Math 180 Spring 2009 Name:

QUIZ #4 @ 50 points

Write neatly. Show all work. Use only information covered up to this point.
Write all responses on separate paper. **Clearly label the exercises.**

1. Find the following:

a) $\int \left(3x - \frac{2}{x^{1/2}} \right) dx$ b) $\int_1^2 (e^{2x} + x^5 - 4^x) dx$ c) $\int_0^p \left(3\cos 2t - \frac{1}{2}\sin\frac{t}{3} \right) dt$

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2. Graph the given function $f(x) = 2x^3$ between $x = 0$ and $x = 1$. Then:

a) Using four rectangles whose height is given by the value of the function at the right-end of each subinterval, estimate the area under the graph.

b) Find the exact area between the graph and the x-axis.

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3. Find the total area between the region and the x-axis.

$$y = 3x^2 - 3, \quad -2 \le x \le 2$$

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4. If $y = \int_{\tan x}^{0} \frac{dt}{1+t^2}$, find $\frac{dy}{dx}$.

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5. Complete the following definitions, properties, theorems, or formulas:

a) The Max – Min Inequality for Definite Integrals:
If $m \le f(x) \le M$ for $x \in [a,b]$, then ____

b) Write the definition of the definite integral using Riemann sums and support your answer with a graph.

c) $\frac{d}{dx}\int_a^x f(t)\,dt =$ ____________________

d) $\int x^n dx =$ ___________________

e) $\int \frac{1}{x} dx =$ ___________________

6. Suppose that *f* is a differentiable function shown in the accompanying graph and that the position at time *t* (sec) of a particle moving along a coordinate axis is

$$s = \int_0^t f(x)\,dx$$

meters. Use the graph to answer the following questions. Give reasons for your answers.

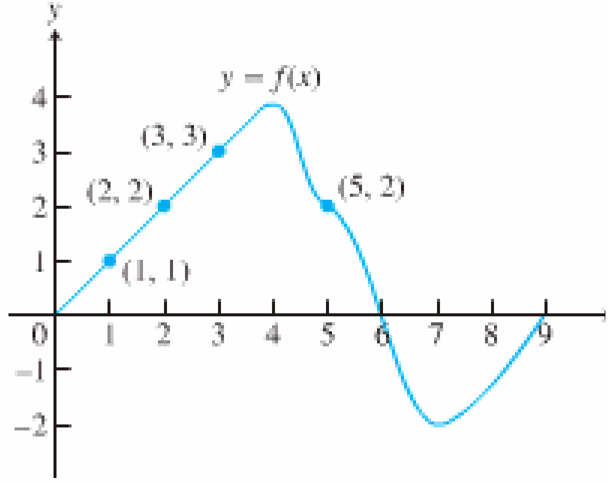
a) What is the particle's velocity at $t = 5$?

b) Is the acceleration at time $t = 5$ positive or negative?

c) What is the particle's position at time $t = 3$?

d) Approximately when is the acceleration zero?

e) On which side of the origin does the particle lie at time $t = 9$?



Quiz #4 – Solutions

① (a) $\int\left(3x - \frac{2}{x^{\frac{1}{2}}}\right)dx =$

$= 3\int x\,dx - 2\int x^{-\frac{1}{2}}\,dx$

$= 3\frac{x^2}{2} - 2\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{3}{2}x^2 - 4x^{\frac{1}{2}} + C$

(b) $\int_1^2 (e^{2x} + x^5 - 4^x)\,dx =$

$= \left(\frac{e^{2x}}{2} + \frac{x^6}{6} - \frac{4^x}{\ln 4}\right)\Big]_1^2$

$= \left(\frac{e^4}{2} + \frac{2^6}{6} - \frac{4^2}{\ln 4}\right) - \left(\frac{e^2}{2} + \frac{1}{6} - \frac{4}{\ln 4}\right)$

$= \frac{1}{2}(e^4 - e^2) + \frac{63}{6} - \frac{12}{\ln 4}$

$= \frac{1}{2}(e^4 - e^2) + \frac{21}{2} - \frac{12}{\ln 4}$

(c) $\int_0^{\pi}\left(3\cos 2t - \frac{1}{2}\sin\frac{t}{3}\right)dt =$

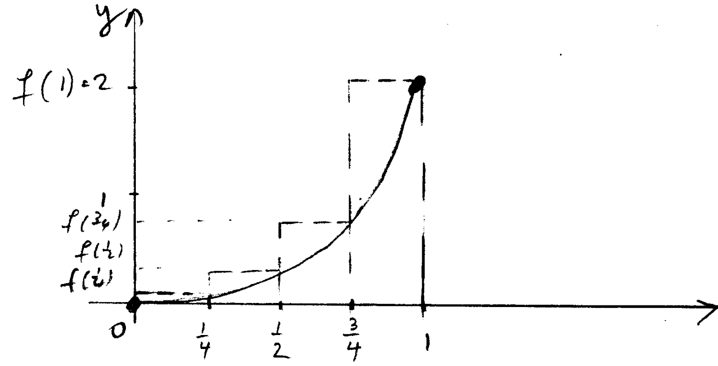
$= 3\int_0^{\pi}\cos 2t\,dt - \frac{1}{2}\int_0^{\pi}\sin\frac{t}{3}\,dt$

$= 3\,\frac{\sin 2t}{2}\Big]_0^{\pi} - \frac{1}{2}\left(\frac{-\cos\frac{t}{3}}{\frac{1}{3}}\right)\Big]_0^{\pi}$

$= \frac{3}{2}(\sin 2\pi - \sin 0) + \frac{3}{2}\left(\cos\frac{\pi}{3} - \cos 0\right)$

$= \frac{3}{2}\cdot\frac{1}{2} - \frac{3}{2} = \frac{3}{4} - \frac{3}{2} = \frac{-3}{4}$

② $f(x) = 2x^3, \quad x \in [0,1]$



let A = area under the graph

(a) $A \approx \frac{1}{4}f(\frac{1}{4}) + \frac{1}{4}f(\frac{1}{2}) + \frac{1}{4}f(\frac{3}{4}) + \frac{1}{4}f(1)$

$A \approx \frac{1}{4}\left[2(\frac{1}{4})^3 + 2\cdot(\frac{1}{2})^3 + 2(\frac{3}{4})^3 + 2(1)^3\right]$

$A \approx \frac{2}{4}\left[\frac{1}{64} + \frac{1}{8} + \frac{27}{64} + 1\right]$

$A \approx \frac{1}{2}\left(\frac{7}{16} + \frac{1}{8} + 1\right) = \frac{1}{2}\cdot\frac{25}{16} = \frac{25}{32}$

$A \approx \frac{25}{32}$

(b) $f(x) \geq 0$ for any $x \in [0,1]$

Therefore,

$A = \int_0^1 2x^3\,dx = 2\int_0^1 x^3\,dx$

$= 2\frac{x^4}{4}\Big]_0^1 = \frac{1}{2}(1^4 - 0) = \frac{1}{2}$

$A = \frac{1}{2}$

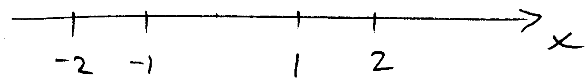
③ $y = 3x^2 - 3, \quad -2 \le x \le 2$

x-n: $3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$x = -1$ or $x = 1$

let A = area between the curve and the x-axis



$$A = \left|\int_{-2}^{-1} y\,dx\right| + \left|\int_{-1}^{1} y\,dx\right| + \left|\int_{1}^{2} y\,dx\right|$$

$$A = \left|\int_{-2}^{-1}(3x^2-3)\,dx\right| + \left|\int_{-1}^{1}(3x^2-3)\,dx\right| + \left|\int_{1}^{2}(3x^2-3)\,dx\right|$$

$$A = \left|(x^3-3x)\Big]_{-2}^{-1}\right| + \left|(x^3-3x)\Big]_{-1}^{1}\right| + \left|(x^3-3x)\Big]_{1}^{2}\right|$$

$$A = \left|(-1-3(-1)) - ((-8)-3(-2))\right| +$$
$$+ \left|(1-3) - (-1-3(-1))\right| +$$
$$+ \left|(8-3(2)) - (1-3(1))\right|$$

$$A = |2+2| + |-2-2| + |2+2|$$

$$A = 12$$

④ $y = \int_{\tan x}^{0} \frac{dt}{1+t^2}$

$$y = -\int_0^{\tan x} \frac{1}{1+t^2}\,dt$$

let $\tan x = u$

then $y = -\int_0^{u} \frac{1}{1+t^2}\,dt$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dt}$$

$$\frac{dy}{dx} = \frac{d}{du}\left(-\int_0^{u} \frac{1}{1+t^2}\,dt\right) \cdot \frac{du}{dt}$$

$$\frac{dy}{dx} = -\frac{d}{du}\left(\int_0^{u} \frac{1}{1+t^2}\,dt\right) \cdot \frac{du}{dt}$$

$$\frac{dy}{dx} = -\frac{1}{1+u^2} \cdot \frac{du}{dt}$$

$$\frac{dy}{dx} = \frac{-1}{1+\tan^2 x} \cdot \sec^2 x$$

$$\frac{dy}{dx} = \frac{-\sec^2 x}{\sec^2 x} = -1$$

$$\frac{dy}{dx} = -1$$

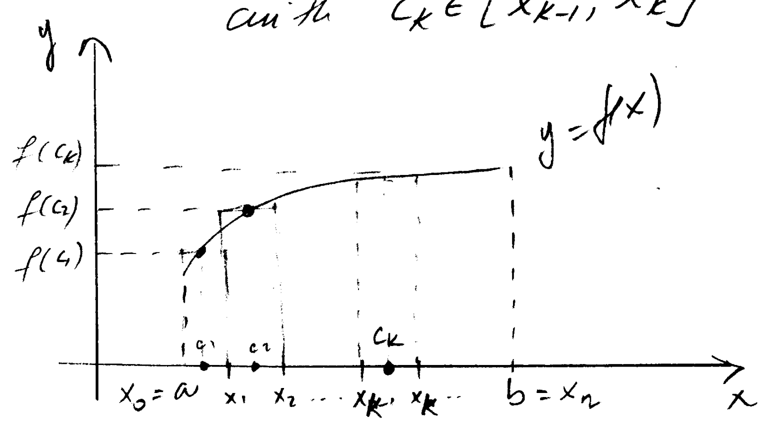
⑤ ⓐ $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

ⓑ $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\,\Delta x$

where f = continuous function on $[a,b]$

$\Delta x = \frac{b-a}{n}$

$c_1, c_2, \dots, c_n$ = sample points with $c_k \in [x_{k-1}, x_k]$



(c) $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

(d) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

(e) $\int \frac{1}{x}\,dx = \ln|x| + C$
$(x \neq 0)$

(6) $s = \int_0^t f(x)\,dx$ - position function

Then $\frac{ds}{dt}$ = velocity function

$\frac{ds}{dt} = \frac{d}{dt}\int_0^t f(x)\,dx = f(t)$

So the graph of $y = f(x)$ represents the velocity function

(a) $t=5$, $f(5)=2$

(b) $\frac{d^2s}{dt^2} = \frac{df}{dt}$: acceleration

At $t=5$, the slope of the tangent to the graph is negative, so the acceleration is negative

(c) $t=3$,
$s = \int_0^3 f(x)\,dx$ - this represents the area under the graph of $y=f(x)$ and the x-axis between $x=0$ and $x=3$

$\int_0^3 f(x)\,dx = \frac{1}{2}\cdot 3\cdot 3 = 4.5$ m

(d) acceleration is zero when the slope of the tangent to the curve is zero <=> tangent is horizontal

At about $t=4$ and $t=7$

(e) $s = \int_0^9 f(x)\,dx$ gives the position of the particle at $t=9$

The area above the x-axis is greater than the area below the x-axis =>

$\int_0^9 f(x)\,dx > 0$

so the particle is on the right side of the origin