Math 61 Spring 2009 Name:

TEST #2 @ 140 points

Write in a neat and organized fashion. Use a pencil. Use a straightedge and compass for your drawings.
Write all the answers and proofs on separate paper.

1) Do the following:

a)
- Draw a scalene triangle and mark an exterior angle.
- Write an equation that gives the measure of the exterior angle in terms of one interior angle.
- Write an equation that gives the measure of the exterior angle in terms of two interior angles of the triangle.

b)
- Draw a scalene triangle.
- Draw a median for the triangle and explain (mathematically) what it means.
- Draw an altitude for the triangle and explain (mathematically) what it means.
- Draw an angle bisector for the triangle and explain (mathematically) what it means.

c) The Triangle Proportionality Theorem
- Draw a scalene triangle ABC.
- Draw the segment $\overline{MN}$, where M is the midpoint of $\overline{AB}$ and N is the midpoint of $\overline{AC}$.
- What do you know about $\overline{MN}$?

d)
- Draw a trapezoid.
- Draw its median.
- What do you know about the median of a trapezoid? To receive full credit, use math notation pertinent to your drawing .

e)
- Draw a parallelogram.
- Write everything you know about the sides, angles, and diagonals of a parallelogram. To receive full credit, use math notation pertinent to your drawing .

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2) "*If two coplanar lines are each perpendicular to a third line, then these lines are parallel to each other*."

a) Make a drawing.
b) Write the hypothesis and conclusion using math notation pertinent to your drawing.
c) Write an indirect proof.

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3) "*If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram*"

a) Make a drawing.
b) Write the hypothesis and conclusion using math notation pertinent to your drawing.
c) Write a formal proof.

4) *" In a rhombus the diagonals are perpendicular."*

a) Write the above statement in an *"if…then…"* form.
b) What is the definition of a rhombus?
c) Make a drawing to illustrate the given statement.
d) Write the hypothesis and conclusion of the given statement using math notation pertinent to your drawing.
e) Write a formal proof of the given statement.

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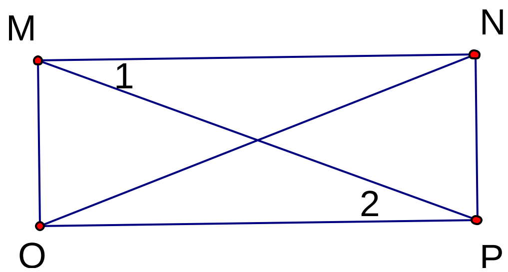
5)



Given $\angle 1 \cong \angle 2$
$\overline{MN} \cong \overline{QP}$

Prove $\overline{MQ} \parallel \overline{NP}$ (formal proof)

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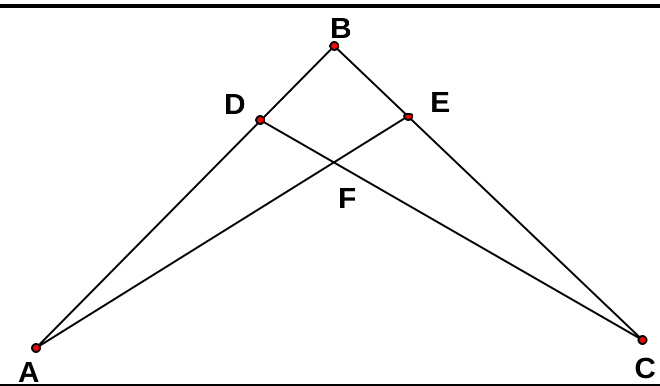
6) Given: $\overline{AB} \cong \overline{CB}$
$\angle A \cong \angle C$
$\angle B$ right angle

Prove: $\overline{BD} \cong \overline{BE}$

(formal proof)



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7) In a triangle MNP, A is the midpoint of side MN and B is the midpoint of side MP. If AB=5 in, find NP. (informal solution).

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8) Let CEMF be a trapezoid with bases $CE = 5x + 3$ and $MF = 13x - 1$. If the median $AB = 6x + 7$, find x.
(informal solution)

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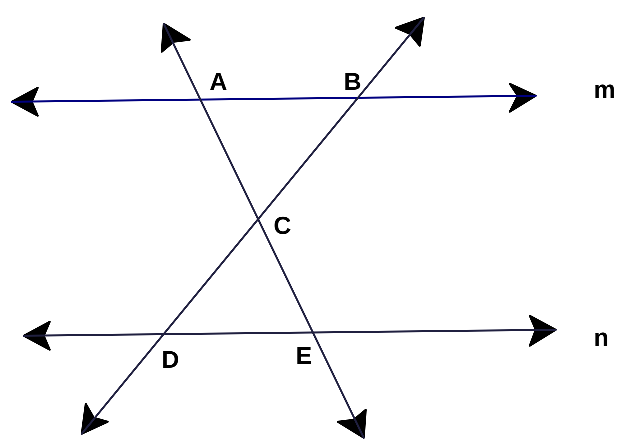
9) Given: $m \parallel n$
$\overline{AB} \cong \overline{DE}$

Prove: $\triangle CBA \cong \triangle CDE$

(formal proof)



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10) Answer TRUE or FALSE. If true, justify your answer. If false, justify the answer or draw a counterexample.

a) The diagonals of a parallelogram are congruent.

b) Any rhombus is a parallelogram.

c) Any parallelogram is a rhombus.

d) If three angles of one triangle are congruent with three angles of a second triangle, then the two triangles are congruent.

e) Triangles can be proved congruent using SSA.

f) If two angles of one triangle are congruent to two angles of a second triangle, the third angles are not necessarily congruent.

g) If two angles of a quadrilateral are right angles, the quadrilateral is a rectangle.

Extra credit ☺

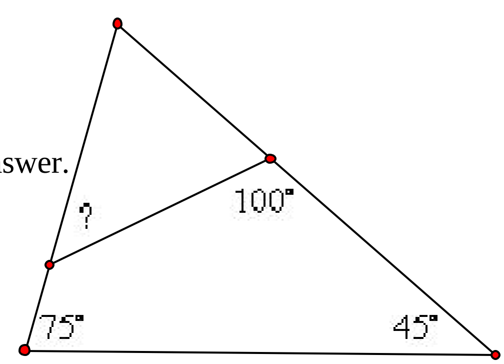
@ 3 points
We have proved the following statement as a theorem:

"In a plane, two lines perpendicular to a third line are parallel to each other."

Would this statement be true if

a) the first three words were omitted? Why?
b) the words "perpendicular" and "parallel" were interchanged? Why?
c) the word "perpendicular" were changed to "parallel"? Why?

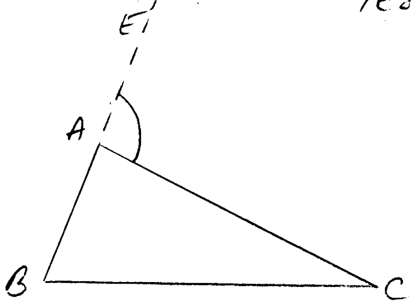
@ 2 points
Find the measure of the indicated angle in the figure below. Justify your answer.



Math 61

TEST 2 - SOLUTIONS

(1)

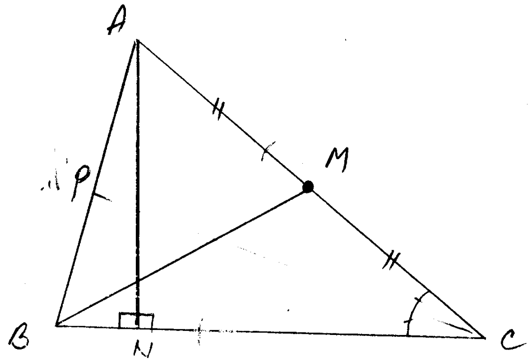


(a) $\triangle ABC$, $\angle CAE$ - exterior

$m\angle CAE = 180° - m\angle BAC$

$m\angle CAE = m\angle ABC + m\angle ACB$

(b)



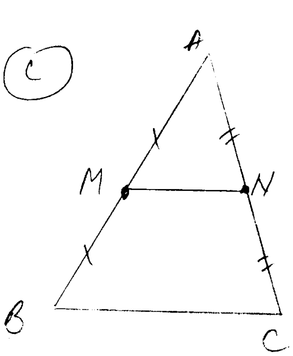
$\triangle ABC$ with

$\overline{BM}$ - median: It connects the vertex B with the midpoint M of opposite side $\overline{AC}$ ($\overline{AM} \cong \overline{MC}$)

$\overline{AN}$ - altitude: $\overline{AN} \perp \overline{BC}$ with $N \in \overleftrightarrow{BC}$

$\overline{CP}$ - angle bisector: $\angle ACP \cong \angle BCP$

(c)



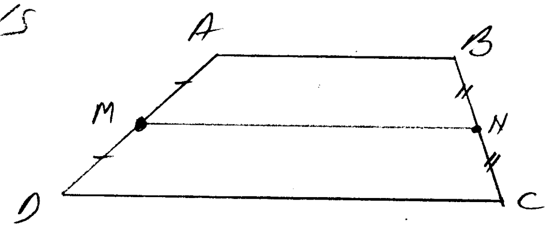
$\triangle ABC$,

M = midpoint $\overline{AB}$

N = midpoint $\overline{AC}$

Then $\overline{MN} \parallel \overline{BC}$ and $MN = \frac{1}{2} BC$

(d)



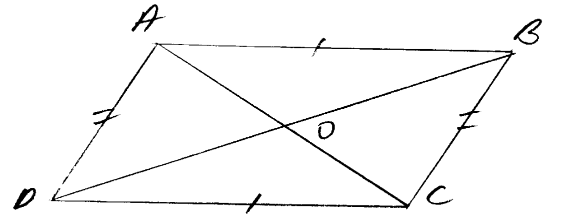
ABCD - trapezoid

$\overline{MN}$ - median (M = midpoint $\overline{AD}$, N = midpoint $\overline{BC}$)

Then $\overline{MN} \parallel \overline{AB} \parallel \overline{DC}$

and $MN = \frac{1}{2}(AB + DC)$

(e)



ABCD - parallelogram

sides: $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

$\overline{AD} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$

angles: $\angle A \cong \angle C$

$\angle B \cong \angle D$

$\angle A$ and $\angle D$ = supplementary

$\angle A$ and $\angle B$ = supplementary

$\angle B$ and $\angle C$ = supplementary

$\angle C$ and $\angle D$ = supplementary

$m\angle A + m\angle B + m\angle C + m\angle D = 360°$

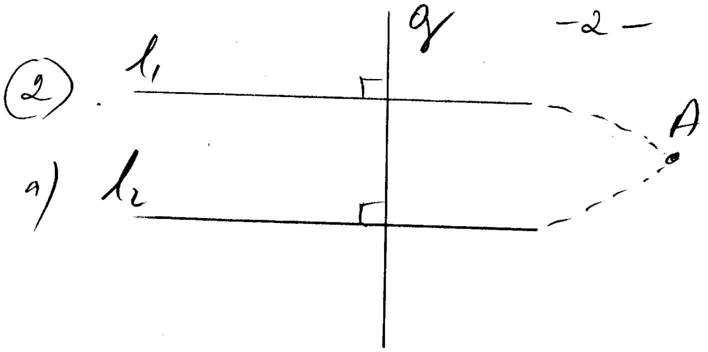
diagonals: $\overline{AC}$, $\overline{BD}$ - diagonals with $\overline{AC} \cap \overline{BD} = O$

Then O = midpoint of $\overline{AC}$ and O = midpoint of $\overline{BD}$

In general, $AC \neq BD$

(2).

a)



b) Given: $l_1 \perp g$

$l_2 \perp g$

l_1, l_2 = coplanar

Prove: $l_1 \parallel l_2$

c) Indirect proof:

Assume $l_1 \nparallel l_2$.
But l_1, l_2 : coplanar } =>

=> l_1 and l_2 have a common point, A

We have:

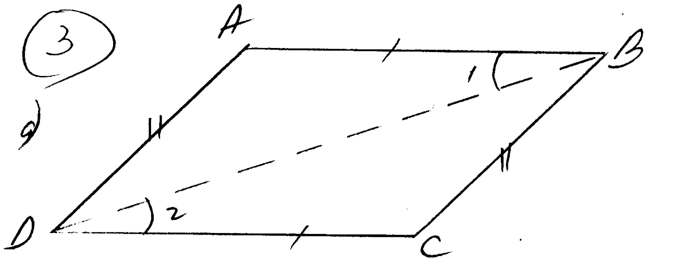
- a line, g
- a point, A
- and two lines, l_1 and l_2, each perpendicular to g, passing through A

This contradicts the Postulate that states that there is only one line perpendicular to a given line through a given point

Therefore, $l_1 \parallel l_2$.

(3)

a)



b) Given ABCD- quadrilateral

$\overline{AB} \cong \overline{DC}$

$\overline{AD} \cong \overline{BC}$

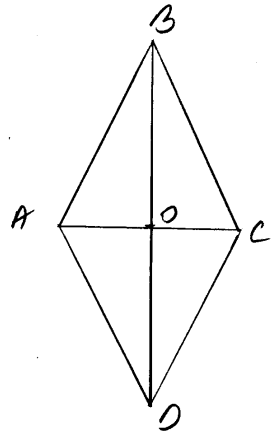
Prove: ABCD- parallelogram

We'll show $\overline{AB} \parallel \overline{DC}$

Proof

Statements	Reasons
1. ABCD- quadrilateral	1. given
2. Draw $\overline{BD}$	2. 2 points determine a line
3. $\triangle ABD$, $\triangle CDB$ { $\overline{BD} \cong \overline{BD}$, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$	3. { reflexive prop. of $\cong$, given, given
4. $\triangle ABD \cong \triangle CDB$	4. SSS
5. $\angle ABD \cong \angle CDB$	5. CPCTC
6. $\overline{AB} \parallel \overline{DC}$	6. ∥ if alternate interior ∡'s ≅ ($\overline{AB}$ and $\overline{DC}$ with transversal $\overline{DB}$)
7. ABCD-parallelogram	7. ▱ if opp. sides ∥ and ≅

(4)



(c)

(a) If a geometric figure is a rhombus, then its diagonals are perpendicular.

(b) A rhombus is a parallelogram with two adjacent sides congruent.

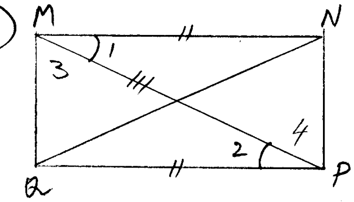
(d) Given: ABCD rhombus
$\overline{AC}, \overline{BD}$ - diagonals

Prove: $\overline{AC} \perp \overline{BD}$

(e) Proof

Statements	Reasons
1. ABCD-◇	1. given
2. $\triangle AOB$, $\triangle COB$: $\overline{AB} \cong \overline{BC}$, $\overline{AO} \cong \overline{CO}$, $\overline{BO} \cong \overline{BO}$	2. sides of ◇ are $\cong$; diag. of ▱ bisect each other; reflexive prop. of $\cong$
3. $\triangle AOB \cong \triangle COB$	3. SSS
4. $\angle AOB \cong \angle COB$	4. CPCTC
5. $\angle AOB$, $\angle COB$ adjacent $\angle$'s	5. definition adj. $\angle$'s
6. $\overline{AC} \perp \overline{BD}$	6. definition $\perp$ lines

(5)



Given: $\angle 1 \cong \angle 2$
$\overline{MN} \cong \overline{QP}$

Prove: $\overline{MQ} \parallel \overline{NP}$

We'll show $\angle 3 \cong \angle 4$

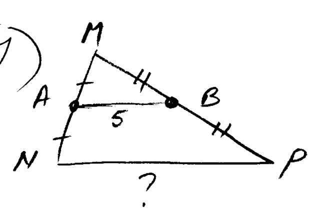
Proof

Statements	Reasons
1. $\triangle MNP$, $\triangle PQM$: $\overline{MN} \cong \overline{QP}$, $\angle 1 \cong \angle 2$, $\overline{MP} \cong \overline{MP}$	1. given; given; reflexive prop. of $\cong$
2. $\triangle MNP \cong \triangle PQM$	2. SAS
3. $\angle 4 \cong \angle 3$	3. CPCTC
4. $\overline{MQ} \parallel \overline{NP}$	4. $\parallel$ if alt. int. $\angle$'s $\cong$ ($\overline{MQ}$ and $\overline{NP}$ with transversal MP)

(6) Proof

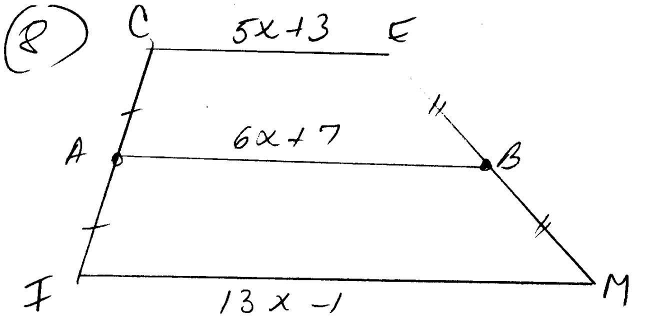
Statements	Reasons
1. $\triangle ABE$, $\triangle CBD$: $\overline{AB} \cong \overline{CB}$, $\angle A \cong \angle C$, $\angle ABE \cong \angle CBE$	1. given; given; reflexive prop. of $\cong$
2. $\triangle ABE \cong \triangle CBD$	2. ASA
3. $\overline{BE} \cong \overline{BD}$	3. CPCTC

(7)



$\triangle MNP$
A, B = midpoints
Then
$AB = \frac{1}{2} NP$
$2AB = NP$
so $NP = 2(5) = 10$ in

(8)



Solution

$\overline{AB}$ - median $\Rightarrow$ $AB = \frac{1}{2}(CE + FM)$

$2AB = CE + FM$

$2(6x+7) = (5x+3) + (13x-1)$

$12x + 14 = 5x + 3 + 13x - 1$

$12x + 14 = 18x + 2$

$14 - 2 = 18x - 12x$

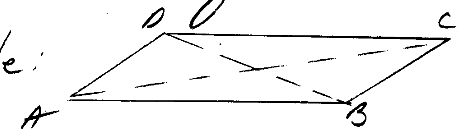
$12 = 6x, \quad x = 2$

(9) Proof

Statements	Reasons
1. $m \parallel n$	1. given
2. $\angle BAC \cong \angle CED$	2. If $\parallel$, then alt. int $\measuredangle$'s $\cong$ ($m \parallel n$, transv. $\overline{AE}$)
3. $\angle ABC \cong \angle EDC$	3. If $\parallel$ then alt. int $\measuredangle$'s $\cong$ ($m \parallel n$, transv. $\overline{BD}$)
4. $\overline{AB} \cong \overline{DE}$	4. given
5 $\triangle ABC \cong \triangle EDC$	5 ASA (2,3,4)

(10) (a) False. The diagonals of a parallelogram are not congruent, in general.

Counterexample: $ABCD$ - $\square$, but $\overline{AC} \ncong \overline{BD}$



(b) True; definition of a rhombus.

(c) False; in order to be a rhombus, a parallelogram must have all sides congruent. In general, in a parallelogram only the opposite sides are congruent, not two adjacent sides.

(d) False; AAA is not a case of triangle congruency.

(e) False; Two triangles can be proven congruent using one of the following: SAS, ASA, SSS, AAS.

(f) False because the sum of the measures of the angles in a triangle is 180°.

(g) False;

Counterexample: trapezoid with $m\angle A = m\angle D = 90°$, which is not a rectangle.