Math 71 Spring 2009 Name:

TEST 1 @ 120 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

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1. Solve the following equations:

a) $\frac{5}{3}(t-1)=\frac{4}{5}(2t+1)+\frac{2}{3}$

c) $\frac{1}{5}=\frac{2t-3}{6}$

b) $50\%(x)+20\%(90-x)=30$

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2. The linear function $f(x)=2x+10$ models the amount, $f(x)$, in billions of dollars, that the drug industry spent on marketing information about drugs x years after 2000.

a) Find the slope of the model. What are the units of the slope?

b) What is the meaning of the slope in the context of this problem?

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3. Let $\frac{3}{2}x-2y=1$ a linear equation in two variables.

a) Graph the equation by the intercepts method. Clearly label the axes and the intercepts.

b) Find the slope of the line.

c) Find an equation for the line perpendicular to the given line and passing through $(-4,5)$.

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4. Four functions are given:

$l(x)=\frac{2x+11}{x-5}$; $g(x)=\sqrt{2+x}$; $h(x)=x^2-3x+2$; $f(x)=3x-7$

Find the following:

a) The domain of each function.

b) $g(3t)$

c) $(h+f)(x)$

d) $(h-f)(-1)$

e) $h(x-1)$

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5. If $f(x)=4x-1$, find $\frac{f(a+h)-f(a)}{h}$.

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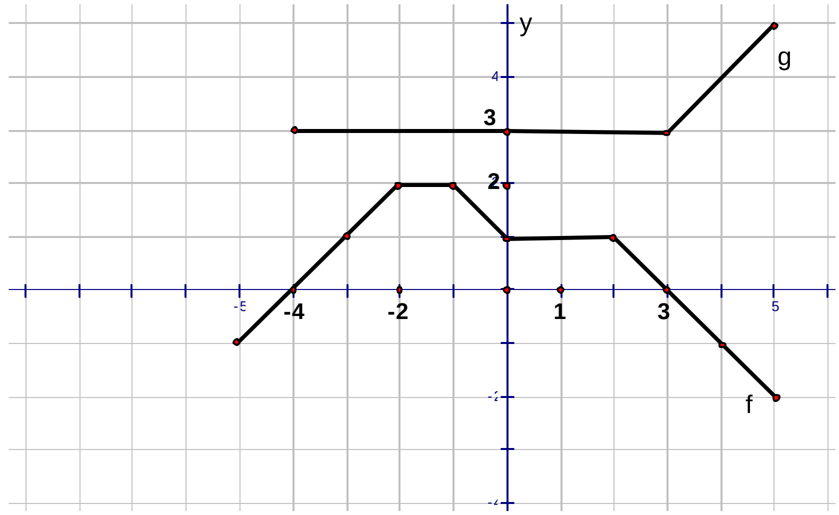
6. Let $f(x)=\begin{cases} 2x+5, & \text{if } x\le 0 \\ -3, & \text{if } 0<x\le 6 \\ x-10, & \text{if } x>6 \end{cases}$ be a piece – wise defined function. Answer the following:

a) What is the domain of the function?

b) Find $f\left(-\frac{1}{2}\right)$, $f(0)$, $f(2)$, $f(7)$, $f(10)$.

7.
Use the graphs of *f* and *g* to answer the following:



a) Are *f* and *g* functions? Why?

b) State the domain and range of f.

c) State the domain and range of *g*.

d) $(f+g)(-4)$

e) $(fg)(3)$

f) $\left(\frac{f}{g}\right)(1)$

g) Solve $f(x)=0$. What do the solutions of this equation represent for the graph of *f* ?

h) Find $g(0)$. What does this represent for the graph of *g?*

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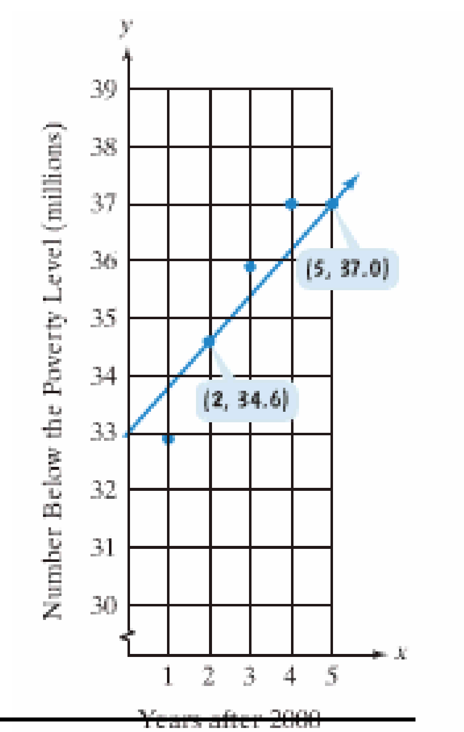
8. The scatter plot shows the number of Americans, in millions, living below the poverty level from 2001 through 2005. Also shown is a line that passes through or near the points.
Answer the following:



a) What are the variables? Which one is independent and which one is dependent?

b) What is the meaning of the point (2, 34.6) ?

c) According to the graph, how many millions of Americans were living below the poverty level in 2001?

c) Find the slope of the line (using the two given points), including units.

d) What is the meaning of the slope?

e) Find an equation of the line that models the number of Americans, *y*, in millions, living below the poverty level *x* years after 2000.

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9. Solve the following system of equation:

$$\begin{cases} 2x-4y+3z=17 \\ x+2y-z=0 \\ 4x-y-z=6 \end{cases}$$

10 & 11

<u>**Choose TWO**</u> of the following word problems. Show clearly what your variables represent. Show clearly the equation(s) you use to solve each problem. You may solve **one problem for extra credit.**

A. Let $s(t) = 11t^2 + t + 100$ be the position, in miles, of a car driving on a straight road at time *t*, in hours.

The card's velocity at any time *t* is given by $v(t) = 22t + 1$.

a) Use function notation to express the car's position after 2 hours. Where is the car then?
b) Use function notation to express the question , "When is the car going 65 mph?" Then find when the velocity of the car was 65 mph.
c) Where is the car when it is going 67 mph?

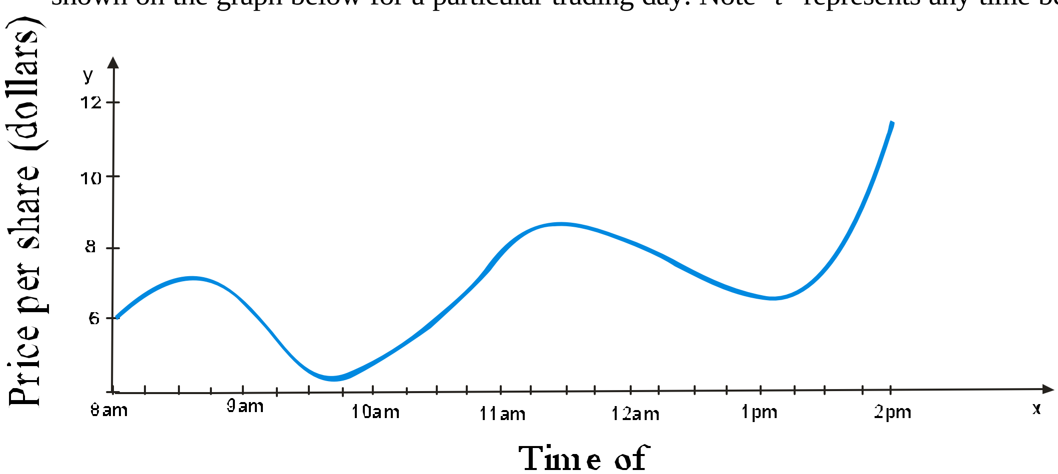
B. The function $A(x) = 0.04x + 5.1$ models the number of women, $A(x)$, in millions, enrolled in U.S. colleges x years after 1980.

The function $B(x) = 0.03x + 3.8$ models the number of men, $B(x)$, in millions, enrolled in U.S. colleges x years after 1980. Use these functions to answer the following questions:

a) Find and interpret $A(2)$.
b) Find the rate of increase of the number of women, in millions, enrolled in U.S. colleges per year. Explain how you have found the answer.
c) Find and interpret $A(10) - B(10)$.
d) Was there a time after 1980 when the number of women enrolled in U.S. colleges was equal to the number of men enrolled in U.S. colleges?

C. A person invested \$6700 for one year , part at 8%, part at 10%, and the remainder at 12%. The total annual income from these investments was \$716. The amount of money invested at 12% was \$300 more than the amounts invested at 8% and 10% combined. Find the amount invested at each rate.

D. The value of a stock varies during the course of any trading day. The price per share "*P*" of a certain stock is shown on the graph below for a particular trading day. Note "*t*" represents any time between 8 am and 2 pm.



a) Is "*P*" a function of "*t*"? Explain using the definition of function.
Using the graph, estimate the answers to the following questions (Use correct units).
b) What is the domain? What is the range?
c) For what value(s) of "*t*" does *P*(t)=8 and what does it mean in practical terms?
d) What is *P*(11) and what does it mean in practical terms?
e) For what value(s) of "*t*" is *P*(t) >5.50?

M71

TEST 1 – SOLUTIONS

(1) (a) $\overset{5/}{\frac{5}{3}}(t-1) = \overset{3/}{\frac{4}{5}}(2t+1) + \overset{5/}{\frac{2}{3}}$

LCD = 15

$25(t-1) = 12(2t+1) + 10$

$25t - 25 = 24t + 12 + 10$

$25t - 25 = 24t + 22$

$25t - 24t = 22 + 25$

$\boxed{t = 47}$

(b) $50\%(x) + 20\%(90-x) = 30$

$\frac{50}{100}x + \frac{20}{100}(90-x) = 30 \quad \Big/ \cdot 100$

$50x + 20(90-x) = 3000$

$50x + 1800 - 20x = 3000$

$30x + 1800 = 3000$

$30x = 3000 - 1800$

$30x = 1200$

$x = \frac{120\cancel{0}}{3\cancel{0}} = \frac{120}{3} = 40$

$\boxed{x = 40}$

(c) $\frac{1}{5} = \frac{2t-3}{6}$

$1 \cdot 6 = 5(2t-3)$

$6 = 10t - 15$

$6 + 15 = 10t$

$21 = 10t$

$\boxed{t = \frac{21}{10}}$

(2) $f(x) = 2x + 10$

x = number of years after 2000

f(x) = cost (billions of \$) of marketing

a) m = 2 billion dollars/year

b) The amount that the drug industry spent on marketing information about drugs increased at a rate of 2 billion dollars per year.

(3) $\frac{3}{2}x - 2y = 1$

(a)

x	y
0	$-\frac{1}{2}$
$\frac{2}{3}$	0

$(0, -\frac{1}{2})$ y-in

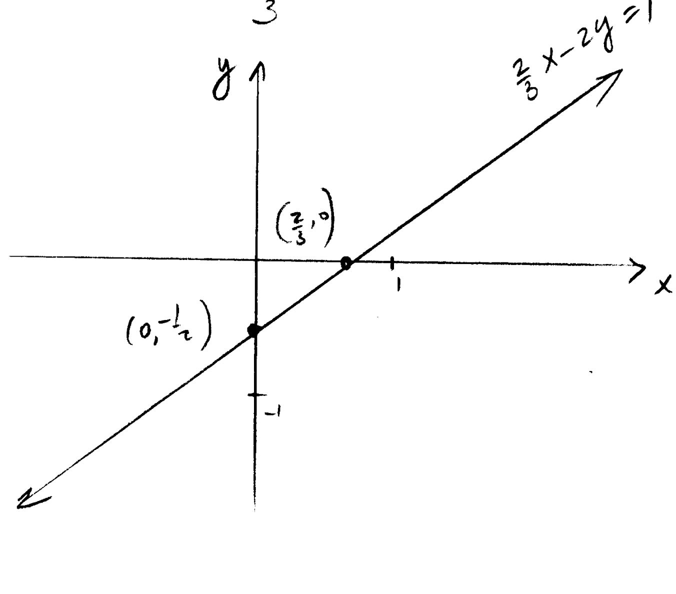
$(\frac{2}{3}, 0)$ x-in

if $x = 0$, $-2y = 1$

$y = -\frac{1}{2}$

if $y = 0$, $\frac{3}{2}x = 1$

$x = \frac{2}{3}$



(b) $\frac{3}{2}x - 2y = 1$

$\frac{3}{2}x - 1 = 2y \quad / \cdot \frac{1}{2}$

$y = \frac{3}{2} \cdot \frac{1}{2} x - \frac{1}{2}$

$y = \frac{3}{4} x - \frac{1}{2}$ so $\boxed{m = \frac{3}{4}}$

(c) then $m_\perp = \frac{-1}{m}$

$m_\perp = \frac{-4}{3}$

We'll use point $(-4,5)$ and slope $-\frac{4}{3}$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-4}{3}(x - (-4))$

$\boxed{y - 5 = \frac{-4}{3}(x+4)}$

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(4) (a) $l(x) = \frac{2x+11}{x-5}$

Condition: $x - 5 \neq 0$

$x \neq 5$

$\boxed{\text{Domain}(l) = \mathbb{R} \setminus \{5\}}$

$g(x) = \sqrt{2+x}$

Condition: $2 + x \geq 0$

$x \geq -2$

$\boxed{\text{Domain}(g) = [-2, \infty)}$

$h(x) = x^2 - 3x + 2, \quad f(x) = 3x - 7$

$\boxed{\text{Domain}(h) = \text{Domain}(f) = \mathbb{R}}$

(b) $g(x) = \sqrt{2+x}$

$\boxed{g(3t) = \sqrt{2+3t}}$

(c) $(h+f)(x) = h(x) + f(x)$

$= (x^2 - 3x + 2) + (3x - 7)$

$= x^2 - 3x + 2 + 3x - 7$

$= x^2 - 5$

$\boxed{(h+f)(x) = x^2 - 5}$

(d) $(h-f)(-1) = h(-1) - f(-1)$

$= ((-1)^2 - 3(-1) + 2) - (3(-1) - 7)$

$= (1 + 3 + 2) - (-3 - 7)$

$= 6 - (-10) = 16$

$\boxed{(h-f)(-1) = 16}$

(e) $h(x-1) = (x-1)^2 - 3(x-1) + 2$

$= x^2 - 2x + 1 - 3x + 3 + 2$

$= x^2 - 5x + 6$

$\boxed{h(x-1) = x^2 - 5x + 6}$

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(5) $f(x) = 4x - 1$

$\frac{f(a+h) - f(a)}{h} = \frac{(4(a+h) - 1) - (4a - 1)}{h}$

$= \frac{\cancel{4a} + 4h - \cancel{1} - \cancel{4a} + \cancel{1}}{h}$

$= \frac{4h}{h} = 4$, so $\boxed{\frac{f(a+h) - f(a)}{h} = 4}$

(6) $f(x)=\begin{cases} 2x+5, & x\le 0 \\ -3, & 0\le x\le 6 \\ x-10, & x>6 \end{cases}$

(a) $\boxed{x \in \mathbb{R}}$

(b) $f(-\frac{1}{2}) = 2(-\frac{1}{2})+5 = -1+5=4$

$\boxed{f(-\frac{1}{2})=4}$

$f(0) = 2(0)+5 = 5$

$\boxed{f(0)=5}$

$\boxed{f(2)= -3}$

$f(7) = 7-10 = -3$

$\boxed{f(7)= -3}$

$f(10) = 10-10 = 0$

$\boxed{f(10)=0}$

(e) $(fg)(3) = f(3)\,g(3)$

$= 0\cdot 3 = 0$

$\boxed{(fg)(3)=0}$

(f) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1}{3}$

$\boxed{\left(\frac{f}{g}\right)(1)=\frac{1}{3}}$

(g) $f(x)=0$ when $x=-4$ and $x=3$

The solutions represent the x-intercepts of the graph: $(-4,0)$ and $(3,0)$

(h) $g(0)=3$

This represents the y-intercept of the graph: $(0,3)$

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(7) (a) Yes, because their graphs pass the Vertical Line Test

(b) Domain: $x\in[-5,5]$

Range: $y\in[-2,2]$

(c) Domain: $x\in[-4,5]$

Range: $y\in[3,5]$

(d) $(f+g)(-4) = f(-4)+g(-4)$

$= 0+3 = 3$

$\boxed{(f+g)(-4)=3}$

(8) (a) x = number of years after 2000

y = number of Americans (in millions) living below the poverty level

x = independent, y = dependent

(b) $(2, 34.6)$

when $x=2$, $y=34.6$

Two years after 2000 (in 2002), there were 34.6 million Americans living below the poverty level

(c) 2001 : $x=1$
when $x=1$, $y \approx 33$
In 2001 there were about 33 million Americans living below the poverty level

(c) $(2, 34.6)$
$(5, 37.0)$

$$m = \frac{\Delta y}{\Delta x} = \frac{37-34.6}{5-2} = \frac{2.4}{3} = 0.8$$

$\boxed{m = 0.8 \text{ million people/year}}$

(d) The number of Americans living below the poverty level increased by 0.8 million people per year after 2001.

(e) We'll use $m=0.8$ and $(5, 37)$

$y - y_1 = m(x - x_1)$

$\boxed{y - 37 = 0.8(x-5)}$

(7) $\begin{cases} 2x - 4y + 3z = 17 & (1) \\ x + 2y - z = 0 & (2) \\ 4x - y - z = 6 & (3) \end{cases}$

Eliminate z:

(1) $\begin{cases} 2x - 4y + 3z = 17 \\ x + 2y - z = 0 \end{cases}$ (2) $\cdot 3$

$\begin{cases} 2x - 4y + 3z = 17 \\ 3x + 6y - 3z = 0 \end{cases}$

(+) $5x + 2y = 17$ (4)

(2) $\begin{cases} x + 2y - z = 0 \\ 4x - y - z = 6 \end{cases}$ (3)

(−) $-3x + 3y = -6$ (5)

(4) $\begin{cases} 5x + 2y = 17 & \cdot 3 \\ -3x + 3y = -6 & \cdot 5 \end{cases}$ (5)

Eliminate x:

$\begin{cases} 15x + 6y = 51 \\ -15x + 15y = -30 \end{cases}$

(+) $21y = 21$, $y = 1$

(4) $5x + 2y = 17$
$5x + 2(1) = 17$
$5x = 15$, $x = 3$

(2) $x + 2y - z = 0$
$3 + 2 - z = 0$, $z = 5$

$\boxed{\text{The solution is } (3, 1, 5)}$

(A) $s(t) = 11t^2 + t + 100$
t = time (in hours)
$s(t)$ = position of car (in miles)
$v(t) = 22t + 1$
$v(t)$ = velocity of car

(a) $s(2)$ = car's position after 2 hours
$s(2) = 11(2)^2 + 2 + 100$
$s(2) = 146$ mi

(b) find $v(t) = 65$ mph
$v(t) = 65$
$22t + 1 = 65$
$22t = 64$
$t = \frac{64}{22} = \frac{32}{11} \approx 2.9$ hours

(c) $v(t) = 67$ mph
$22t + 1 = 67$
$22t = 66$
$t = 3$ hrs
so $s(3) = 11(3)^2 + 3 + 100$
$s(3) = 202$ mi

(B) $A(x) = 0.04x + 5.1$
women (in millions) enrolled in US colleges
$B(x) = 0.03x + 3.8$
men (in millions) enrolled in U.S. colleges
x = number of years after 1980

(a) $A(2) = 0.04(2) + 5.1$
$A(2) = 5.18$ million women
in 1982 there were 5.18 mill. women enrolled

(b) $m = 0.04$ million women/year
This represents the slope of the line
$A(x) = 0.04x + 5.1$
$m = \frac{\Delta A}{\Delta x}$ rate of change of A with respect to x.

(c) $A(10) - B(10) =$
$(0.04(10) + 5.1) - (0.03(10) + 3.8) =$
$= 1.4$ million
in 1990, there were 1.4 million more women than men enrolled in US colleges.

(d) $0.04x + 5.1 = 0.03x + 3.8$
$0.04x - 0.03x = 3.8 - 5.1$
$0.01x = -1.3$
$x < 0$ so the number of women enrolled was never equal to the number of men after 1980

(c)

$6700\$ \begin{cases} x\$ \text{ at } 8\% \\ y\$ \text{ at } 10\% \\ z\$ \text{ at } 12\% \end{cases}$

\$ 716 = total income

Let x = amount invested at 8%

y = amount invested at 10%

z = amount invested at 12%

$$\begin{cases} x+y+z = 6700 \\ 8\% x + 10\% y + 12\% z = 716 \\ z = (x+y) + 300 \end{cases}$$

$$\begin{cases} x+y+z = 6700 \\ \frac{8}{100}x + \frac{10}{100}y + \frac{12}{100}z = 716 \quad \Big| \cdot 100 \\ z = x+y+300 \end{cases}$$

$$\begin{cases} x+y+z = 6700 & (1) \\ 8x+10y+12z = 71600 & (2) \\ z = x+y+300 & (3) \end{cases}$$

Substitution (3) in (1) and (2)

$$\begin{cases} x+y+(x+y+300) = 6700 \\ 8x+10y+12(x+y+300) = 71600 \end{cases}$$

$$\begin{cases} 2x+2y = 6400 \quad \Big| \cdot(-10) \\ 20x+22y = 68{,}000 \end{cases}$$

$$\begin{cases} -20x-20y = -64000 \\ 20x+22y = 68{,}000 \end{cases}$$

$2y = 4000$, $y = 2000\$$

$2x+2y = 6400 \quad |:2$

$x+y = 3200$

$x+2000 = 3200$, $x = 1200\$$

$z = x+y+300$

$z = 1200+2000+300$, $z = 3500\$$

The person invested 1200\$ at 8%, 2000 \$ at 10%, and 3500 \$ at 12%.

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(1) (a) Yes, for every time there is only one price

(b) $t \in [8am, 2pm]$ - domain

$P \in [2\$, 12\$]$ - range

(c) $P(t) = 8\$$

The times when the stock was 8 dollars

$t \approx$ 11am, 12:15pm, 1:30pm

(d) $P(11) = 8\$$

The price of stock at 11am

(e) $P(t) > 5.50$

when $t \in [8am, 9{:}10am) \cup (10{:}15am, 2pm]$