TEST #1 @ 140 points

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.
Write all answers on separate paper.

1. Complete the following Postulates and make a drawing to illustrate each Postulate:

a) *Segment – Addition Postulate:*
If R is a point on a segment AH, then ____________________________________

b) *Angle – Addition Postulate:*
If W is a point in the interior of the angle ANG, then ____________________________

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2. Do the following:

a) How many midpoints does a line segment have?

b) Draw a segment $\overline{BK}$ with A midpoint. Write using math notation that A is the midpoint of the line segment.

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3. Do the following:

a) Draw two vertical angles. Label the drawing using correct math notation. Name the two vertical angles.

b) What do you know about any two vertical angles?

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4. A triangle ABC is given. All the questions below refer to the triangle ABC.

a) Draw a scalene triangle ABC (draw a relatively big triangle).

b) Name the following:
- the angle opposite side $\overline{AB}$:
- the side opposite angle $\angle ABC$:
- the angle included by $\overline{AC}$ and $\overline{BC}$:
- an exterior angle of the triangle (make sure to mark it on the drawing) :

c) Using your figure, draw the bisector of angle B , name it $\overline{BD}$, and state , using mathematical notation, that $\overline{BD}$ is the bisector of angle B (what does it mean?).

d) Using your above triangle, draw the altitude from vertex A to the opposite side, name it $\overline{AE}$, and state, using mathematical notation, that $\overline{AE}$ is an altitude (what does it mean?).

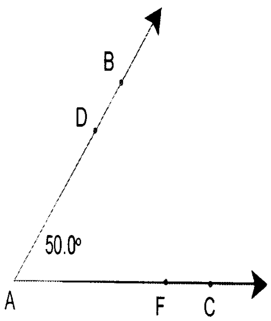
e) Using your above triangle , draw the median from vertex C, name it $\overline{CF}$, and state, using mathematical notation, that $\overline{CF}$ is a median (what does it mean?).

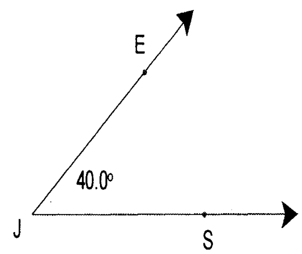
5. Given an angle $\angle AMP$,

PART I : Construct using only a compass and a straightedge, the bisector $\overrightarrow{ME}$ of the given angle. Explain in words the steps (how you are constructing it).

PART II: Prove that, indeed, the ray constructed is the bisector of the angle.

6.





Refer to the figure to answer true or false.

a) $\angle BAC$ is the same angle as $\angle BAF$ ________

b) $\angle SJE$ is complementary to $\angle CAD$ ________

c) $\angle EJS$ is an obtuse angle ________

d) $m\angle DAC + m\angle EJS = 90^{\circ}$ ________

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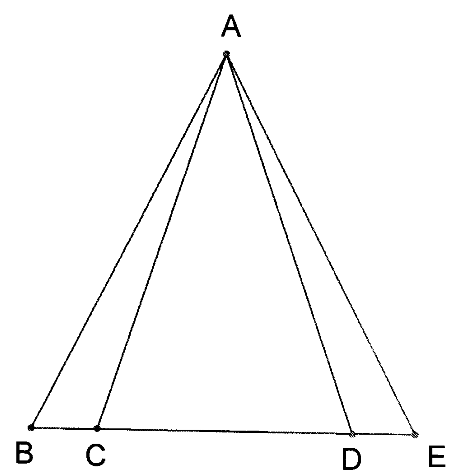
7. Two statements are given. If possible, write a third statement that can be deduced from these statements. Otherwise, write " no deduction possible".

a) All night owls hoot it up.
Fred never gives a hoot.
Therefore, ...

b) Tom would be a gardener if he had a green thumb.
If Tom were a gardener, he would raise bonsai trees.
Therefore,...

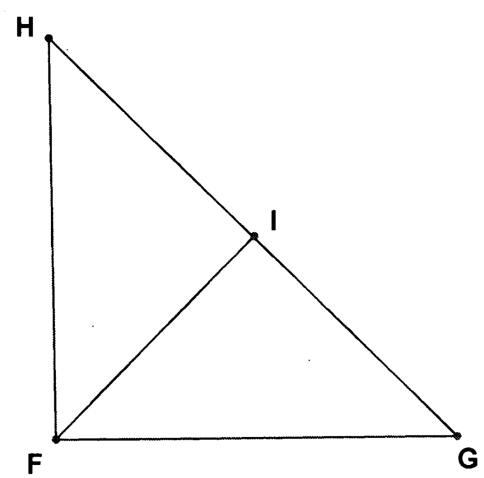
8.

In the given figure, $\overline{AC} \cong \overline{AD}$ and $\overline{BD} \cong \overline{CE}$.
Prove that $\overline{AB} \cong \overline{AE}$.



9. Show the **formal proof** of the following theorem: "Supplements of equal angles are equal." (Make sure you write the hypothesis and conclusion; make a drawing to illustrate the theorem.)

10.



Given: $\overline{FH} \cong \overline{FG}$
I midpoint of segment HG

Prove: $\overline{FI}$ bisects $\angle HFG$

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11. In a triangle ACD, $\overline{AB}$ is an altitude and also an angle bisector (B is on $\overline{CD}$). Show that the triangle ACD is isosceles.

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6. $\overrightarrow{ME}$ bisector of $\angle AMD$

Now we'll prove that, indeed, $\overrightarrow{ME}$ is an angle bisector

Statements	Reasons
1. Connect B with E, connect C with E	1. Two points determine a line
2. $\overline{MB} \cong \overline{MC}$	2. by construction
3. $\overline{BE} \cong \overline{CE}$	3. by construction (radii in congruent circles)
4. $\overline{ME} \cong \overline{ME}$	4. reflexive prop.
5. $\triangle MBE \cong \triangle MCE$ (2,3,4)	5. SSS
6. $\angle BME \cong \angle CME$	6. CPCTC
7. $\overrightarrow{ME}$ bisector	7. definition angle bisector

(6) a) True

b) True

c) False

d) True

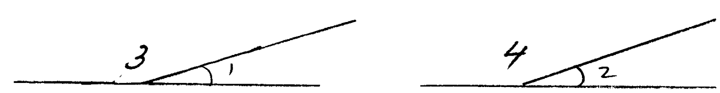
(7) (a) Fred is not a night owl.

(b) If Tom had a green thumb, then he would raise bonsai trees.

(8) Proof

Statements	Reasons
1. $\overline{AC} \cong \overline{AD}$	1. given
2. $\angle ADC \cong \angle ACD$	2. In $\triangle ACD$, if 2 sides $\cong$, opp. $\angle$'s $\cong$.
3. $\triangle ABD$, $\triangle AEC$: $\overline{AC} \cong \overline{AD}$, $\overline{BD} \cong \overline{EC}$, $\angle ADB \cong \angle ACE$	3. given, given, (2) above
4. $\triangle ABD \cong \triangle AEC$	4. SAS
5. $\overline{AB} \cong \overline{AE}$	5. CPCTC

(9)



Given $m\angle 1 = m\angle 2$

$\angle 1, \angle 3$ = supplementary

$\angle 2, \angle 4$ = supplementary

Prove $m\angle 3 = m\angle 4$

Proof

Statements	Reasons
1. $\angle 1, \angle 3$ = supplem.; $\angle 2, \angle 4$ = supplem.	1. given
2. $m\angle 1 + m\angle 3 = 180°$; $m\angle 2 + m\angle 4 = 180°$	2. definition of supplementary angles
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$ (2)	3. substitution or transitivity

4. $m\angle 1 = m\angle 2$	4. given
5. $m\angle 1 + m\angle 3 = m\angle 1 + m\angle 4$ (3,4)	5. substitution
6. $m\angle 3 = m\angle 4$	6. +/- property of equality

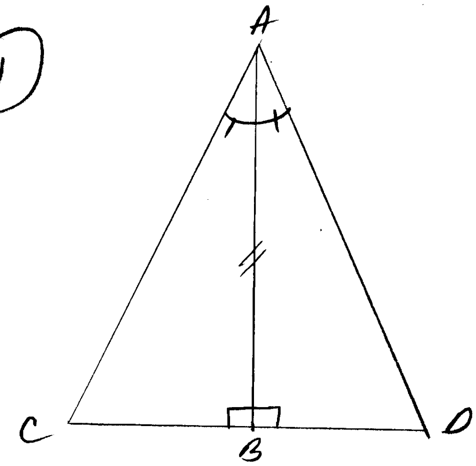
Q.E.D.

(10) Proof

Statements	Reasons
1. $\overline{FH} \cong \overline{FG}$	1. given
2. I = midpoint $\overline{HG}$	2. given
3. $\overline{HI} \cong \overline{IG}$	3. definition midpoint
4. $\overline{FI} \cong \overline{FI}$	4. reflexive prop.
5. $\triangle HFI \cong \triangle GFI$ (1,3,4)	5 SSS
6. $\angle GFI \cong \angle HFI$	6. CPCTC
7. $\overline{FI}$ bisects $\angle HFG$	7. definition angle bisector

Q.E.D.

(11)



Given $\triangle ACD$
$\overline{AB}$ - altitude
$\overline{AB}$ - bisector $\angle A$

Prove $\triangle ACD$ - isosceles

We'll prove $\overline{AC} \cong \overline{AD}$

For that, we'll show

$\triangle ACB \cong \triangle ADB$

(LA OR ASA)

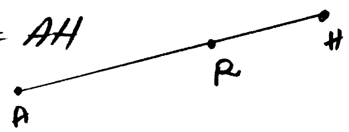
Proof

Statements	Reasons
1. $\triangle ACD$, $\overline{AB}$ - altitude	1. given
2. $\overline{AB} \perp \overline{CD}$	2. definition altitude
3. $\angle ABC$, $\angle ABD$ = right $\angle$'s	3. $\perp$ its right $\angle$'s
4. $\triangle ABC$, $\triangle ABD$ = right $\triangle$'s	4. definition right $\triangle$
5. $\overline{AB}$ bisector	5. given
6. $\angle CAB \cong \angle DAB$	6. definition angle bisector
7. $\overline{AB} \cong \overline{AB}$	7. reflexive prop.
8. $\triangle ABC \cong \triangle ABD$ (4,6,7)	8. LA
9. $\overline{AC} \cong \overline{AD}$	9. CPCTC
10. $\triangle ACD$ isosceles	10. definition isosceles $\triangle$

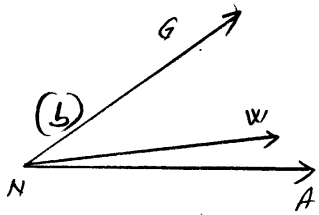
Math 61

TEST 1 - SOLUTIONS

① (a) $AR + RH = AH$



(b)



$m\angle ANW + m\angle WNG =$
$= m\angle ANG$

c) $\overline{BD}$ bisector of $\angle B$ iff
$\angle CBD \cong \angle DBA$
$(m\angle CBD = m\angle DBA)$

d) $\overline{AE}$ - altitude iff
$\overline{AE} \perp \overline{BC}$, $E \in \overline{BC}$

e) $\overline{CF}$ - median iff
$F \in \overline{AB}$, F = midpoint of $\overline{AB}$
$(\overline{AF} \cong \overline{FB})$

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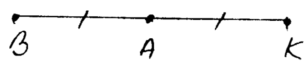
② (a) One and only one

(b)



$\begin{cases} A \in \overline{BK} \\ \overline{BA} \cong \overline{AK} \end{cases}$
$(BA = AK)$

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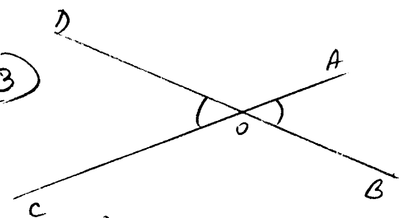
③



a) $\angle AOB$ and $\angle COD$

b) $\angle AOB \cong \angle COD$
$(m\angle AOB = m\angle COD)$

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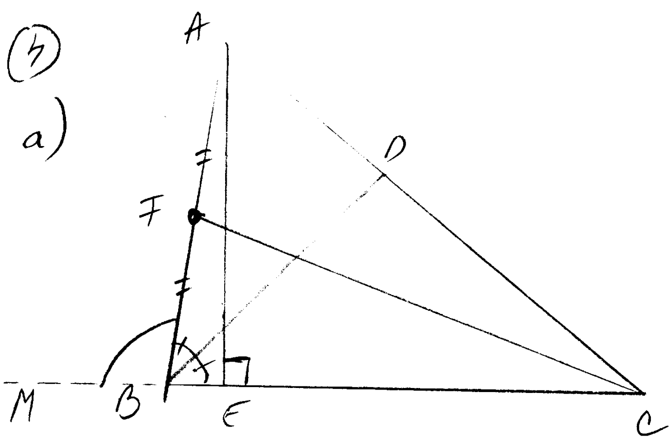
④ a)



b) $\angle C$ opposite $\overline{AB}$
$\overline{AC}$ opposite $\angle ABC$
$\angle C$ included by $\overline{AC}$ and $\overline{BC}$
$\angle ABM$ - exterior angle

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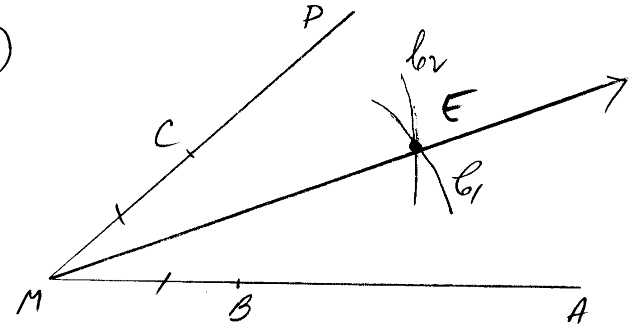
⑤



Given $\angle AMP$

Construct $\overrightarrow{ME}$ = bisector

(condition: $\angle AME \cong \angle EMP$)

Solution

1. Let $B \in \overrightarrow{MA}$
2. Mark off $C \in \overrightarrow{MP}$ such that $MB = MC$
3. Construct circle $\mathcal{C}_1$ with center B and radius r
4. Construct circle $\mathcal{C}_2$ with center C and radius r
5. Let $E = \mathcal{C}_1 \cap \mathcal{C}_2$
(the intersection of the two circles)