SOLUTIONS

Math 180 Spring 09 Name:

QUIZ #5 @ 50 points

Write neatly. Show all work. **Write all responses on separate paper.** **Clearly label the exercises.**

Find the following:

1) $\displaystyle\int_0^2 \sqrt{2r+1}\,dr$

2) $\displaystyle\int_0^{\pi} 3\cos^2 x \sin x\,dx$

3) $\displaystyle\int_0^1 \frac{5x}{\left(4+x^2\right)^2}\,dx$

4) $\displaystyle\int_0^1 \frac{8r}{4r^2-5}\,dr$

5) $\displaystyle\int_0^{\pi/4} \left(1+e^{\tan\theta}\right)\sec^2\theta\,d\theta$

6) $\displaystyle\int \cot t\,dt$

7) $\displaystyle\int e^x \sin x\,dx$

8) $\displaystyle\int_0^{2\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\,dx$

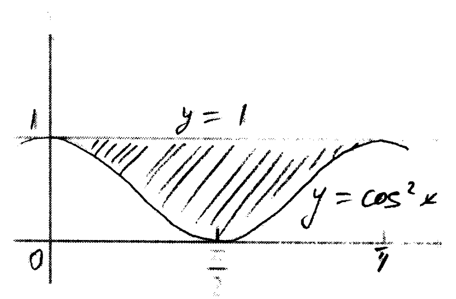
9) $\displaystyle\int \ln x\,dx$

10) $\displaystyle\int 2r\sqrt{1+r^2}\,dr$

11) $\displaystyle\int \sin^2 x\,dx$

Find the total areas of the shaded regions:

12) the area between the curves: $y=1$ and $y=\cos^2 x$ between $x=0$ and $x=\pi$.



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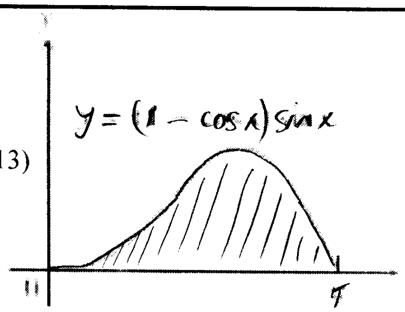
13)



the area between the curve $y=(1-\cos x)\sin x$ and the x-axis between $x=0$ and $x=\pi$.

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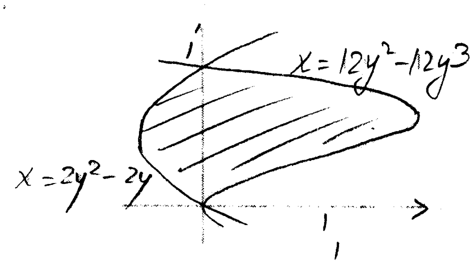
14) the area between the curves $x=2y^2-2y$ and $x=12y^2-12y^3$



Math 180

Quiz 5 Solutions

① $\int_0^2 \sqrt{2r+1}\, dr$

let $2r+1 = u$
$2\,dr = du$
$dr = \frac{1}{2}\, du$

when $r=0$, $u = 2(0)+1 = 1$
$r = 2$, $u = 2(2)+1 = 5$

$$\int_0^2 \sqrt{2r+1}\, dr = \int_1^5 \frac{1}{2}\sqrt{u}\, du$$
$$= \frac{1}{2}\int_1^5 u^{\frac{1}{2}}\, du$$
$$= \frac{1}{2}\cdot \frac{u^{\frac{1}{2}+1}}{\frac{3}{2}}\Big]_1^5$$
$$= \frac{1}{2}\cdot\frac{2}{3}\, u^{\frac{3}{2}}\Big]_1^5$$
$$= \frac{1}{3}\left(5^{\frac{3}{2}}-1\right) = \boxed{\frac{1}{3}\left(5\sqrt{5}-1\right)}$$

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② $\int_0^{\pi} 3\cos^2 x \sin x\, dx$

let $\cos x = u$
$-\sin x\, dx = du$

when $x=0$, $u = \cos 0 = 1$
$x = \pi$, $u = \cos\pi = -1$

$$\int_0^{\pi} 3\cos^2 x \sin x\, dx = \int_1^{-1} -3u^2\, du$$
$$= 3\int_{-1}^{1} u^2\, du$$
$$= 3\,\frac{u^3}{3}\Big]_{-1}^{1}$$
$$= u^3\Big]_{-1}^{1} = 1^3 - (-1)^3$$
$$= 1-(-1) = \boxed{2}$$

③ $\int_0^1 \frac{5x}{(4+x^2)^2}\, dx$

let $4+x^2 = u$
$2x\, dx = du$
$x\, dx = \frac{1}{2}\, du$

when $x=0$, $u = 4$
$x = 1$, $u = 4+1 = 5$

$$\int_0^1 \frac{5x}{(4+x^2)^2}\, dx = 5\int_4^5 \frac{1}{2}\cdot\frac{1}{u^2}\, du$$
$$= \frac{5}{2}\int_4^5 u^{-2}\, du$$
$$= \frac{5}{2}\,\frac{u^{-2+1}}{-1}\Big]_4^5$$
$$= \frac{-5}{2}\, u^{-1}\Big]_4^5$$
$$= \frac{-5}{2}\left(5^{-1}-4^{-1}\right) = \frac{-5}{2}\left(\frac{1}{5}-\frac{1}{4}\right)$$
$$= \frac{-5}{2}\cdot\frac{-1}{20} = \boxed{\frac{1}{8}}$$

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④ $\int_0^1 \frac{8r}{4r^2-5}\, dr$ let $4r^2-5 = u$
$8r\,dr = du$
when $r=0$, $u=-5$
$r=1$, $u=-1$

so $\int_0^1 \frac{8r}{4r^2-5}\, dr = \int_{-5}^{-1} \frac{du}{u}$
$$= \ln|u|\Big]_{-5}^{-1}$$
$$= \ln|-1| - \ln|-5|$$
$$= \ln 1 - \ln 5$$
$$= \boxed{-\ln 5}$$

(5) $\int_0^{\frac{\pi}{4}} \left(1+e^{\tan\theta}\right) \sec^2\theta \, d\theta = I$

let $\tan\theta = u$

$\sec^2\theta \, d\theta = du$

when $\theta = 0$, $u = \tan 0 = 0$

$\theta = \frac{\pi}{4}$, $u = \tan\frac{\pi}{4} = 1$

$I = \int_0^1 (1+e^u) \, du$

$= \int_0^1 du + \int_0^1 e^u \, du$

$= u\Big]_0^1 + e^u\Big]_0^1$

$= (1-0) + (e^1 - e^0)$

$= 1 + e - 1 = \boxed{e}$

(6) $I = \int \cot t \, dt = \int \frac{\cos t}{\sin t} dt$

let $\sin t = u$

$\cos t \, dt = du$

$I = \int \frac{du}{u} = \ln|u| + C$

$= \boxed{\ln|\sin t| + C}$

(7) $\int e^x \sin x \, dx$

let $\left[f = e^x \quad g' = \sin x \right.$

then $\left[f' = e^x \quad g = -\cos x \right.$

Then, $I = \int e^x \sin x \, dx =$

$I = -e^x \cos x - \int -e^x \cos x \, dx$

$I = -e^x \cos x + \int e^x \cos x \, dx$

let $\left[f = e^x \quad g' = \cos x \right.$

then $\left[f' = e^x \quad g = \sin x \right.$

$I = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$

$I = -e^x \cos x + e^x \sin x - I$

$2I = -e^x \cos x + e^x \sin x$

$\boxed{I = -\frac{e^x}{2} \cos x + \frac{e^x}{2} \sin x + C}$

(8) $\int_0^{2\pi} \frac{\cos x}{\sqrt{4+3\sin x}} dx$

let $4 + 3\sin x = u$

$3\cos x \, dx = du$

$\cos x \, dx = \frac{1}{3} du$

when $x = 0$, $u = 4$

$x = 2\pi$, $u = 4$

So $\int_0^{2\pi} \frac{\cos x}{\sqrt{4+3\sin x}} dx = \int_4^4 \frac{du}{3\sqrt{u}}$

$= \boxed{0}$

(9) $\int \ln x \, dx$

let $f = \ln x \searrow g' = 1$
then $f' = \frac{1}{x} \leftarrow g = x$

$$\int \ln x \, dx = x\ln x - \int \frac{1}{x} x \, dx$$
$$= x\ln x - \int dx$$
$$= \boxed{x\ln x - x + C}$$

(10) $\int 2y\sqrt{1+y^2}\, dy$

let $1+y^2 = u$
$2y\,dy = du$

$$\int 2y\sqrt{1+y^2}\, dy = \int \sqrt{u}\, du$$
$$= \int u^{\frac{1}{2}} du$$
$$= \frac{2u^{\frac{3}{2}}}{3} + C$$
$$= \boxed{\frac{2}{3}(1+y^2)^{\frac{3}{2}} + C}$$

(11) $\int \sin^2 x \, dx$

Method I - Integration by Parts

let $f = \sin x \searrow g' = \sin x$
then $f' = \cos x \leftarrow g = -\cos x$

$$I = \int \sin^2 x \, dx$$
$$= -\sin x \cos x - \int -\cos^2 x \, dx$$
$$I = -\sin x \cos x + \int \cos^2 x \, dx$$
$$I = -\sin x \cos x + \int (1-\sin^2 x)\, dx$$
$$I = -\sin x \cos x + \int dx - \int \sin^2 x \, dx$$
$$I = -\sin x \cos x + x - I$$
$$2I = -\sin x \cos x + x$$
$$\boxed{I = \frac{-1}{2}\sin x \cos x + \frac{x}{2} + C}$$

Method II

$$I = \int \sin^2 x \, dx$$
$$\cos 2x = 1 - 2\sin^2 x$$
$$\sin^2 x = \frac{1-\cos 2x}{2}$$
$$I = \int \frac{1-\cos 2x}{2} dx$$
$$= \int \frac{1}{2} dx - \int \frac{\cos 2x}{2} dx$$
$$= \frac{1}{2}x - \frac{1}{2}\cdot\frac{\sin 2x}{2} + C$$
$$= \boxed{\frac{x}{2} - \frac{\sin 2x}{4} + C}$$

(12) Boundary curves are:

$$\begin{cases} y = 1 \text{ and} \\ y = \cos^2 x \end{cases}$$

Limits of integration are:

$x = 0$ and $x = \pi$

Let A = area

$$A = \int_0^{\pi} (1 - \cos^2 x)\, dx$$

$$A = \int_0^{\pi} \sin^2 x \, dx$$

$$A = \int_0^{\pi} \frac{1-\cos 2x}{2}\, dx$$

$$A = \frac{1}{2} x \Big]_0^{\pi} - \frac{1}{2} \frac{\sin 2x}{2} \Big]_0^{\pi}$$

$$A = \frac{\pi}{2} - \frac{1}{4} \sin 2\pi$$

$$\boxed{A = \frac{\pi}{2}}$$

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(13) Let A = area

Note $y = (1-\cos x)\sin x \geq 0$ for any $x \in [0, \pi]$

So

$$A = \int_0^{\pi} y\, dx$$

$$A = \int_0^{\pi} (1-\cos x) \sin x \, dx$$

let $1 - \cos x = u$

$-(-\sin x)dx = du$

$\sin x \, dx = du$

when $x = 0$, $u = 0$

$x = \pi$, $u = 2$

Therefore,

$$A = \int_0^2 u\, du$$

$$A = \frac{u^2}{2}\Big]_0^2 = \frac{1}{2}(2^2 - 0)$$

$$\boxed{A = 2}$$

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(14) Boundary curves are:

$$\begin{cases} x = 2y^2 - 2y \\ x = 12y^2 - 12y^3 \end{cases}$$

The limits of integration are:

$$\begin{cases} y = 0 \\ y = 1 \end{cases}$$

Let A = area

$$A = \int_0^1 \left((12y^2 - 12y^3) - (2y^2 - 2y) \right) dy$$

$$A = \int_0^1 (12y^2 - 12y^3 - 2y^2 + 2y)\, dy$$

$$A = \int_0^1 (10y^2 - 12y^3 + 2y)\, dy$$

$$= \left[10 \frac{y^3}{3} - 12 \cdot \frac{y^4}{4} + y^2 \right]_0^1$$

$$= \frac{10}{3} - 3 + 1$$

$$= \frac{10}{3} - 2 = \frac{4}{3}$$

So $\boxed{A = \frac{4}{3}}$