Math 180 Spring 2009 Name:

QUIZ #1 @ 50 points

Write neatly. Show all work. **Write all responses on separate paper.** **<u>Clearly label the exercises.</u>**

1. The graph of the function $y = f(t)$ is shown.
   Find the following limits or explain why they do not exist.

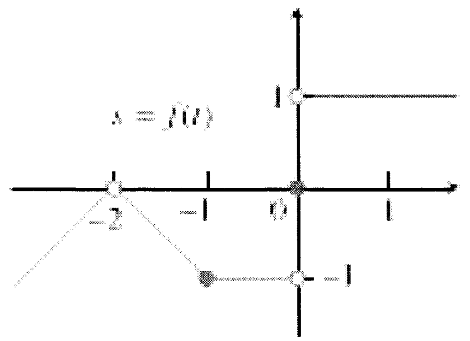
   a) $\lim\limits_{t\to -2} f(t)$

   b) $\lim\limits_{t\to -1} f(t)$

   c) $\lim\limits_{t\to 0} f(t)$

   d) $\lim\limits_{t\to 0^-} f(t)$

   e) $\lim\limits_{t\to 0^+} f(t)$



2. Find each limit. Support your answer by showing the work. Do not just write down an answer. No work, no credit given.

   a) $\lim\limits_{a\to 1}\left(2a^4 - a^3 + 5a^2 - a - 1\right)$

   b) $\lim\limits_{x\to -2}\dfrac{2x^3}{x-1}$

   c) $\lim\limits_{t\to 2}\dfrac{t^2 - 7t + 10}{t-2}$

   d) $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+100} - 10}{x^2}$

   e) $\lim\limits_{x\to 0}\dfrac{x\csc 3x}{\cos 6x}$

   f) $\lim\limits_{t\to 0}\dfrac{\tan 5t}{2t}$

   g) $\lim\limits_{y\to\infty}\left(4 - \dfrac{2}{y}\right)$

   h) $\lim\limits_{x\to -\infty}\dfrac{7x^5 - 4x^3 + 21}{2x^5 + 3x^2 + 5x}$

   i) $\lim\limits_{x\to 0} x^2 \sin\dfrac{1}{x}$

   j) $\lim\limits_{a\to\infty}\left(a - \sqrt{a^2 + 5a}\right)$

3. Find $\lim\limits_{h\to 0}\dfrac{f(x+h) - f(x)}{h}$ for the given values of x and function f:

   $f(x) = \sqrt{x}$, $x = 1$.

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Quiz 1 - Solutions

① (a) $\lim_{t\to -2} f(t) = 0$

(b) $\lim_{t\to -1} f(t) = -1$

(c) $\lim_{t\to 0} f(t)$ does not exist because

$\lim_{t\to 0^-} f(t) = -1$ and

$\lim_{t\to 0^+} f(t) = 1$

(d) $\lim_{t\to 0^-} f(t) = -1$

(e) $\lim_{t\to 0^+} f(t) = 1$

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(2)

(a) $\lim_{a\to 1}(2a^4 - a^3 + 5a^2 - a - 1) =$

$2(1)^4 - 1^3 + 5(1)^2 - 1 - 1 =$

$2 - 1 + 5 - 2 = \boxed{4}$

(b) $\lim_{x\to -2} \frac{2x^3}{x-1} = \frac{2(-2)^3}{-2-1}$

$= \frac{2(-8)}{-3}$

$= \boxed{\frac{16}{3}}$

(c) $\lim_{t\to 2} \frac{t^2 - 7t + 10}{t-2} = \left(\frac{0}{0} \text{ special case}\right)$

$\lim_{t\to 2} \frac{(t-5)\cancel{(t-2)}}{\cancel{t-2}} =$

$\lim_{t\to 2}(t-5) = 2 - 5 = \boxed{-3}$

(d) $\lim_{x\to 0} \frac{\sqrt{x^2+100} - 10}{x^2} = \left(\frac{0}{0} \text{ special case}\right)$

$\lim_{x\to 0} \frac{(\sqrt{x^2+100} - 10)(\sqrt{x^2+100} + 10)}{x^2(\sqrt{x^2+100} + 10)} =$

$\lim_{x\to 0} \frac{(x^2+100) - 10^2}{x^2(\sqrt{x^2+100} + 10)} =$

$\lim_{x\to 0} \frac{1}{\sqrt{x^2+100} + 10} =$

$\frac{1}{\sqrt{100} + 10} = \boxed{\frac{1}{20}}$

(e) $\lim_{x\to 0} \frac{x \csc 3x}{\cos 6x} =$

$\lim_{x\to 0} \frac{x}{(\cos 6x)\sin 3x} =$

$\lim_{x\to 0} \frac{1}{\cos 6x} \cdot \lim_{x\to 0} \frac{x}{\sin 3x} =$

$\frac{1}{1} \cdot \lim_{x\to 0} \frac{1}{\frac{\sin 3x}{3x} \cdot 3} =$

$\frac{1}{3} \lim_{x\to 0} \frac{1}{\frac{\sin 3x}{3x}} =$

$\frac{1}{3} \cdot \frac{1}{1} = \boxed{\frac{1}{3}}$

(f) $\lim_{t\to 0} \frac{\tan 5t}{2t} = \left(\frac{0}{0} \text{ special case}\right)$

$\lim_{t\to 0} \frac{\sin 5t}{2t\cos 5t} =$

$\frac{1}{2} \lim_{t\to 0} \frac{1}{\cos 5t} \cdot \lim_{t\to 0} \frac{\sin 5t}{t} =$

$\frac{1}{2} \cdot \frac{1}{1} \cdot \lim_{t\to 0} \frac{\sin 5t}{5t} \cdot 5 =$

$\frac{1}{2} \cdot 5 \lim_{t\to 0} \frac{\sin 5t}{5t} =$

$\frac{1}{2} \cdot 5 \cdot 1 = \boxed{\frac{5}{2}}$

(g) $\lim_{y\to\infty} \left(4 - \frac{2}{y}\right) = 4 - \frac{2}{\infty}$
$= 4 - 0$
$= \boxed{4}$

(h) $\lim_{x\to -\infty} \frac{7x^5 - 4x^3 + 21}{2x^5 + 3x^2 + 5x} = \left(\frac{\infty}{\infty} \text{ special case}\right)$

$\lim_{x\to -\infty} \frac{7 - \frac{4x^3}{x^5} + \frac{21}{x^5}}{2 + \frac{3x^2}{x^5} + \frac{5x}{x^5}} =$

$\lim_{x\to -\infty} \frac{7 - \frac{4}{x^2} + \frac{21}{x^5}}{2 + \frac{3}{x^3} + \frac{5}{x^4}} =$

$\frac{7 - 0 + 0}{2 + 0 + 0} = \boxed{\frac{7}{2}}$

(i) $\lim_{x\to 0} x^2 \sin\frac{1}{x}$

Note that the Product Rule cannot be applied because $\lim_{x\to 0} \sin\frac{1}{x}$ does not exist

We know that

$-1 \le \sin\frac{1}{x} \le 1$ for any $x \ne 0$

Note that when $x \to 0$, $x \ne 0$
so $x^2 > 0$ for any $x \to 0$

$-x^2 \le x^2 \sin\frac{1}{x} \le x^2$

$\lim_{x\to 0} (-x^2) = 0$

$\lim_{x\to 0} x^2 = 0$

By the Squeeze Theorem $\Rightarrow$

$\Rightarrow \boxed{\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0}$

(j) $\lim_{a\to\infty} \left(a - \sqrt{a^2+5a}\right) = \left(\infty - \infty \text{ special case}\right)$

$\lim_{a\to\infty} \frac{\left(a - \sqrt{a^2+5a}\right)\left(a + \sqrt{a^2+5a}\right)}{a + \sqrt{a^2+5a}} =$

$\lim_{a\to\infty} \frac{a^2 - (a^2+5a)}{a + \sqrt{a^2(1+\frac{5}{a})}} =$

$\lim_{a\to\infty} \frac{-5a}{a + |a|\sqrt{1+5/a}} =$

Note that when $a \to \infty$, $|a| = a$

$\lim_{a\to\infty} \frac{-5a}{a + a\sqrt{1+\frac{5}{a}}} =$

$$\lim_{a \to \infty} \frac{-5\cancel{a}}{\cancel{a}\left(1+\sqrt{1+5/a}\right)} =$$

$$-5 \lim_{a \to \infty} \frac{1}{1+\sqrt{1+\frac{5}{a}}} = \frac{-5}{1+\sqrt{1+0}}$$

$$= \boxed{\frac{-5}{2}}$$

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(3) $f(x) = \sqrt{x}, \quad x = 1$

Method I

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h} = \left(\frac{0}{0} \text{ special case}\right)$$

$$\lim_{h \to 0} \frac{(\sqrt{1+h} - 1)(\sqrt{1+h} + 1)}{h(\sqrt{1+h} + 1)} =$$

$$\lim_{h \to 0} \frac{1+h - 1}{h(\sqrt{1+h} + 1)} =$$

$$\lim_{h \to 0} \frac{1}{\sqrt{1+h} + 1} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

Method II

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \left(\frac{0}{0} \text{ special case}\right)$$

$$\lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} =$$

$$\lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} =$$

$$\lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

When $x = 1$, $\frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2}}$