REVIEW TEST 1
Chapters 1& 2

Chapter 2 – Limits and Continuity
After completing this section, you should:
- be able to find the rate of change of a function over an interval
- know the definition of the limit of a function at a point
- be able to find limits given a graph
- be able to calculate limits using The Limit Laws
- know and be able to apply The Sandwich (Squeeze) Theorem in finding limits
- be able to calculate one-sided limits and infinite limits
- know the definition of a function continuous at a point
- be able to recognize points of discontinuity
- be able to find the slope of the tangent to a curve at a given point
- be able to find an equation of the tangent line to a graph at a point
- be able to find vertical and horizontal tangents to a graph
- know the epsilon-delta definition of a limit of a function at a point
- be able to find delta knowing epsilon
- be able to recognize the special cases $\dfrac{0}{0}, 0\cdot\infty, \infty-\infty, \dfrac{\infty}{\infty}$
- know how to find special – case limits

To prepare for the test, you should study **all quizzes** and all **examples done in class**, as well as your **homework** (check website for solutions to selected homework exercises).

More practice:

Textbook – Practice Exercises page 139 : # 1, 3, 9 – 23 odd, 29 – 41 odd

1. Let $f(x) = \begin{cases} \sqrt{-x}, x < 0 \\ 3-x, 0 \le x < 3 \\ (x-3)^2, x > 3 \end{cases}$.

a) Evaluate each limit, if it exists.

(i) $\lim\limits_{x\to 0^+} f(x)$ (ii) $\lim\limits_{x\to 0^-} f(x)$ (iii) $\lim\limits_{x\to 0} f(x)$ (iv) $\lim\limits_{x\to 3^-} f(x)$ (v) $\lim\limits_{x\to 3^+} f(x)$ (vi) $\lim\limits_{x\to 3} f(x)$

b) Where is f discontinuous?

2. Find a equation of the tangent line to the curve $y = x^3 - 2x$ at the point $(2, 4)$

3. Let $f(x) = \begin{cases} x+1, x \le a \\ x^2, x > a \end{cases}$ Find an a such that f is continuous everywhere.

4-11. Find the limits.

4. $\lim\limits_{x\to 1} e^{x^3 - x}$

5. $\lim\limits_{x\to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

6. $\lim\limits_{h\to 0} \dfrac{(h-1)^3 + 1}{h}$

7. $\lim\limits_{r\to 9} \dfrac{\sqrt{r}}{(r-9)^4}$

8. $\lim\limits_{v\to 4^+}\dfrac{4-v}{|4-v|}$

9. $\lim\limits_{x\to\infty}e^{-3x}$

10. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{1-x^2}}{x}$

11. $\lim\limits_{x\to\infty}\dfrac{\sqrt{3x^2-1}}{x-1}$

12. Find $\lim\limits_{x\to 0}\sqrt{x^3+x^2}\,\sin\dfrac{p}{x}$.

13. Prove that $\lim\limits_{x\to 0}x^4\cos\dfrac{2}{x}=0$

14. Prove that $\lim\limits_{x\to 0^+}\sqrt{x}\,e^{\sin(p/x)}=0$

15-16. Find the limit, if it exists. If the limit does not exist, explain way.

15. $\lim\limits_{x\to -4}|x+4|$

16. $\lim\limits_{x\to 2}\dfrac{|x-2|}{x-2}$

17 – 22 Find the limit.

17. $\lim\limits_{x\to 5^-}\dfrac{e^x}{(x-5)^3}$

18. $\lim\limits_{x\to\infty}\dfrac{x+2}{\sqrt{9x^2+1}}$

19. $\lim\limits_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

20. $\lim\limits_{x\to\infty}\dfrac{\sin^2 x}{x^2}$

21. $\lim\limits_{x\to\infty}\cos x$

22. $\lim\limits_{x\to -\infty}x^3-5x^2$

23 – 25 Find :

23. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x}$

24. $\lim\limits_{x\to -1}\dfrac{\sin\left(x^2-x-2\right)}{x+1}$

25. $\lim\limits_{x\to 1}\dfrac{\sin\left(1-\sqrt{x}\right)}{x-1}$

26. Find $\lim\limits_{x\to\infty}f(x)$ if $\dfrac{4x-1}{x}<f(x)<\dfrac{4x^2+3x}{x^2}$ for all $x>5$.

Answers

1) a) (i) 3   (ii) 0   (iii) DNE   (iv) 0   (v) 0   (vi) 0 ;   b) 0, -3

2) $y-4=10(x-2)$; 3) $\dfrac{1\pm\sqrt{5}}{2}$; 4) 1; 5) $\dfrac{3}{2}$; 6) 3; 7) ∞; 8) -1; 9) 0; 10) 0; 11) $\sqrt{3}$; 12) 0; 13); 14); 15) 0

16) DNE; 17) $-\infty$; 18) $\dfrac{1}{3}$; 19) $\dfrac{1}{6}$; 20) 0; 21) DNE; 22) $-\infty$; 23) 0; 24) -3; 25) $-\dfrac{1}{2}$; 26) 4