REVIEW TEST 3 - Chapters 8 and 9 (9.1 – 9.4)

To prepare for the test, you may study the following:

- Quiz #3
- Handout: 8.3 Quadratic Functions and Their Graphs –all exercises and applications (see website for handout)
- Handout: Sections 9.1 Exponential Functions; Exponential Growth and Decay – all exercises and applications except example 2 (page 4) and exercise 2 (page 6)
- Homework Chapters 8 & 9

More practice

Chapter 8

Solve (in $\mathbb{C}$) by extracting roots:

1) $9x^2 = 25$; **2)** $\frac{2x^2}{3} = 4$; **3)** $\left(x - \frac{1}{2}\right)^2 = \frac{3}{4}$;

4) $3(t-2)^2 + 38 = 0$ **5)** $4(x+2)^2 = 12$ **6)** $1 - 3(y-1)^2 = 10$

7) $81\left(x + \frac{1}{3}\right)^2 + 1 = 0$ **8)** $3(8x-7)^2 - 24 = 0$

Solve the following (in $\mathbb{C}$) by completing the square:

9) $x^2 - 6x - 7 = 0$; **10)** $2a^2 - 6a - 5 = 0$; **11)** $-4x^2 - 36x - 65 = 0$;

12) $3x^2 = 5x + 21$ **13)** $x^2 + \frac{4}{7}x + \frac{3}{49} = 0$ **14)** $9v^2 + 12v - 41 = 0$

15) $6t^2 + 4t + 18 = 0$

Solve the **f**ollowing (in $\mathbb{C}$) by the quadratic formula:

16) $2x^2 + 1 = 4x$; **17)** $t^2 - \frac{t}{2} + 1 = 0$; **18)** $\frac{1}{2}x^2 + 1 = \frac{3}{2}x$;

19) Write a quadratic equation with rational coefficients that has:

a) $1 - \sqrt{2}$ as a solution;
b) -2 and 3 as solutions;
c)Write (in standard form) a quadratic equation with real coefficients that has 1-2i as a solution.
d) Write a quadratic equation in standard form with integer coefficients that has the solutions of $\frac{2}{7}, -\frac{4}{3}$.

20) Solve each equation for the indicated variable:

a) $3x^2 + xy + y^2 = 2$ solve for y in terms of x; b) $A = 2w^2 + 4lw$ solve for w in terms of A and l;

c) $a^2 + b^2 = c^2$ solve for b in terms of a and c; d) $h = -16t^2 + \frac{23}{3}t$ solve for t in terms of h.

e) $h = \frac{v^2}{2g}$ for v f) $s = \frac{kwd^2}{l}$ for d.

21) Show in two different ways that 3-2i is a solution of $x^2 - 6x + 13 = 0$.

22) Solve the following equations: a) $x^4 - 3x^2 = -2$; b) $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 3 = 0$; c) $x + \sqrt{x} - 6 = 0$;

23) Solve:

a) $\frac{19x+14}{3}=x^2$ b) Solve for p: $r=5p-mp^2$ c) Solve for w in terms of A: $A=\frac{1}{2}w(100-2w)$

d) $6x-13=\frac{5}{x}$ e) $\frac{1}{x^2-3x+2}=\frac{1}{x+2}+\frac{5}{x^2-4}$ (#67/576) f) $\sqrt{2}x^2+3x-2\sqrt{2}=0$ (#69/586)

g) $\left|x^2+2x\right|=3$ (#71/586) (A: c) $w=25\pm\sqrt{625-A}$; d) $-1/3, 5/2$)

24) Answer all questions; show all work. Let $y=\frac{1}{3}(x+3)^2-2$ be a parabola.

a) What type of curve is this?; b) y-intercept?; c) Vertex ; d) x- intercept(s)? ; e) sketch its graph; f) What is the standard form of the equation? g) Domain? h) Range? i) Is this function one-to-one? Does it have an inverse?

25) Answer all questions for each parabola.

i) $y=-2x^2+x+3$ ii) $y=-10x^2-2x+1$ iii) $y=\frac{1}{7}x^2-8x+66$

a) What type of curve is this?; b) y-intercept?; c) Vertex ; d) x- intercept(s)? ; e) sketch its graph; f)What is the vertex form of the above equation? g) Domain? h) Range? i) Is this function one-to-one? Does it have an inverse?

26) A model rocket launched with an upward velocity of 3.75 meters per second. The height of the rocket after *t* seconds if given by the formula: $h=-4.9t^2+3.75t+12.25$.

a) How high is the rocket off the ground to start with?
b) How long does it take the rocket to hit the ground?
c) When does the rocket reach a height of 16 meters?
d) During what time intervals is the rocket at a height greater than 15 feet?

27) A baseball thrown vertically reaches a height *h* in feet given by $h=56t-16t^2$, where *t* is measured in seconds. During what intervals is the height of the ball greater than 40 feet?

28) Solve the following inequalities. Write the solution set in interval notation:

a) $x^2-6x-7\le 0$; b) $6x-x^2\ge 7$; c) $x(2-3x)(x-3)\ge 0$; d) $\frac{3}{x+3}>\frac{5}{x-2}$;

e) $-x(x+1)^2(x^2+5x+6)(2x^2+3x+10)>0$

29) Hortense is investing $2600 in an account where interest is calculated according to the formula $A=P(1+r)^t$ where P is the original principal, r is the interest rate and t is the time measured in years. If Hortense wants her money to grow to double in two years, what interest rate must the account have? (Approximate the answer to the nearest hundredth of a percent) (A: 41.42%)

30) Earl borrowed $5500 from his uncle for 2 years with interest compounded annually according to the formula $A=p(1+r)^t$ where "p" is the original principal, "r" is the interest rate and "t" is the time measured in years. At the end of 2 years he owed his uncle $6474.74. What was the interest rate on the loan? (A: 8.5%)

31) When Maria serves in volleyball, the ball leaves her hand with an upward velocity of 20 feet per second. The height "h" of the volleyball above the ground after "t" seconds is given by: $h=-16t^2+20t+5$.

a) If nobody hits the ball, how long will it take the ball to hit the ground? (A:1.46 sec)
b) If nobody hits the ball, how long will it take the ball to reach its initial height again? (A: 5/4 sec)

32) The fish population in a certain lake rises and falls according to the formula:

$$F = 2000\left(15 + \frac{17}{2}t - \frac{1}{2}t^2\right).$$

Here "*F*" is the number of fish at the time "*t*" where "*t*" is measured in years since January 1,1997 when the fish population was first estimated.

a) On what date will the fish population again be the same as on January 1,1998?

b) By what date will all the fish in the lake have died? (Approximate your answer in years to one decimal place).

CHAPTER 9 – Exponential and Logarithmic Functions 9.1 – 9.4

1) Find the domain of each function:

a) $f(x) = \log_{10}(12 - 4x)$; b) $g(x) = \ln\left(x^2 - 25\right)$; c) $h(x) = \log\left(\frac{3-4x}{x+2}\right)$

2) Simplify:

a) $\log_2\left(\log_4 16\right)$ b) $\log_{10}\left(\log_3\left(\log_5 125\right)\right)$ c) $2^{\log_2 5} - 3\log_5 \sqrt[3]{5}$

3) Find the following:

a) $\log_3 27$ b) $\log_4 \frac{1}{16}$ c) $\log_{1/2} 8$ d) $\log_2 \sqrt{2}$

e) $\log_2\left(\log_4 16\right)$ f) $\log\left(\ln e\right)$ g) $\log\left(\log_3\left(\log_5 125\right)\right)$ h) $\log 70 - \log 7$

i) $2^{\log_2 5} - 3\log_5 \sqrt[3]{5}$ j) $\frac{\log_3 81 - \log_p 1}{\log_{2\sqrt{2}} 8 - \log 0.001}$ k) $\left(\log_2 10\right)\left(\log 2\right)$

l) $5e^{\ln\left(A^2\right)}$ m) $\ln\left(e^{2ab}\right)$

4) Expand as much as possible. Simplify the result if possible. Assume all variables represent positive real numbers

a) $\log_3 \frac{4p}{q}$ b) $\log_5 \frac{5\sqrt{7}}{3}$ c) $\log_6\left(7m + 3q\right)$ d) $\log_m \sqrt{\frac{5r^3}{z^5}}$

e) $\log_3 \frac{\sqrt{x} \cdot \sqrt[3]{y}}{w^2\sqrt{z}}$ f) $\ln \frac{5x\sqrt{1+3x}}{(x-4)^3}$

5) Write as a single logarithm with coefficient 1. Assume all variables represent positive real numbers

a) $\log_a x + \log_a y - \log_a m$ b) $2\log_m a - 3\log_m b^2$ c) $\log_b\left(2y+5\right) - \frac{1}{2}\log_b\left(y+3\right)$

6) Let $f(x) = 1 - 2x$ and $g(x) = \frac{2-x}{x+3}$. Answer the following questions:

a) Find $(g \circ f)(x)$. b) $(f \circ g)(2)$ c) Find $f^{-1}(x)$. d) Find $g^{-1}(x)$.

7) Simplify the following expressions.

a) $4\ln x + 7\ln y - 3\ln z$

b) $\frac{1}{2}\left(\log_5 x + \log_5 y\right) - 2\log_5\left(x+1\right)$

c) $\log_3 405 - \log_3 5 + \log 5 + \log 2$

d) $\log_4\left(\log_2 16\right)$

8) Graph $f\left(x\right) = 3^x$ and $f^{-1}\left(x\right) = \log_3 x$ on the same coordinate system showing the symmetry about the bisector line $y = x$. Label the axes and all the points.

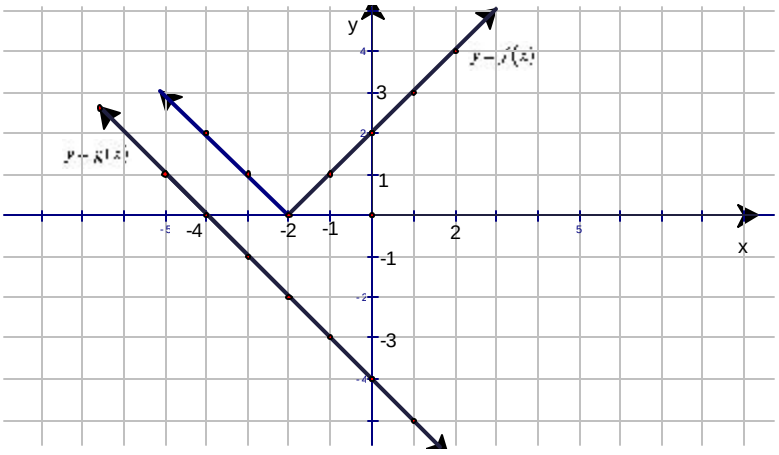
9) Graph the function $f\left(x\right) = 4^x$. Label the axes and show clearly how you graph (label all the points you use). Answer the following questions:

a) What is the domain of f?
b) What is the range of f?
c) What is the y-intercept?
d) What is the x-intercept (if any)?
e) Does the graph have an asymptote? What kind? What is its equation?
f) If this function one-to –one? Explain.
g) Does f have an inverse? Why?
h) What is the inverse function (Do not prove).
i) Show on the above coordinate system how you obtain the graph of f^{-1} from the graph of f. That is, sketch the graph of f^{-1} showing the symmetry about the line $y = x$.

10) Repeat all questions from 9) for

a) $f\left(x\right) = \log_2 x$; b) $g\left(x\right) = \ln x$

11) Use the graphs of f and g to evaluate each composite function.



a) $\left(f \circ g\right)(-1)$

b) $\left(g \circ f\right)(0)$