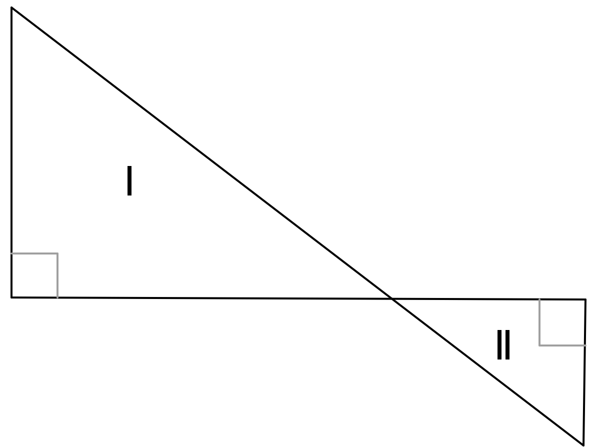
TEST #3 @ 140 points

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.
Write all solutions and proofs on separate paper. Label each exercise.

1. a) State whether $\triangle I \sim \triangle II$.

   b) If so, write what case of similarity applies.



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2. a) The Triangle Proportionality Theorem
   - Draw a scalene triangle ABC.
   - Draw a segment $\overline{MN}$ parallel to $\overline{BC}$, where M is on $\overline{AB}$ and N is on $\overline{AC}$.
   - Write everything you know in this given situation; that is: What triangles are similar? What sides are proportional? What other segments are proportional? To receive full credit, use math notation pertinent to your drawing .

   b)
   - Draw a right triangle.
   - Draw the altitude to the hypotenuse.
   - Write a formula for the altitude.
   - Write a formula for each leg.

   c) Draw a right triangle.
   - Write the Pythagorean theorem. To receive full credit, use math notation pertinent to your drawing .

   d)
   - Draw a triangle and an angle bisector .
   - What conclusion can be drawn from the Triangle Angle Bisector Theorem? To receive full credit, use math notation pertinent to your drawing

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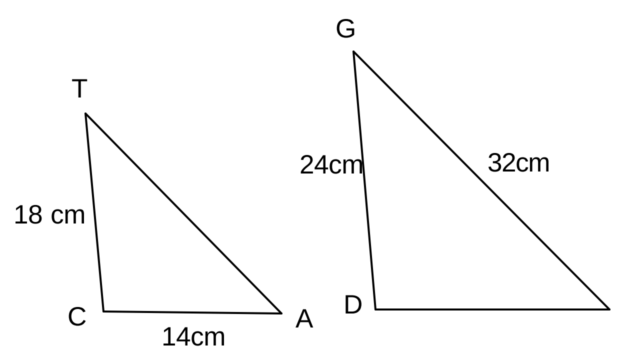
3. Given $\triangle TCA \sim \triangle GDO$, $m\angle D = 96^\circ$, $m\angle A = 48^\circ$, $GD = 24$ cm, $TC = 18$, $CA = 14$, $GO = 32$. Find $m\angle G$, TA, and DO.



4. Prove the following theorem using a formal proof. Make a drawing and state the hypothesis (given) and conclusion(to prove) using math notation pertinent to your drawing – that is, do not state the hypothesis and conclusion in words!

Two tangent segments to a circle from the same point have equal lengths.

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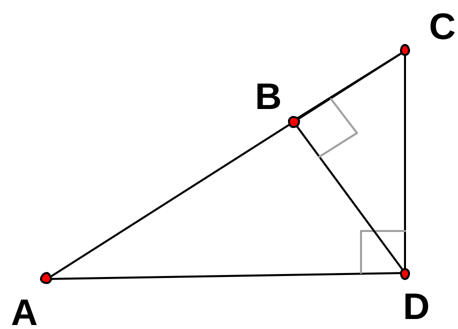
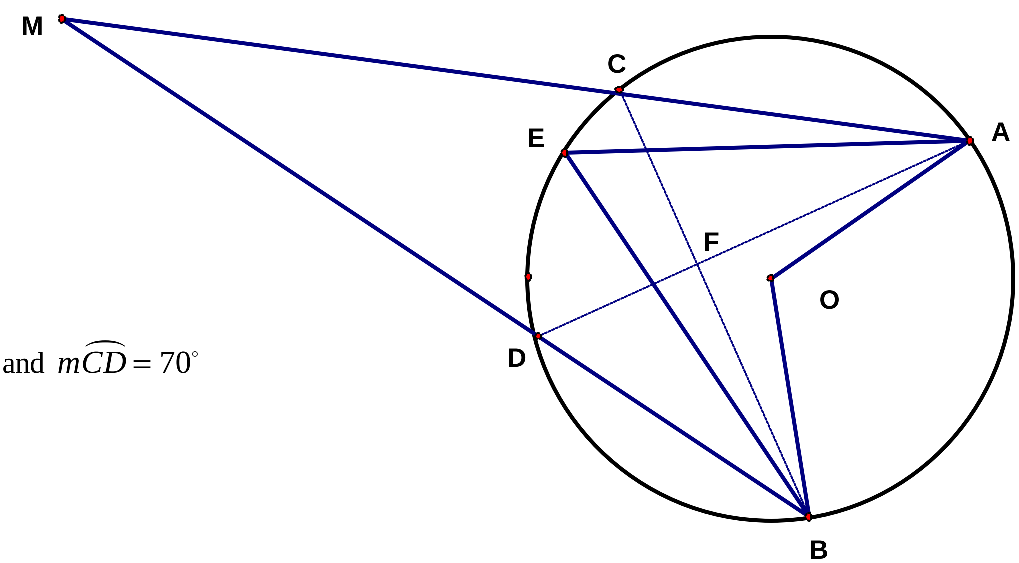
5. Given: $\triangle ADC$ right at D
$\overline{BD} \perp \overline{AC}$
$DC = 6$
$BC = 2$

Find: AC and AB
(Informal solution)



6.



Given arcs: $m\overset{\frown}{AB} = 110^\circ$ and $m\overset{\frown}{CD} = 70^\circ$

Find:

a) $m\angle AOB =$

b) $m\angle CFD =$

Name another angle that is congruent with $\angle CFD$: ________________

c) $m\angle CBD =$

Name another angle that is congruent with $\angle CBD$: _________________

d) $m\angle AEB =$

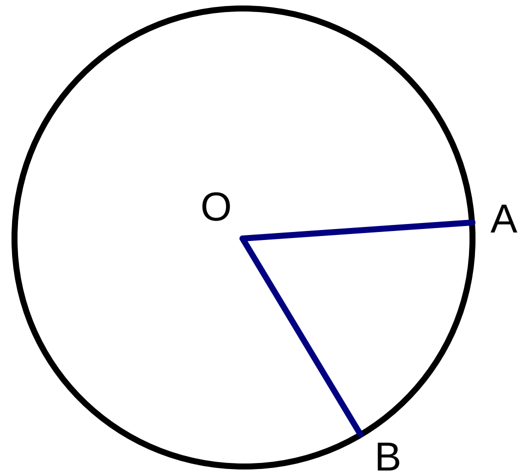
Name two other angles that are congruent with $\angle AEB$: ______________ and __________________

e) $m\angle AMB =$

f) Complete the following formulas: $MC \cdot MA =$____________
$AF \cdot FD =$ ____________

7. Given $\odot O$ with $m\angle AOB = 50^\circ$ and $OA = 10in$,

find the following (exact answers) and <u>use correct units</u>:



a) $m\widehat{AB}$

b) $l\widehat{AB}$

c) Circumference of the circle

d) Area of the circle

e) Area of the sector AOB

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8. Find the area of an equilateral triangle with sides measuring 10 ft. (Do not just write an answer. Justify your steps).

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9. a) Draw a right triangle ABC with right angle C.

b) Draw the altitude to hypotenuse CD.

c) Draw the median to hypotenuse CE.

e) If AB = 50 cm and BD = 32 cm, find BC.

f) If CE is 5 ft, find AB.

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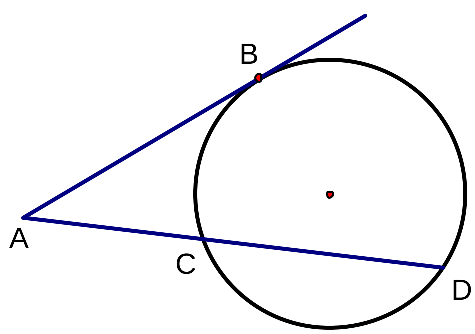


Given: $\overline{AB}$ tangent to the circle at B
AB=8, AD=12

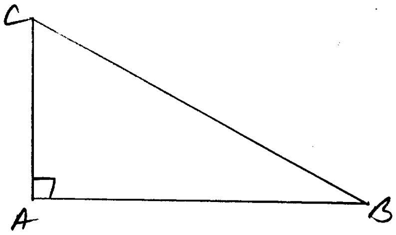
Find: AC

MATH61

TEST 3 – SOLUTIONS

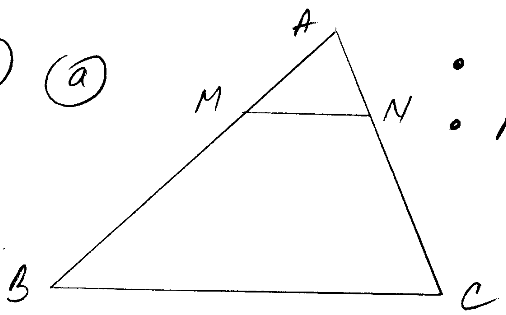
① yes, by the case AA.
(one pair of vertical angles
two right angles)

ⓒ



$(AB)^2 + (AC)^2 = (BC)^2$

② ⓐ



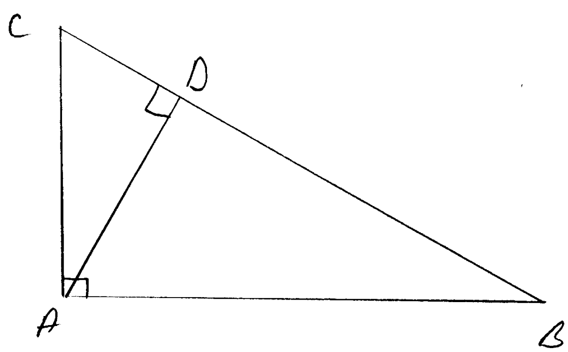
- $\triangle ABC$
- $\overline{MN} \parallel \overline{BC}$

- $\triangle AMN \sim \triangle ABC$

$$\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$$

$$\frac{AM}{MB} = \frac{AN}{NC}$$

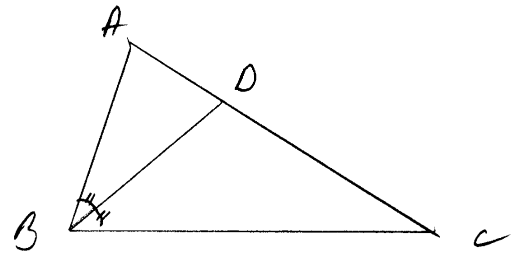
ⓑ



- $\triangle ABC$, $\angle A$ = right
- $\overline{AD}$ - altitude ($\overline{AD} \perp \overline{BC}$)
- $AD^2 = CD \cdot DB$
- $AB^2 = BD \cdot BC$

$AC^2 = CD \cdot BC$

ⓓ



- $\triangle ABC$, $\overline{BD}$ - angle bisector
- $\frac{AD}{DC} = \frac{AB}{BC}$

③ Solution

$\triangle TCA \sim \triangle GDO \Rightarrow$

$$\frac{TC}{GD} = \frac{CA}{DO} = \frac{TA}{GO}$$

$$\frac{18}{24} = \frac{14}{DO} = \frac{TA}{32}$$

$\frac{18}{24} = \frac{14}{DO} \Rightarrow DO = \frac{\cancel{24}^{4} \cdot 14}{\cancel{18}_{3}} = \frac{56}{3}$

$DO = 18\frac{2}{3}$

$\frac{18}{24} = \frac{TA}{32} \Rightarrow TA = \frac{{}^{3}\cancel{18} \cdot 32}{\cancel{24}_{4}} = 24$

$TA = 24$

Also

$\triangle TCA \sim \triangle GDO \Rightarrow$

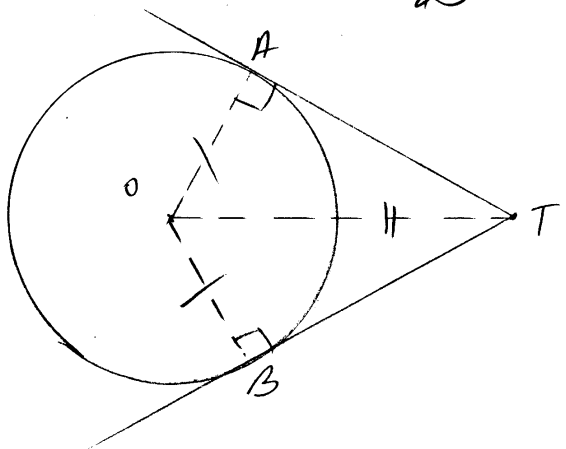
$\angle A \cong \angle O \Rightarrow m\angle O = 48°$

in $\triangle DOG$: $m\angle D + m\angle O + m\angle G = 180°$

so $m\angle G = 180° - 48° - 96°$

$m\angle G = 36°$

(4)



Given $\odot O$, $T \in ext \odot O$
$\overline{TA}, \overline{TB}$ - tangents
$A, B \in \odot O$

Prove: $TA = TB$

Proof

Statements	Reasons
1. $\odot O$, $\overline{TA}, \overline{TB}$ tg.	1. given
2. Construct $\overline{OA}, \overline{OB}$ radii and $\overline{OT}$	2. Two points determine a line
3. $\overline{OA} \perp \overline{AT}$, $\overline{OB} \perp \overline{BT}$	3. radii are $\perp$ to tangents
4. $\angle OAT$, $\angle OBT$ are right $\angle$'s	4 $\perp$ iff right $\angle$'s
5. $\triangle OAT$, $\triangle OBT$ are right $\triangle$'s	5. definition right $\triangle$'s
6. $\overline{OA} \cong \overline{OB}$	6. radii are $\cong$
7. $\overline{OT} \cong \overline{OT}$	7. reflexive prop. of $\cong$
8. $\triangle OAT \cong \triangle OBT$	8. HL
9. $\overline{TA} \cong \overline{TB}$	9. CPCTC
10. $TA = TB$	10. def. $\cong$ segments

(5) Solution

$\triangle BCD$: apply Pythagorean th.

$BC^2 + BD^2 = DC^2$

$BD^2 = DC^2 - BC^2$

$BD^2 = 36 - 4 = 32$

$BD = \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$

$BD = 4\sqrt{2}$

$\overline{BD}$ - altitude to hypotenuse

$BD^2 = BC \cdot AB$

$32 = 2 \cdot AB \Rightarrow AB = 16$

b $AC = AB + BC \Rightarrow AC = 18$

(6) (a) $m\angle AOB = m\overset{\frown}{AB} = 110^\circ$
(central $\angle$)

(b) $m\angle CID = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB})$

$m\angle CID = \frac{1}{2}(180^\circ) = 90^\circ$

$\angle CID \cong \angle BIA$
(vertical $\angle$s)

(c) $m\angle CBD = \frac{1}{2} m\overset{\frown}{CD} = 35^\circ$
(inscribed $\angle$)

$\angle CBD \cong \angle CAD$
(subtend the same arc $\overset{\frown}{CD}$)

(d) $m\angle AEB = \frac{1}{2} m\overset{\frown}{AB} = 55^\circ$
(inscribed $\angle$)

$\angle AEB \cong \angle ACB \cong \angle ADB$
(subtend the same arc $\overset{\frown}{AB}$)

(e) $m\angle AMB = \frac{1}{2}\left(m\widehat{AB} - m\widehat{CD}\right)$

$m\angle AMB = \frac{1}{2}(110^\circ - 70^\circ) = 20^\circ$

(f) $MC \cdot MA = MD \cdot MB$

$AI \cdot ID = CI \cdot IB$

(7) (a) $m\widehat{AB} = m\angle AOB$ (central $\angle$)

$m\widehat{AB} = 50^\circ$

(b) $\frac{\ell(\widehat{AB})}{50^\circ} = \frac{C\odot}{360^\circ}$

$\frac{\ell(\widehat{AB})}{50^\circ} = \frac{2\pi \cdot 10\text{ in}}{360^\circ}$

$\ell(\widehat{AB}) = \frac{2\pi \cdot 10\text{ in} \cdot 50^\circ}{360^\circ} = \frac{50\pi}{18}$

$\ell(\widehat{AB}) = \frac{25\pi}{9}$ in

(c) $C = 2\pi r = \pi(10\text{ in})$

$C = 20\pi$ in

(d) $A = \pi r^2 = \pi(10\text{ in})^2$

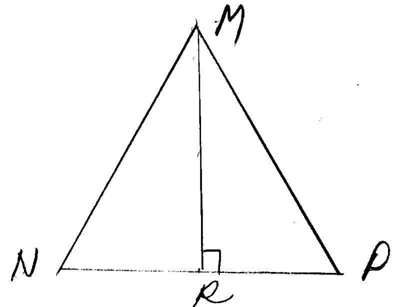
$A = 100\pi\text{ in}^2$

(e) $\frac{A(AOB)}{50^\circ} = \frac{A\odot}{360^\circ}$

$A(AOB) = \frac{(100\pi \cdot \text{in}^2)\cdot 50^\circ}{360^\circ}$

$A(AOB) = \frac{125\pi}{9}\text{ in}^2$

(8)



Given $\triangle MNP$ equilateral

$MN = NP = PM = 10$ ft

Find Area $(\triangle MNP)$

Solution

let A = area of $\triangle MNP$

$A = \frac{1}{2}(\text{base}) \cdot \text{height}$

Let $\overline{MR} \perp \overline{NP}$

Then $A = \frac{1}{2} \cdot NP \cdot MR$

$\triangle MRP$: $PR = \frac{1}{2} NP = 5$ ft

$MP = 10$ ft

$PR^2 + RM^2 = MP^2 \Rightarrow$

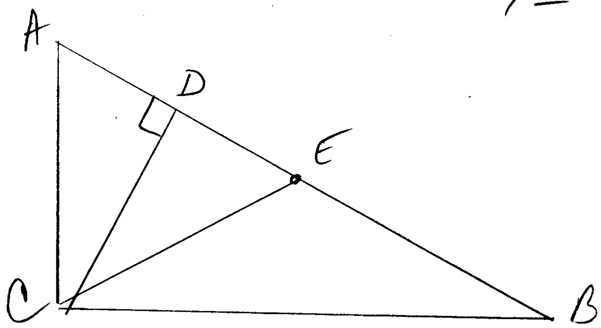
$MR^2 = 100 - 25 = 75$

$MR = 5\sqrt{3}$ ft

$A = \frac{1}{2} \cdot 10 \cdot 5\sqrt{3} = 25\sqrt{3}\text{ ft}^2$

$A = 25\sqrt{3}\text{ ft}^2$

(9)



a) $\triangle ABC$ right at C

b) $\overline{CD} \perp \overline{AB}$, $\overline{CD}$ - alt

c) E - midpoint of $\overline{AB}$, so
$\overline{CE}$ = median

e) $AB = 50$ cm
$BD = 32$ cm

find $BC = ?$

$BC^2 = BD \cdot AB$

$BC^2 = (32\text{ cm})(50\text{ cm})$

$BC = \sqrt{32 \cdot 50} = \sqrt{16 \cdot 2 \cdot 25 \cdot 2}$

$BC = 4 \cdot 5 \cdot 2$

so $BC = 40$ cm

f) $CE = 5$ ft

find $AB = ?$

$CE = \frac{1}{2} AB \Rightarrow$

$AB = 2CE = 2(5\text{ ft})$

$AB = 10$ ft

(10) Solution

$AB^2 = AC \cdot AD$

$8^2 = AC \cdot 12$

$AC = \frac{64}{12} = \frac{16}{3}$

$AC = \frac{16}{3}$