Math 71 Spring 09 | QUIZ #4 @ 55 points | Name:

Write neatly. Show all work. Write all the solutions on separate paper.

Ex 1. Find the domain of each logarithmic function.

a) $f(x)=\log_5(x+6)$ b) $f(x)=\log(7-x)$

Ex 2. Use logarithmic properties to expand the expression as much as possible: $\log_6\left(\frac{\sqrt[3]{x}}{36y^4}\right)$

Ex 3. Write as a single logarithm: $4\log_b x-2\log_b 6-\frac{1}{2}\log_b y$

Ex 4. Evaluate the expression $\log(\ln e)$ without using a calculator.

Ex 5. Solve:

a) $\log_5(3x-1)-2=0$

b) $\log_7(x+4)-\log_7 3=1$

c) $4^x=20$

d) $3-\log_5(2-x)=5$

e) $40e^{0.6x}-3=237$

f) $\log_4(x+3)=2$

Ex 6. The formula $A=P\left(1+\frac{r}{n}\right)^{nt}$ describes the accumulated value, A, of a sum of money, P, the principal, after t years at annual percentage rate r in decimal form) compounded n times a year. How long will it take \$25,000 to grow to 500,000 at 9% annual interest compounded monthly?

Ex 7. Graph the following:

a) $f(x)=5^x$. State the domain, range, and asymptote. What is its inverse function? Graph it on the same coordinate system, showing the relationship between the two graphs.

b) $g(x)=\ln x$. State the domain, range, and asymptote. What is its inverse function? Graph it on the same coordinate system, showing the relationship between the two graphs.

Ex 8.

Let $2x^2+3x+2y^2-y-2=0$ be the equation of a circle.
Find the center and radius of the circle.

Ex 9.

a) Find the distance between the points $(1,-7)$ and $(-8,-2)$.

b) Find the midpoint of the line segment with these two endpoints.

Ex 10.

a) Write the standard form of the equation of the circle with center $(5,-6)$ and radius 10.

b) Find the exact x,y – intercepts.

M71 - SOLUTIONS

Quiz 4

① (a) $f(x) = \log_5(x+6)$

condition: $[x+6>0$

$x>-6$

$\boxed{\text{Domain} = (-6,\infty)}$

(b) $f(x) = \log(7-x)$

condition: $[7-x>0$

$7>x$

$x<7$

$\boxed{\text{Domain} = (-\infty,7)}$

② $\log_6\left(\frac{\sqrt[3]{x}}{36y^4}\right) = \log_6\sqrt[3]{x} - \log_6(36y^4)$

$= \log_6 x^{\frac{1}{3}} - (\log_6 36 + \log_6 y^4)$

$= \frac{1}{3}\log_6 x - \log_6 36 - 4\log_6 y$

$= \boxed{\frac{1}{3}\log_6 x - 2 - 4\log_6 y}$

③ $4\log_6 x - 2\log_6 6 - \frac{1}{2}\log_6 y$

$= \log_6 x^4 - \log_6 6^2 - \log_6 y^{\frac{1}{2}}$

$= \log_6\frac{x^4}{36} - \log_6\sqrt{y}$

$= \log_6\frac{\frac{x^4}{36}}{\sqrt{y}}$

$= \boxed{\log_6\frac{x^4}{36\sqrt{y}}}$

④ $\log(\ln e) = \log 1 = \boxed{0}$

⑤ (a) $\log_5(3x-1) - 2 = 0$

Condition: $3x-1>0 \Rightarrow 3x>1$

$x>\frac{1}{3}$

$\log_5(3x-1) = 2$

$3x-1 = 5^2$

$3x = 26 \Rightarrow x = \frac{26}{3} > \frac{1}{3}$

Therefore, $\boxed{x \in \{\frac{26}{3}\}}$

(b) $\log_7(x+4) - \log_7 3 = 1$

Condition: $x+4>0 \Rightarrow x>-4$

$\log_7\frac{x+4}{3} = 1$

$\frac{x+4}{3} = 7^1$

$x+4 = 21$

$x = 17 > -4$, so $\boxed{x \in \{17\}}$

(c) $4^x = 20$ $/\ln$

$\ln 4^x = \ln 20$

$x\ln 4 = \ln 20$

$\boxed{x = \frac{\ln 20}{\ln 4}}$, $x \approx 2.16$

(d) $3 - \log_5(2-x) = 5$

Condition: $2 - x > 0 \Rightarrow x < 2$

$3 - 5 = \log_5(2-x)$

$-2 = \log_5(2-x)$

$5^{-2} = 2 - x$

$2 - x = \frac{1}{25}$

$x = 2 - \frac{1}{25} = \frac{49}{25} > 2$

so $\boxed{x \in \emptyset}$ (no solutions)

(e) $40e^{0.6x} - 3 = 237$

$40e^{0.6x} = 240$

$e^{0.6x} = \frac{24\cancel{0}}{4\cancel{0}}$

$e^{0.6x} = 6 \quad // \ln$

$\ln e^{0.6x} = \ln 6$

$0.6x = \ln 6$

$x = \frac{\ln 6}{0.6} = \frac{10 \ln 6}{6} = \frac{5 \ln 6}{3}$

$\boxed{x = \frac{5 \ln 6}{3}}$, $x \simeq 2.99$

(f) $\log_4(x+3) = 2$

Condition: $x + 3 > 0 \Rightarrow x > -3$

$4^2 = x + 3$

$16 = x + 3 \Rightarrow x = 13 > -3$

so $\boxed{x \in \{13\}}$

(6) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = 25{,}000$ \$

$A = 500{,}000$ \$

$r = 0.09$

$n = 12$

$t = ?$

$500{,}0\cancel{00} = 25{,}\cancel{000}\left(1 + \frac{0.09}{12}\right)^{12 \cdot t}$

$\frac{500}{25} = (1.0075)^{12t}$

$(1.0075)^{12t} = 20 \quad // \ln$

$\ln(1.0075)^{12t} = \ln 20$

$12t \ln(1.0075) = \ln 20$

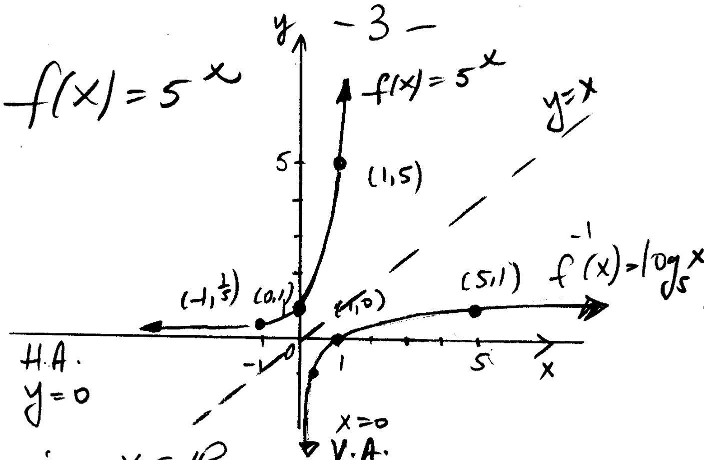
$t = \frac{\ln 20}{12 \ln(1.0075)} \approx 33.4$

It will take about $\boxed{33.4 \text{ years}}$

⑦ (a) $f(x) = 5^x$

$x \in \mathbb{R}$

x	y
-1	$\frac{1}{5}$
0	1
1	5



- Domain: $x \in \mathbb{R}$
- Range: $y \in (0, \infty)$
- H.A $y = 0$
- $f^{-1}(x) = \log_5 x$

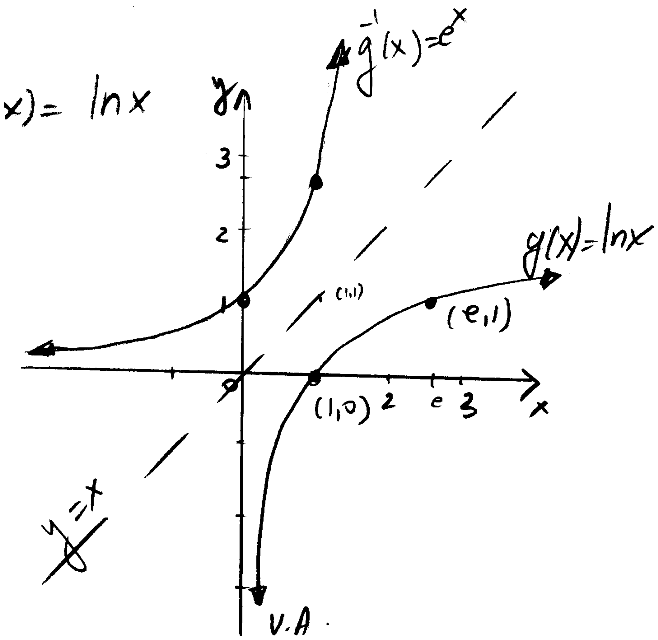
x	$\log_5 x$
$\frac{1}{5}$	-1
1	0
5	1

- The two graphs are symmetric about the line $y = x$

(b) $g(x) = \ln x$

$x > 0$

x	y
$\frac{1}{e}$	-1
1	0
e	1



- Domain: $x \in (0, \infty)$
- Range: $y \in \mathbb{R}$
- V.A $x = 0$
- $g^{-1}(x) = e^x$

x	e^x
-1	$\frac{1}{e}$
0	1
1	e

- The graphs are symmetric about the line $y = x$

⑧ $2x^2 + 3x + 2y^2 - y - 2 = 0$

1st $2x^2 + 3x + 2y^2 - y = 2$

2nd $x^2 + \frac{3}{2}x + y^2 - \frac{1}{2}y = 1$

3rd $\left(\frac{1}{2}\text{ coef } x\right)^2 = \left(\frac{1}{2} \cdot \frac{3}{2}\right)^2 = \frac{9}{16}$

$\left(\frac{1}{2}\text{ coef } y\right)^2 = \left(\frac{1}{2} \cdot \frac{-1}{2}\right)^2 = \frac{1}{16}$

$x^2 + \frac{3}{2}x + \frac{9}{16} + y^2 - \frac{1}{2}y + \frac{1}{16} = 1 + \frac{9}{16} + \frac{1}{16}$

$\left(x + \frac{3}{4}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \frac{26}{16}$

$$\boxed{\begin{aligned} \text{center} &= \left(-\frac{3}{4}, \frac{1}{4}\right) \\ r &= \sqrt{\frac{26}{16}} = \frac{\sqrt{26}}{4} \end{aligned}}$$

⑨ (a) $(1,-7)$ and $(-8,-2)$

let d = the distance between the points

$d^2 = (\Delta x)^2 + (\Delta y)^2$

$d^2 = (1-(-8))^2 + (-7-(-2))^2$

$d^2 = 81 + 25$

$d^2 = 106$

$d = \sqrt{106}$ or $d = -\sqrt{106}$ not possible

$$\boxed{d = \sqrt{106}}$$

(b) let $M(x_M, y_M)$ be the midpoint of the segment

$$x_M = \frac{x_1+x_2}{2}, \quad y_M = \frac{y_1+y_2}{2}$$

$$x_M = \frac{1-8}{2} = \frac{-7}{2}$$

$$y_M = \frac{-7-2}{2} = \frac{-9}{2}$$

so $\boxed{M\left(-\frac{7}{2}, -\frac{9}{2}\right)}$

(10) (a) $(x-h)^2+(y-k)^2 = r^2$

center $(5, -6)$

$r = 10$

$$\boxed{(x-5)^2 + (y+6)^2 = 100}$$

(b) x-in: let $y=0$

$$(x-5)^2 + 36 = 100$$

$$(x-5)^2 = 64 \quad \Big| \sqrt{\ }$$

$$x-5 = \pm\sqrt{64}$$

$$x = 5 \pm 8$$

$x = 13$ or $x = -3$

so, the x-in are:

$\boxed{(13,0) \text{ and } (-3,0)}$

y-in: let $x=0$

$$25 + (y+6)^2 = 100$$

$$(y+6)^2 = 75 \quad \Big| \sqrt{\ }$$

$$y+6 = \pm\sqrt{75}$$

$$y = -6 \pm \sqrt{75}$$

$$y = -6 \pm 5\sqrt{3}$$

The y-in are:

$\boxed{(0, -6 \pm 5\sqrt{3})}$