Math 180 Spring 2009 Name:

QUIZ #3 @ 50 points

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1. Find the derivative of each function:

   a) $y = \sin^{-1}\left(x^2\right)$

   b) $y = \tan^{-1}\left(\ln x\right)$

   c) $r = x\cos^{-1} x + \sqrt{1-x^2}$

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2. A particle moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1}\sqrt{t}$.
   What is the velocity of the particle when $t = 16$?

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3. The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30-t)^2$.
   a) How fast is the water running out at the end of 10 minutes?

   b) What is the average rate at which the water flows out during the first 10 minutes?

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4. The radius r and height h of a right circular cylinder are related to the cylinder's volume V by the formula $V = \pi r^2 h$.
   a) What is dV/dt if r is constant?

   b) What is dV/dt if h is constant?

   c) What is dV/dt if neither r nor h is constant?

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5. When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.01 cm/min.
   At what rate is the plate's area increasing when the radius is 50 cm?

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6. The position of a particle is given by the equation
   $s = f(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s in meters
   a) Find the velocity at time t.

   b) When is the particle at rest?

   c) When is the particle moving forward (that is, in the positive direction)? When is it moving backward?

   d) Find the total distance traveled by the particle during the first five seconds.

   e) Find the acceleration at time t.

   f) When is the particle speeding up? When is it slowing down?

MATH 180

QUIZ 3 - SOLUTIONS

① (a) $y = \sin^{-1}(x^2)$

$$y' = \frac{dy}{dx} = \frac{1}{\sqrt{1-(x^2)^2}}\,(x^2)'$$

$$\boxed{y' = \frac{2x}{\sqrt{1-x^4}}}$$

(b) $y = \tan^{-1}(\ln x)$

$$y' = \frac{1}{1+(\ln x)^2}\,(\ln x)'$$

$$\boxed{y' = \frac{1}{x(1+\ln^2 x)}}$$

(c) $r = x\cos^{-1}x + \sqrt{1-x^2}$

$$r' = x'\cos^{-1}x + x(\cos^{-1}x)' + \left(\sqrt{1-x^2}\right)'$$

$$r' = \cos^{-1}x + x\,\frac{-1}{\sqrt{1-x^2}} + \frac{1\cdot(1-x^2)'}{2\sqrt{1-x^2}}$$

$$r' = \cos^{-1}x - \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$$

$$\boxed{r' = \cos^{-1}x - \frac{2x}{\sqrt{1-x^2}}}$$

② (Example 5 - SECTION 3.8)

$x(t) = \tan^{-1}\sqrt{t}$, $t \geq 0$

(position function)

let $v(t)$ = velocity

$$v(t) = x'(t) = \frac{1}{1+(\sqrt{t})^2}\cdot(\sqrt{t})'$$

$$v(t) = \frac{1}{2\sqrt{t}(1+t)}$$

$$v(16) = \frac{1}{2\sqrt{16}(1+16)} = \frac{1}{2\cdot 4\cdot 17} = \frac{1}{136}$$

The velocity at $t=16$ is $\frac{1}{136}$ $\quad\boxed{v(16) = \frac{1}{136}}$

③ (Problem 26 - SECTION 3.4)

$Q(t) = 200(30-t)^2$

\# of gallons of water

t = time (in min)

How fast is the water running?

we need to find the inst. rate of change of Q with respect to time, $Q'(t)$

$$Q'(t) = 200(2)(30-t)(-1)$$

$$Q'(t) = -400(30-t)$$

$$Q'(10) = -400(30-10)$$

$\boxed{Q'(10) = -8000 \text{ gallons/min}}$

the rate the water is running at the end of 10 min.

Average rate $= \dfrac{Q(10) - Q(0)}{10 - 0}$

$= \dfrac{200(30-10)^2 - 200(30)^2}{10}$

$= \dfrac{200(400 - 900)}{10} = \boxed{-10{,}000}$

$\boxed{\text{Average rate} = -10{,}000 \text{ gallons/min}}$ during the first 10 min

(4) (Problem 3 - SECTION 3.9)

$V = \pi r^2 h$

(a) $r =$ constant $\Rightarrow \boxed{\dfrac{dV}{dt} = \pi r^2 \dfrac{dh}{dt}}$

(b) $h =$ constant $\Rightarrow$

$\dfrac{dV}{dt} = \pi h \cdot \dfrac{d}{dt}(r^2)$

$\boxed{\dfrac{dV}{dt} = 2\pi h r \dfrac{dr}{dt}}$

(c) $\dfrac{dV}{dt} = \pi h \dfrac{d}{dt}(r^2) + \pi r^2 \dfrac{d}{dt}(h)$

$\boxed{\dfrac{dV}{dt} = 2\pi h r \dfrac{dr}{dt} + \pi r^2 \dfrac{dh}{dt}}$

(5) (Problem 10 - SECTION 3.9)

let $A =$ area of circular plate

$r =$ radius

given: $\dfrac{dr}{dt} = 0.01$ cm/min

find: $\dfrac{dA}{dt}$ when $r = 50$cm

Solution

$A = \pi r^2$

$\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$

$\left.\dfrac{dA}{dt}\right|_{r=50\text{cm}} = 2\pi(50\text{cm}) \cdot (0.01 \text{ cm/min})$

$\boxed{\dfrac{dA}{dt} = \pi \text{ cm}^2/\text{min}}$

(6) (Exercise 1 - Handout 3.3 & 3.9)

$s = f(t) = t^3 - 6t^2 + 9t$ (position at time t)

(a) let $v(t) =$ velocity

$v(t) = \dfrac{ds}{dt} = 3t^2 - 12t + 9$

(b) particle at rest when $v(t) = 0$

$3t^2 - 12t + 9 = 0 \quad / \div 3$

$t^2 - 4t + 3 = 0$

$(t-1)(t-3) = 0$

$t = 1$ sec and $t = 3$ sec particle at rest

(c) Particle is moving forward when

$v(t) > 0$

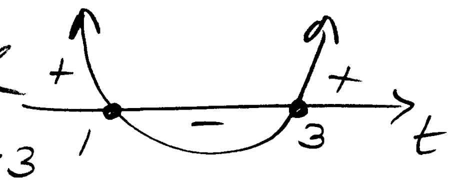
Particle is moving backward when

$v(t) < 0$

$v(t) = 3t^2 - 12t + 9$

$v(t) = 3(t^2 - 4t + 3)$

parabola opening upward



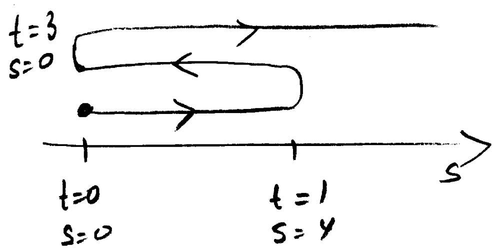
$v(t) > 0$ when $0 \leq t < 1$ and $t > 3$

$v(t) < 0$ when $1 < t < 3$

Particle is moving forward when $0 < t < 1$ and $t > 3$

Particle is moving backward when $1 < t < 3$.

(d) if $d =$ distance traveled on a time interval $[t_1, t_2]$, then $d = |f(t_2) - f(t_1)|$



Total distance traveled is

$d = |f(1) - f(0)| + |f(3) - f(1)| + |f(5) - f(3)|$

$d = |4 - 0| + |0 - 4| + |20 - 0|$

$\boxed{d = 20\ m}$

(e) let $a(t) =$ acceleration

$a(t) = \dfrac{dv}{dt}$, $\boxed{a(t) = 6t - 12}$, $a(t) = 0$ when $t = 2$

(f)

t	0		1		2		3		∞
$v(t)$	+	+	0	−	−	−	0	+	+
$a(t)$	−	−	−	−	0	+	+	+	+

Particle is speeding up when $v(t)$ and $a(t)$ have same sign

$1 < t < 2$ and $t > 3$

Particle is slowing down when $v(t)$ and $a(t)$ have opposite signs

$0 \leq t < 1$ and $2 < t < 3$