
QUIZ #2 @ 50 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. If $f(x) = x + 1$, and $g(x) = x^2 - 2x + 3$. Find the following and simplify:

a) $f(a+h) - f(a)$

b) $(f+g)(x)$

c) $(f-g)(x)$

d) $(fg)(-1)$

2. Factor each polynomial completely:

a) $9t^2 - 15t + 4$

b) $x^2 - 5x - 24$

c) $a^3 + 8b^3$

d) $2x^4 - 6x - x^3y + 3y$

e) $1 - 81x^4$

3. Solve each equation by factoring:

a) $t^3 = 64t$

b) $(x-3)(x+8) = -30$

d) $-5(4x+7)\left(2x - \frac{1}{5}\right)(3x^2 - 7x) = 0$

e) $\frac{1}{4}y^2 - \frac{5}{2}y + 6 = 0$

f) $x(x-2)^3 - 35(x-2)^2 = 0$

4. A tree is supported by a wire anchored in the ground 15 feet from its base. The wire is 4 feet longer than the height that it reaches on the tree. Find the length of the wire.

Quiz 2 - SOLUTIONS

$$\begin{aligned} (1) \quad f(x) &= x+1 \\ g(x) &= x^2-2x+3 \end{aligned}$$

$$\begin{aligned} (a) \quad f(a+h) - f(a) &= \\ (a+h+1) - (a+1) &= \\ a+h+1 - a-1 &= h \end{aligned}$$

$$\begin{aligned} (b) \quad (f+g)(x) &= f(x) + g(x) \\ &= (x+1) + (x^2-2x+3) \\ &= x+1+x^2-2x+3 \\ &= x^2-x+4 \end{aligned}$$

$$\begin{aligned} (c) \quad (f-g)(x) &= f(x) - g(x) \\ &= (x+1) - (x^2-2x+3) \\ &= x+1-x^2+2x-3 \\ &= -x^2+3x-2 \end{aligned}$$

$$\begin{aligned} (d) \quad (fg)(-1) &= f(-1)g(-1) \\ &= (-1+1)((-1)^2-2(-1)+3) \\ &= 0 \end{aligned}$$

$$(b) \quad x^2 - 5x - 24 = (x+3)(x-8)$$

$$\begin{aligned} \text{product} &= c = -24 < \begin{matrix} +3 \\ -8 \end{matrix} \\ \text{sum} &= b = -5 \\ \hline 24 &= 8 \cdot 3 \end{aligned}$$

$$\begin{aligned} (c) \quad a^3 + 8b^3 &= a^3 + (2b)^3 \\ &= (a+2b)(a^2 - a(2b) + (2b)^2) \\ &= (a+2b)(a^2 - 2ab + 4b^2) \end{aligned}$$

$$\begin{aligned} (d) \quad 2x^4 - 6x - x^3y + 3y &= \\ &= 2x(x^3-3) - y(x^3-3) \\ &= (x^3-3)(2x-y) \end{aligned}$$

$$\begin{aligned} (e) \quad 1 - 9x^4 &= 1 - (3x^2)^2 \\ &= (1-9x^2)(1+9x^2) \\ &= (1-(3x)^2)(1+9x^2) \\ &= (1-3x)(1+3x)(1+9x^2) \end{aligned}$$

$$\begin{aligned} (2) \quad (a) \quad 9t^2 - 15t + 4 &= \% \\ \text{we'll split middle term:} & \\ \text{product} &= ac = 36 < \begin{matrix} -12 \\ -3 \end{matrix} \\ \text{sum} &= b = -15 \\ \hline 36 &= 12 \cdot 3 \end{aligned}$$

$$\begin{aligned} \% &= 9t^2 - 12t - 3t + 4 \\ &= 3t(3t-4) - (3t-4) \\ &= (3t-4)(3t-1) \end{aligned}$$

$$(3) \quad (a) \quad t^3 = 64t$$

$$t^3 - 64t = 0$$

$$t(t^2 - 64) = 0$$

$$t(t-8)(t+8) = 0$$

$$t = 0 \quad \text{OR}$$

$$t-8 = 0 \Rightarrow t = 8 \quad \text{OR}$$

$$t+8 = 0 \Rightarrow t = -8$$

$$t \in \{0, 8, -8\}$$

$$\textcircled{b} (x-3)(x+8) = -30$$

$$x^2 + 8x - 3x - 24 + 30 = 0$$

$$x^2 + 5x + 6 = 0$$

$$(x+3)(x+2) = 0$$

$$x+3=0 \quad \text{OR} \quad x+2=0$$

$$x=-3 \quad \quad \quad x=-2$$

$$x \in \{-3, -2\}$$

$$\textcircled{d} -5(4x+7)(2x-\frac{1}{5})(3x^2-7x) = 0$$

$$4x+7=0 \Rightarrow 4x=-7$$

$$\text{OR} \quad x = -\frac{7}{4}$$

$$2x-\frac{1}{5}=0 \Rightarrow 2x=\frac{1}{5}$$

$$\text{OR} \quad x = \frac{1}{10}$$

$$3x^2-7x=0 \Rightarrow x(3x-7)=0$$

$$x=0 \quad \text{OR} \quad 3x-7=0$$

$$\quad \quad \quad \quad \quad \quad x = \frac{7}{3}$$

$$x \in \left\{ -\frac{7}{4}, \frac{1}{10}, 0, \frac{7}{3} \right\}$$

$$\textcircled{e} \frac{1}{4}y^2 - \frac{5}{2}y + 6 = 0 \quad / \cdot 4$$

$$y^2 - 10y + 24 = 0$$

$$(y-6)(y-4) = 0$$

$$y-6=0 \quad \text{OR} \quad y-4=0$$

$$y=6 \quad \quad \quad y=4$$

$$y \in \{6, 4\}$$

$$\textcircled{f} x(x-2)^3 - 35(x-2)^2 = 0$$

$$(x-2)^2 (x(x-2) - 35) = 0$$

$$(x-2)^2 (x^2 - 2x - 35) = 0$$

$$(x-2)^2 (x-7)(x+5) = 0$$

$$x-2=0 \Rightarrow x=2$$

$$\text{OR}$$

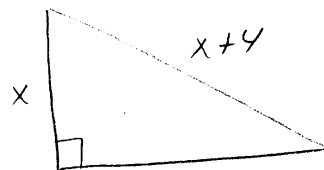
$$x-7=0 \Rightarrow x=7$$

$$\text{OR}$$

$$x+5=0 \Rightarrow x=-5$$

$$x \in \{2, 7, -5\}$$

(4)



let $x =$ height of tree

Pythagorean theorem \Rightarrow

$$x^2 + 15^2 = (x+4)^2$$

$$x^2 + 225 = x^2 + 8x + 16$$

$$8x = 225 - 16$$

$$8x = 209$$

$$x = \frac{209}{8}$$

Then, the length of the wire is

$$x+4 = \frac{209}{8} + 4 = \frac{209+32}{8}$$

$$= \frac{241}{8} = 30\frac{1}{8} \text{ ft}$$