Math 61 Spring 2009 Name:

QUIZ #3 @ 70 points

Write in a neat and organized fashion. Use a pencil. Show all work to get credit.
Write all the solutions on separate paper.

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1. Simplify the following trigonometric expressions:

a) $\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{1+\sin x}$ b) $\cos^3\theta - \sin^2\theta\cos\theta$ c) $\tan\alpha\cos\alpha\csc\alpha$

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2. Sketch a right triangle that has one acute angle θ, and find the other five trigonometric ratios of θ knowing that

$\sin\theta = \dfrac{4}{9}$

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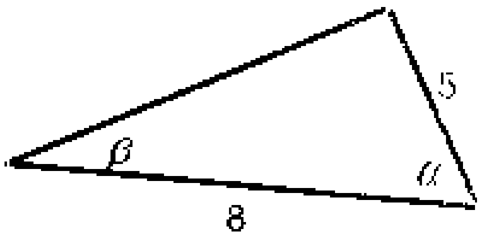
3. Find $\sin\alpha$, $\cos\beta$, $\tan\alpha$, $\cot\beta$ if



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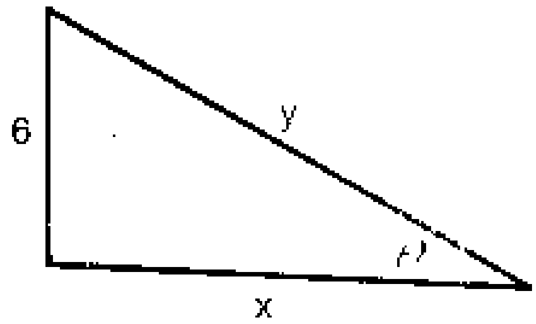
4. Express x and y in terms of trigonometric ratios of θ.



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5. Prove the following identities:

a) $\sin\alpha\cot\alpha = \cos\alpha$ b) $\dfrac{\cos x}{\sec x}+\dfrac{\sin x}{\csc x}=1$ c) $\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$

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6) From the top of a 165-ft lighthouse, the angle of depression to a ship in the ocean is 29°. How far is the ship from the base of the lighthouse?

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Quiz 3 - Solutions

① (a) $\frac{\overset{1+\sin x}{\sin x}}{\cos x} + \frac{\overset{\cos x}{\cos x}}{1+\sin x} =$

LCD $= \cos x(1+\sin x)$

$$= \frac{\sin x(1+\sin x)+\cos^2 x}{\cos x(1+\sin x)}$$

$$= \frac{\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$$

$$= \frac{\cancel{\sin x+1}}{\cos x(\cancel{1+\sin x})} = \frac{1}{\cos x} = \boxed{\sec x}$$

(b) $\cos^3\theta + \sin^2\theta\cos\theta =$

$= \cos\theta(\cos^2\theta+\sin^2\theta)$

$= \cos\theta \cdot 1 = \boxed{\cos\theta}$

(c) $\tan a \cos a \csc a =$

$$= \frac{\cancel{\sin a}}{\cancel{\cos a}} \cdot \frac{\cancel{\cos a}}{1} \cdot \frac{1}{\cancel{\sin a}} = \boxed{1}$$

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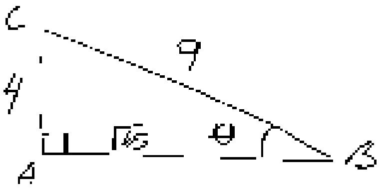
② 

$\sin\theta = \frac{4}{9}$ (given)

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{BC}$ } ⟹

⟹ let $AC = 4$, $BC = 9$

Then, $AB^2 + AC^2 = BC^2$

$AB^2 = 9^2 - 4^2$

$AB^2 = 65$, so $AB = \sqrt{65}$

Therefore,

$\sin\theta = \frac{4}{9}$ given

$\cos\theta = \frac{\sqrt{65}}{9}$

$\tan\theta = \frac{4}{\sqrt{65}}$, $\tan\theta = \frac{4\sqrt{65}}{65}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{\sqrt{65}}{4}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{9}{4}$

$\sec\theta \quad \frac{1}{\cos\theta} = \frac{9}{\sqrt{65}} = \frac{9\sqrt{65}}{65}$

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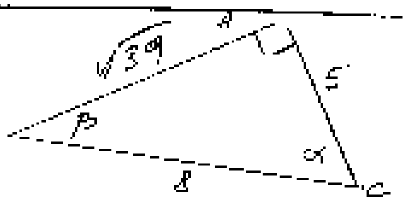
③ Let ΔABC with $AC = 5$, $BC = 8$ (given)



Then $AB^2 + AC^2 = BC^2$

$AB^2 = 8^2 - 5^2$

$AB^2 = 39$, so $AB = \sqrt{39}$

$\sin\alpha = \frac{AB}{BC}$

$\boxed{\sin\alpha = \frac{\sqrt{39}}{8}}$

$\cos\beta = \frac{AB}{BC}$

$\boxed{\cos\beta = \frac{\sqrt{39}}{8}}$

$\cot\beta = \frac{AB}{AC}$

$\boxed{\cot\beta = \frac{\sqrt{39}}{5}}$

$\tan\alpha = \frac{AB}{AC}$

$\boxed{\tan\alpha = \frac{\sqrt{39}}{5}}$

④ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan\theta = \frac{6}{x} \Rightarrow$

$x\tan\theta = 6 \Rightarrow \boxed{x = \frac{6}{\tan\theta}}$

or $x = 6\cot\theta$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin\theta = \frac{6}{y} \Rightarrow$

$y\sin\theta = 6 \Rightarrow \boxed{y = \frac{6}{\sin\theta}}$

or $y = 6\csc\theta$

⑤ (a) $\sin a \cot a = \cos a$

Proof

LHS $= \sin a \cot a$

$= \cancel{\sin a} \cdot \frac{\cos a}{\cancel{\sin a}}$

$= \cos a = RHS$

Therefore, the given equation is an identity

(b) $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$

Proof

LHS $= \frac{\cos x}{\sec x} + \frac{\sin x}{\csc x}$

$= \frac{\cos x}{\frac{1}{\cos x}} + \frac{\sin x}{\frac{1}{\sin x}}$

$= \cos^2 x + \sin^2 x$

$= 1 = RHS$

So, the given equation is an identity

(c) $\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$

Proof

LHS $= \cos\theta(\sec\theta - \cos\theta)$

$= \cos\theta\left(\frac{1}{\cos\theta} - \cos\theta\right)$

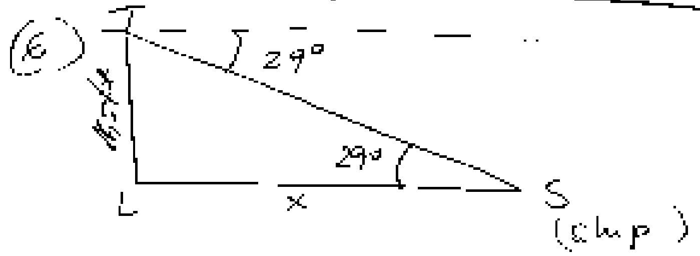
$= \frac{\cos\theta}{\cos\theta} - \cos^2\theta$

$= 1 - \cos^2\theta = \sin^2\theta$

$= RHS$

So, the given equation is an identity

⑥



Let TL = the lighthouse

then ∡ TSL = 29°

Let x = distance between ship and base of lighthouse.

$\tan 29° = \frac{165\ ft}{x} \Rightarrow$

$x = \frac{165\ ft}{\tan 29°}$

$x \approx 298$ ft