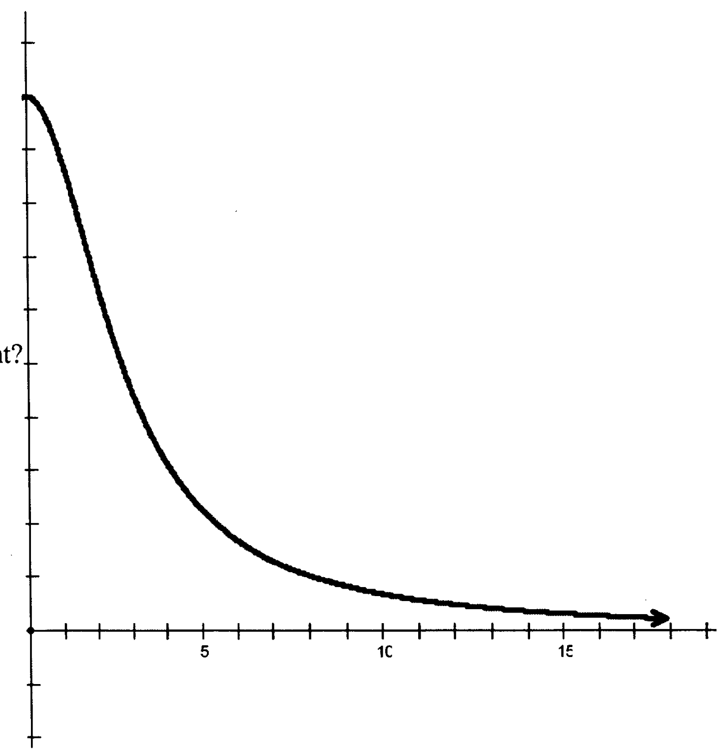
12 The rational function $P(x) = \frac{72,900}{100x^2 + 729}$ models the percentage of people in the U.S., $P(x)$, with $x$ years of education who are unemployed. The graph of the function is shown below. Answer the following:

a) Identify the independent and dependent variables and label the axes accordingly.

b) What is the domain of the function? What is the range?

c) Find and interpret $P(10)$. Identify the point on the graph and label it.

d) Describe the end behavior of the graph. Is there an education level that leads to guaranteed employment? How is this indicated by the graph?



MATH 71

# TEST 2 - SOLUTIONS

(1) (a) $x^5 - x^3 + 27x^2 - 27 =$

$= x^3(x^2-1) + 27(x^2-1)$

$= (x^2-1)(x^3+27)$

$= (x-1)(x+1)(x+3)(x^2-3x+9)$

(b) $216x - x^4 =$

$= x(216 - x^3)$

$= x(6^3 - x^3)$

$= x(6-x)(36+6x+x^2)$

(c) $10a^2 + 19a + 6 =$

$= 10a^2 + 15a + 4a + 6$

product $= 60 \langle {15 \atop 4}$

sum $= 19$

$= 5a(2a+3) + 2(2a+3)$

$= (2a+3)(5a+2)$

(b) $\dfrac{\frac{1}{x} + \frac{y}{x^2}}{\frac{1}{y} + \frac{x}{y^2}} = \dfrac{\frac{x+y}{x^2}}{\frac{y+x}{y^2}}$

$= \dfrac{x+y}{x^2} \div \dfrac{x+y}{y^2}$

$= \dfrac{x+y}{x^2} \cdot \dfrac{y^2}{x+y} = \dfrac{y^2}{x^2}$

(c) $\left(4 - \dfrac{3}{x+2}\right)\left(1 + \dfrac{5}{x-1}\right) =$

$= \dfrac{4(x+2)-3}{x+2} \cdot \dfrac{x-1+5}{x-1}$

$= \dfrac{4x+8-3}{x+2} \cdot \dfrac{x+4}{x-1}$

$= \dfrac{4x+5}{x+2} \cdot \dfrac{x+4}{x-1}$

$= \dfrac{(4x+5)(x+4)}{(x+2)(x-1)}$

(2)

(a) $\dfrac{1}{x^2+x-2} - \dfrac{3}{2x^2+3x-2} + \dfrac{2}{2x^2-3x+1} =$

$= \dfrac{\overset{2x-1}{1}}{(x+2)(x-1)} - \dfrac{\overset{x-1}{3}}{(2x-1)(x+2)} + \dfrac{\overset{x+2}{2}}{(2x-1)(x-1)}$

LCD $= (x+2)(x-1)(2x-1)$

$= \dfrac{2x-1 - 3(x-1) + 2(x+2)}{(x+2)(x-1)(2x-1)}$

$= \dfrac{2x-1-3x+3+2x+4}{(x+2)(x-1)(2x-1)}$

$= \dfrac{x+6}{(x+2)(x-1)(2x-1)}$

(d) $\left(\dfrac{x^{-\frac{5}{4}}\, y^{\frac{1}{3}}}{x^{-\frac{3}{4}}}\right)^{-6} = \left(x^{-\frac{5}{4}+\frac{3}{4}}\, y^{\frac{1}{3}}\right)^{-6}$

$= \left(x^{-\frac{1}{2}}\, y^{\frac{1}{3}}\right)^{-6}$

$= \left(x^{-\frac{1}{2}}\right)^{-6}\left(y^{\frac{1}{3}}\right)^{-6} = x^3 y^{-2}$

$= \dfrac{x^3}{y^2}$

(e)
$$2\sqrt{45} - 3\sqrt{20} - (\sqrt{5}-3)(\sqrt{5}+3) =$$
$$= 2\sqrt{9\cdot 5} - 3\sqrt{4\cdot 5} - \left((\sqrt{5})^2 - 3^2\right)$$
$$= 2\cdot 3\sqrt{5} - 3\cdot 2\sqrt{5} - (5-9)$$
$$= 6\sqrt{5} - 6\sqrt{5} - (-4) = 4$$

(h)
$$\left(\sqrt{x} + \sqrt{x-3}\right)^2 =$$
$$= (\sqrt{x})^2 + 2\sqrt{x}\cdot\sqrt{x-3} + (\sqrt{x-3})^2$$
$$= x + 2\sqrt{x(x-3)} + x - 3$$
$$= 2x + 2\sqrt{x(x-3)} - 3$$

(f)
$$\frac{2-5i}{1+2i} = \frac{(2-5i)(1-2i)}{(1+2i)(1-2i)}$$
$$= \frac{2-4i-5i+10i^2}{1^2-(2i)^2}$$
$$= \frac{2-9i+10(-1)}{1-4i^2}$$
$$= \frac{-8-9i}{1-4(-1)} = \frac{-8-9i}{5}$$

(g)
$$\frac{-14+\sqrt{-128}}{16} =$$

$$2\,|\,128 \quad 64$$

$$= \frac{-14+\sqrt{(-1)2\cdot 64}}{16}$$
$$= \frac{-14+8i\sqrt{2}}{16}$$
$$= \frac{2(-7+4i\sqrt{2})}{16}$$
$$= \frac{-7+4i\sqrt{2}}{8}$$

(3) $f(x) = x^2 + 3x - 2$
$$\frac{f(a+h) - f(a)}{h} =$$
$$\frac{\left((a+h)^2 + 3(a+h) - 2\right) - (a^2+3a-2)}{h} =$$
$$= \frac{\cancel{a^2}+2ah+h^2+\cancel{3a}+3h\cancel{-2}\cancel{-a^2}\cancel{-3a}\cancel{+2}}{h}$$
$$= \frac{2ah+h^2+3h}{h}$$
$$= \frac{h(2a+h+3)}{h} = 2a+h+3$$

(5) $f(x) = -x^2 + 2x + 3$

(a)
$$f(i) = -(i)^2 + 2i + 3$$
$$= -(-1) + 2i + 3$$
$$= 4 + 2i$$

(b)
$$f(3-\sqrt{2}) = -(3-\sqrt{2})^2 + 2(3-\sqrt{2}) + 3$$
$$= -(9 - 6\sqrt{2} + 2) + 6 - 2\sqrt{2} + 3$$
$$= \cancel{-9} + 6\sqrt{2} - 2 \cancel{+9} - 2\sqrt{2}$$
$$= -2 + 4\sqrt{2}$$

(5) $(f+g)(a) = h(a)$

$f(a) + g(a) = h(a)$

$$\frac{5}{a-4} + \frac{3}{a-3} = \frac{a^2-20}{a^2-7a+12}$$

$$\frac{\overset{a-3}{5}}{a-4} + \frac{\overset{a-4}{3}}{a-3} = \frac{a^2-20}{(a-3)(a-4)}$$

conditions: $\begin{cases} a \neq 4 \\ a \neq 3 \end{cases}$

$LCD = (a-3)(a-4)$

$5(a-3) + 3(a-4) = a^2-20$

$5a - 15 + 3a - 12 = a^2 - 20$

$8a - 27 = a^2 - 20$

$a^2 - 20 - 8a + 27 = 0$

$a^2 - 8a + 7 = 0$

$(a-1)(a-7) = 0$

$a-1=0$ OR $a-7=0$

$a=1$ $\quad a=7$

Therefore, $a \in \{1, 7\}$

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(6) $f(x) = \sqrt{x+2}$

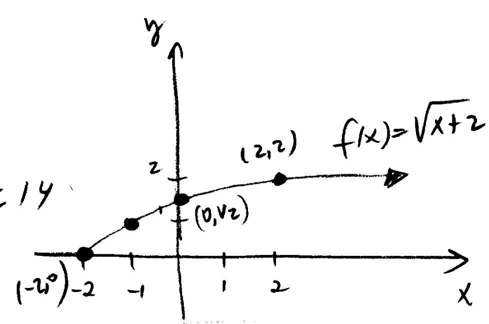
(a) Condition: $x+2 \geq 0$

$x \geq -2$

Domain: $x \in [-2, \infty)$

(b)

| $x$ | $y$ |
|---|---|
| $-2$ | $0$ |
| $-1$ | $1$ |
| $0$ | $\sqrt{2} \approx 1.4$ |
| $2$ | $2$ |




(c) Range: $y \in [0, \infty)$

(7) (a) $x^3 - 4x^2 - x + 4 = 0$

$x^2(x-4) - (x-4) = 0$

$(x-4)(x^2-1) = 0$

$(x-4)(x-1)(x+1) = 0$

$x-4=0 \Rightarrow x=4$

OR

$x-1=0 \Rightarrow x=1$

OR

$x+1=0 \Rightarrow x=-1$

so $x \in \{4, -1, 1\}$

(b) $(5x+4)(x-1) = 2$

$5x^2 - 5x + 4x - 4 - 2 = 0$

$5x^2 - x - 6 = 0$

product $= -30 < \begin{smallmatrix} +5 \\ -6 \end{smallmatrix}$

sum $= -1$

$30 = 6 \cdot 5$

$5x^2 + 5x - 6x - 6 = 0$

$5x(x+1) - 6(x+1) = 0$

$(x+1)(5x-6) = 0$

$x+1=0$ OR $5x-6=0$

$x=-1$ $\quad x = \frac{6}{5}$

so $x \in \{-1, \frac{6}{5}\}$

(c) $\sqrt{x+16} - \sqrt{x} - 2 = 0$

$\sqrt{x+16} = 2 + \sqrt{x} \quad |^2$

$(\sqrt{x+16})^2 = (2+\sqrt{x})^2$

$\cancel{x} + 16 = 4 + 4\sqrt{x} + \cancel{x}$

$16 - 4 = 4\sqrt{x}$

$12 = 4\sqrt{x} \quad | \div 4$

$3 = \sqrt{x}$

$3 = \sqrt{x} \quad |^2$

$9 = x$

check:

$\sqrt{9+16} - \sqrt{9} - 2 \stackrel{?}{=} 0$

$5 - 3 - 2 = 0$ TRUE

so, $x \in \{9\}$

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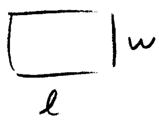
(8) 


let $l$ = length
$w$ = width

$w = 3\sqrt{28} = 3\sqrt{7 \cdot 4} = 6\sqrt{7}$ ft

$l = \sqrt{63} = \sqrt{9 \cdot 7} = 3\sqrt{7}$ ft

Let $P$ = perimeter

$P = 2l + 2w$

$= 2 \cdot 3\sqrt{7} + 2 \cdot 6\sqrt{7}$

$= 6\sqrt{7} + 12\sqrt{7}$

$P = 18\sqrt{7}$ ft

Let $A$ = area

$A = lw$

$A = (3\sqrt{7})(6\sqrt{7})$

$A = 18 \cdot 7$ $\quad A = 126 \text{ ft}^2$

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(9) $t$ = time (seconds)
$h$ = height (ft)

$t = ?$ if $h = 64$ ft

$64 = -16t^2 + 64t + 4$

$16t^2 - 64t + 64 - 4 = 0$

$16t^2 - 64t + 60 = 0 \quad |\div 4$

$4t^2 - 16t + 15 = 0$

$4t^2 - 10t - 6t + 15 = 0$

$2t(2t-5) - 3(2t-5) = 0$

$(2t-5)(2t-3) = 0$

$2t - 5 = 0$ or $2t - 3 = 0$

$t = \frac{5}{2}$ sec $\quad t = \frac{3}{2}$ sec.

The ball will reach 64 ft after 1.5 seconds and after 2.5 seconds.

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(10)

$$\begin{array}{r} 3x^3 + 5x + 4 \\ x^2 - 2 \enclose{longdiv}{3x^5 - x^3 + 4x^2 - 12x - 8} \\ -3x^5 + 6x^3 \\ \hline 5x^3 + 4x^2 - 12x - 8 \\ -5x^3 + 10x \\ \hline 4x^2 - 2x - 8 \\ -4x^2 + 8 \\ \hline -2x \end{array}$$

$$\frac{3x^5 - x^3 + 4x^2 - 12x - 8}{x^2 - 2} =$$

$$= 3x^3 + 5x + 4 - \frac{2x}{x^2 - 2}$$

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(11) $f(x) = -\frac{1}{4}x^2 + 3x + 17$

$x$ = number of years after 1990

$f(x)$ = number of people (in million)

(a) $x = ?$ if $f(x) = 26$

$-\frac{1}{4}x^2 + 3x + 17 = 26$

$\frac{1}{4}x^2 - 3x - 17 + 26 = 0$

$\frac{1}{4}x^2 - 3x + 9 = 0 \quad |\cdot 4$

$x^2 - 12x + 36 = 0$

$x^2 - 12x + 36 = 0$

product $= 36$ $<$ $-6$, $-6$

sum $= -12$

$36 = 6 \cdot 6$

$(x-6)^2 = 0$ so $x = 6$ years after 1990

Therefore, in 1996, 26 million people received food stamps.

(b) 1995: $x = 5$ so

$f(5) = \frac{-1}{4}(5)^2 + 3(5) + 17$

$f(5) = \frac{-25}{4} + 32$

$f(5) = \frac{103}{4} = 25.75$ million people

Therefore, in 1995, there were 25.75 million people who received food stamps.

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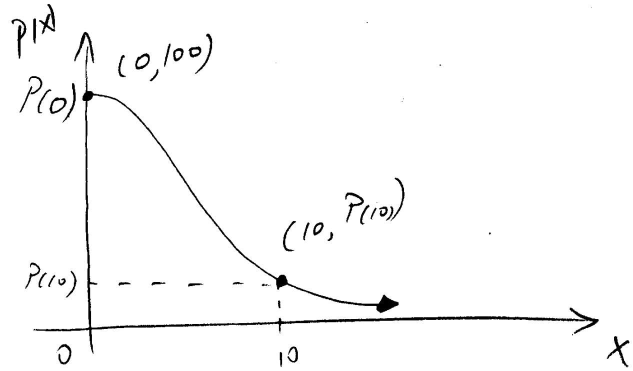
(12) $P(x) = \dfrac{72{,}900}{100x^2 + 729}$

$x$ = number of years of education

$P(x)$ = percentage of people in the USA who are unemployed

(a) $x$ = independent variable

$P(x)$ = dependent variable




(b) Domain: $x \in [0, \infty)$

Range: $y \in (0, P(0)]$

where $P(0) = \dfrac{72{,}900}{729} = 100$

so $y \in (0, 100]$

(c) $P(10) = \dfrac{72{,}900}{100(10)^2 + 729}$

$P(10) = \dfrac{72{,}900}{10{,}729} \approx 6.8$

6.8% of people in the U.S with 10 years of educations are unemployed

(d) As $x \to \infty$, $P(x) \to 0$

The graph has a horizontal asymptote $P = 0$.

There is no education level that guarantees employment

$P(x) \to 0$, but $P(x) \neq 0$