Math 130 Spring 2009 Name:

TEST #2 @ 160 points

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1. Consider the polynomial function $f(x) = 2x^4 + x^3 - 11x^2 + 11x - 3$.
Questions $a - e$ below relate to this polynomial function.

a) Describe the long-term behavior of this function; that is, the end-behavior. Give reasons for your answer.

b) Using Descartes' rule of signs, determine the number of positive real zeros and the number of negative real zeros for $f(x)$.

c) Find all the real zeros of $f(x)$ and factor $f(x)$ completely.

d) What are the intercepts of the graph of $f(x)$? Write each intercept as an ordered pair.

e) Sketch a graph of $f(x)$ showing how it passes through its intercepts. Plot additional points (if necessary) to get the shape of the graph. Clearly label all the points.

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2. Consider $f(x) = \dfrac{2x^2 + x - 3}{x^3 + 2x^2 - 9x - 18}$.
Questions $a - f$ below relate to this polynomial function.

a) Factor the numerator and the denominator.

b) What is the domain of the function?

c) What are the vertical asymptotes?

d) What is the horizontal asymptote?

e) What are the intercepts for this function? Write them as ordered pairs.

f) Plot additional points (if necessary) to get the shape of this function and sketch a graph.

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3. Find the polynomial of least degree with leading coefficient 1 and rational coefficients whose zeros are $\sqrt{5}$ and $3i$.

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4. Do the following:

a) Write the expression as a single logarithm with coefficient 1. Assume all variables are positive real numbers:

$$\log_b (2y+5) - \frac{1}{2}\log_b (y+3)$$

b) Expand the expression as much as possible. Simplify the result if possible. All variables are positive real numbers:

$$\log_3 \frac{\sqrt{x} \cdot \sqrt[3]{y}}{w^2 \sqrt{z}}$$

c)Find the following:

$\ln\left(e^{3xy}\right)$ $3^{\log_3 4} - 5\log_2 \sqrt[5]{8}$ $(\log_3 10)(\log 3)$

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5. Let $f(x) = e^{x+1} - 3$.

a) Graph the function using transformations. Clearly show how you're obtaining the graph, that is, show all equations, their meaning, and the corresponding graphs.

b) State the domain, range, and asymptote.

c) Find the exact x- and y-intercepts (if any).

d) Does the function have an inverse? Explain.

e) Find $f^{-1}(x)$.

f) State the domain, range, and asymptote for the inverse function $f^{-1}(x)$.

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6. Solve the following equations. Give exact answer(s) as well as approximations (when appropriate). Write conditions (if any).

a) $\log_3(x+1) + \log_3(x-1) = 2$

b) $5^{x+1} = 2^{3x-1}$

c) $\sqrt{\ln x} = \ln x$

d) $\log_2(\log_3 x) = -1$

e) Solve for t: $P = P_0 e^{kt}$

f) Solve for t: $ae^{kt} = e^{bt}$ where $k \neq b$

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7. Find the domain of each function and graph it:

a) $f(x) = 3^x + 1$

b) $g(x) = \log(x+1)$

8. Identify all the asymptotes of the following functions:

a) $f(x)=\dfrac{8x^2+2x-10}{2x^2-3x-5}$

b) $g(x)=\dfrac{2x^2+5}{x-3}$

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9. Newton's law of cooling says that the rate at which a body cools is proportional to the difference C in temperature between the body and the environment around it. The temperature $f(t)$ of the body at time t in appropriate units after being introduced into an environment having constant temperature T_0 is

$$f(t)=T_0+Ce^{-kt}$$, where C and k are constants.

A pot of coffee with a temperature of 100°C is set down in a room with a temperature of 20°C. The coffee cools to 60°C after 1 hr.

a) Write an equation to model the date

b) Find the temperature after half an hour.

c) How long will it take for the coffee to cool to 50°C ?

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10. The number of bacteria present in a culture after t hours is given by the formula $P=500e^{0.5t}$.

a) How many bacteria will be there after 2 hours?

b) How long will it be before there are 2,500 bacteria?

c) What is the doubling time?

MATH130

TEST 2 - SOLUTIONS

① $f(x) = 2x^4 + x^3 - 11x^2 + 11x - 3$

(a) The long-term behavior is given by the leading term $2x^4$

when $x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to \infty$

(b) $f(x)$ has 3 variations in sign $\Rightarrow$ $f(x)$ has 3 or 1 positive real zeros

$f(-x) = 2x^4 - x^3 - 11x^2 - 11x - 3$

$f(-x)$ has 1 variation in sign

$\Rightarrow$ $f(x)$ has 1 negative zero

(c) All the coefficients are integers and constant term is nonzero $\Rightarrow$ we can apply the Rational zeros th.

Possible rational zeros:

$$\frac{p}{q} = \frac{\text{factors of } 3}{\text{factors of } 2} = \frac{\pm 1, \pm 3}{\pm 1, \pm 2}$$

$$\frac{p}{q} \in \left\{ \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2} \right\}$$

	2	1	-11	11	-3
1	2	3	-8	3	0

$f(x) = (x-1)(2x^3 + 3x^2 - 8x + 3)$

	2	3	-8	+3
1	2	5	-3	0

$f(x) = (x-1)(x-1)(2x^2 + 5x - 3)$

$f(x) = (x-1)^2(2x-1)(x+3)$

complete factorization of $f(x)$

All the zeros are:

$x = 1$ with multiplicity 2

$x = \frac{1}{2}$ multiplicity 1

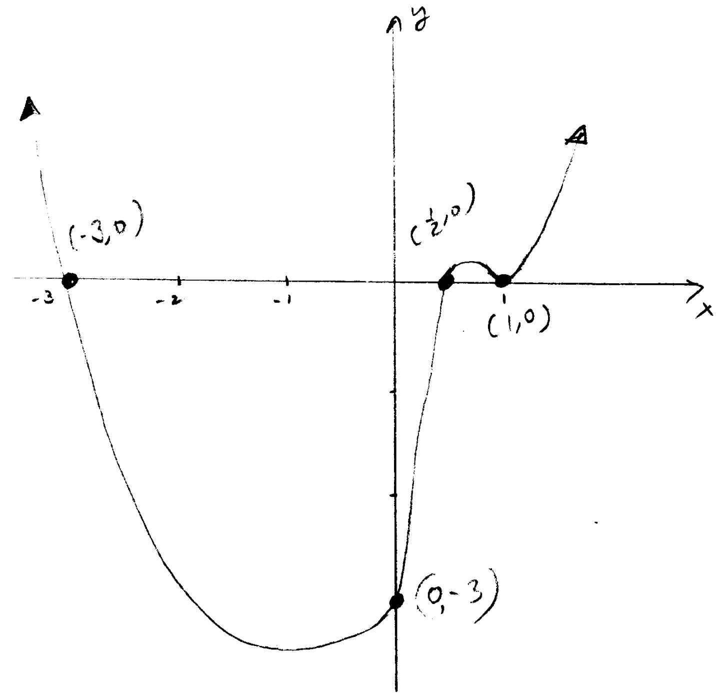
$x = -3$ multiplicity 1

(d) x-int: $(1,0)$, $(\frac{1}{2}, 0)$, $(-3, 0)$

y-int: $(0, -3)$

(e)

x	$-\infty$	-3	0	$\frac{1}{2}$	1	∞
$f(x)$	∞	0	-3	0	0	∞
		$m=1$		$m=1$	$m=2$	



(2) $f(x) = \dfrac{2x^2+x-3}{x^3+2x^2-9x-18}$

(a) $2x^2+x-3 = (2x+3)(x-1)$

$x^3+2x^2-9x-18 = x^2(x+2)-9(x+2)$

$= (x+2)(x^2-9)$

$= (x+2)(x+3)(x-3)$

$f(x) = \dfrac{(2x+3)(x-1)}{(x+2)(x+3)(x-3)}$

(b) $x \neq -2$
$x \neq -3$
$x \neq 3$ so $x \in \mathbb{R} \setminus \{-2,-3,3\}$

(c) V.A. $x=-2$, $x=-3$, $x=3$

(d) H.A $y=0$
(degree of numerator is less than the degree of the denominator)

(e) x-int: $y=0$ when $2x^2+x-3=0$
$(2x+3)(x-1)=0$
$x=-\frac{3}{2}$, $x=1$

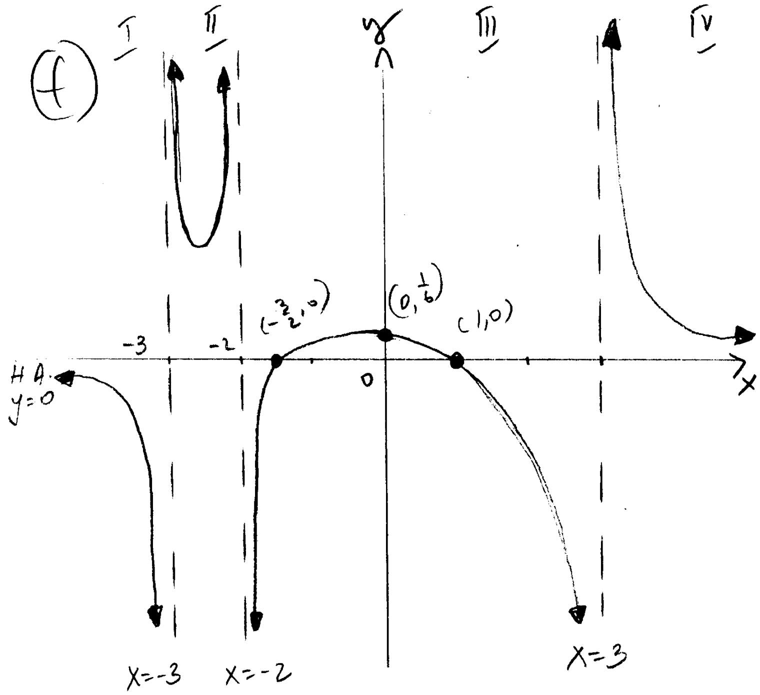
x-int: $(-\frac{3}{2},0)$ and $(1,0)$

y-int: $x=0$, $y=\frac{-3}{-18}=\frac{1}{6}$

y-int: $(0,\frac{1}{6})$

(f) Test points:
I $x=-10$, $y=\dfrac{(-)(-)}{(-)(-)(-)}=(-)$
II $x=-2.5$, $y=\dfrac{(-)(-)}{(-)(+)(-)}=(+)$
IV $x=10$, $y=(+)$

(f)



(3) f has rational coeff.
$\Rightarrow$ $x=\sqrt{5}$ and $x=-\sqrt{5}$
$x=3i$ and $x=-3i$
are the zeros of $f(x)$

$f(x)=(x-\sqrt{5})(x+\sqrt{5})(x-3i)(x+3i)$

$f(x)=(x^2-5)(x^2-9i^2)$

$f(x)=(x^2-5)(x^2+9)$

$f(x)=x^4+4x^2-45$

(4)
(a) $\log_b(2y+5)-\frac{1}{2}\log_b(y+3) =$

$\log_b(2y+5)-\log_b(y+3)^{\frac{1}{2}} =$

$\log_b \dfrac{2y+5}{\sqrt{y+3}}$

(b) $\log_3 \frac{\sqrt{x}\cdot\sqrt[3]{y}}{w^2\sqrt{z}} =$

$\log_3(\sqrt{x}\,\sqrt[3]{y}) - \log_3(w^2\sqrt{z}) =$

$\log_3\sqrt{x} + \log_3\sqrt[3]{y} - \log_3 w^2 - \log_3\sqrt{z} =$

$\log_3 x^{\frac{1}{2}} + \log_3 y^{\frac{1}{3}} - 2\log_3 w - \log_3 z^{\frac{1}{2}} =$

$\frac{1}{2}\log_3 x + \frac{1}{3}\log_3 y - 2\log_3 w - \frac{1}{2}\log_3 z$

(c) • $\ln(e^{3xy}) = 3xy$

• $3^{\log_3 4} - 5\log_2\sqrt[5]{8} =$

$4 - \log_2(\sqrt[5]{8})^5 =$

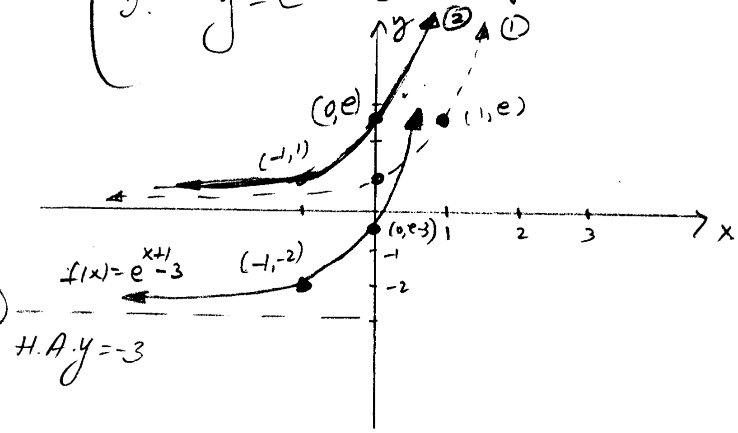
$4 - \log_2 8 = 4 - 3 = \boxed{1}$

• $(\log_3 10)(\log 3) =$

$\frac{\log 10}{\log 3}\cdot\log 3 = \log 10 = \boxed{1}$

(5) $f(x) = e^{x+1} - 3$ (Handout 4.3-4.6)

(a)
1. let $y = e^x$
2. $y = e^{x+1}$ shift left 1
3. $y = e^{x+1} - 3$ shift down 3



(b)
Domain: $x \in \mathbb{R}$
Range: $y \in (-3, \infty)$
H.A. $y = -3$

(c) x-n: let $y = 0$

$e^{x+1} - 3 = 0$

$e^{x+1} = 3$ $\quad \| \ln$

$\ln e^{x+1} = \ln 3$

$x + 1 = \ln 3$

$x = \ln 3 - 1$

x-n: $(\ln 3 - 1, 0)$

y-n: let $x = 0$,

$y = e^1 - 3$

y-n: $(0, e-3)$

(d) $y = f(x)$ is one-to-one, therefore it has an inverse

(e) - $y = e^{x+1} - 3$

- Solve for x

$$y+3 = e^{x+1} \quad \Big| \ln$$

$$\ln(y+3) = \ln e^{x+1}$$

$$\ln(y+3) = x+1$$

$$x = \ln(y+3) - 1$$

- $x \longleftrightarrow y$

$$y = \ln(x+3) - 1$$

so $\boxed{f^{-1}(x) = \ln(x+3) - 1}$

(f) $\begin{cases} \text{Domain: } & x \in (-3, \infty) \\ \text{Range: } & y \in \mathbb{R} \\ \text{V.A } & x = -3 \end{cases}$

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(6) (a) $\log_3(x+1) + \log_3(x-1) = 2$

Conditions:

$$\begin{cases} x+1 > 0 \\ \text{and} \\ x-1 > 0 \end{cases} \Rightarrow \begin{cases} x > -1 \\ \text{and} \\ x > 1 \end{cases}$$

so $x > 1$

$$\log_3 (x+1)(x-1) = 2$$

$$\Longleftrightarrow$$

$$3^2 = (x+1)(x-1)$$

$$9 = x^2 - 1$$

$$x^2 = 10$$

$$x = \pm\sqrt{10}$$

but $-\sqrt{10} < 1$ so

$$\boxed{x = \sqrt{10}}$$

(b) $5^{x+1} = 2^{3x-1} \quad \Big| \ln$

$$\ln 5^{x+1} = \ln 2^{3x-1}$$

$$(x+1)\ln 5 = (3x-1)\ln 2$$

$$x\ln 5 + \ln 5 = 3x\ln 2 - \ln 2$$

$$\ln 5 + \ln 2 = 3x\ln 2 - x\ln 5$$

$$\ln 5 \cdot 2 = x(3\ln 2 - \ln 5)$$

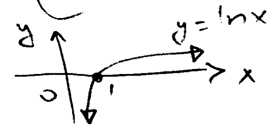
$$\ln 10 = x(\ln 8 - \ln 5)$$

$$\boxed{x = \frac{\ln 10}{\ln(\frac{8}{5})}}$$

(c) $\sqrt{\ln x} = \ln x$ (Handout 4.3-46)

Conditions:

$$\begin{cases} x > 0 \\ \text{and} \\ \ln x \geq 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ \text{and} \\ x \geq 1 \end{cases}$$



so $x \geq 1$

let $\ln x = t$

$\sqrt{t} = t \;\Big|^2$

$t = t^2$

$t^2 - t = 0$

$t(t-1) = 0 \quad \left\langle \begin{array}{l} t = 0 \\ \text{OR} \\ t = 1 \end{array} \right.$

If $t = 0$, then $\ln x = 0$

$x = 1$

if $t = 1$, then $\ln x = 1$

$x = e$

so $\boxed{x \in \{1, e\}}$

(d) $\log_2(\log_3 x) = -1$ (Handout 4.3–4.6)

Conditions:

$\left\{ \begin{array}{l} x > 0 \\ \text{and} \\ \log_3 x > 0 \end{array} \right. \Rightarrow \left. \begin{array}{l} x > 0 \\ \text{and} \\ x > 1 \end{array} \right\}$

so $x > 1$

$2^{-1} = \log_3 x$

$\frac{1}{2} = \log_3 x$

$x = 3^{\frac{1}{2}}$

$\boxed{x = \sqrt{3}}$

(e) $P = P_0 e^{kt}$ (Handout 4.3–4.6)

$\frac{P}{P_0} = e^{kt} \quad \Big| \cdot \ln$

$\ln\left(\frac{P}{P_0}\right) = \ln\left(e^{kt}\right)$

$\ln\left(\frac{P}{P_0}\right) = kt \Rightarrow$

$\boxed{t = \frac{\ln\left(\frac{P}{P_0}\right)}{k}}$

(Handout 4.3–4.6)

(f) $a e^{kt} = e^{bt} \quad \Big/ \div e^{kt} \neq 0$

$a = \frac{e^{bt}}{e^{kt}}$

$a = e^{bt - kt}$

$a = e^{t(b-k)} \quad // \ln$

$\ln a = \ln\left(e^{t(b-k)}\right)$

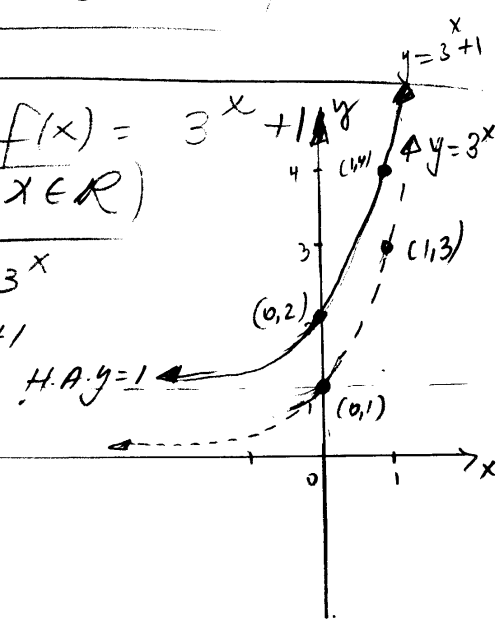
$\ln a = t(b-k)$

$\boxed{t = \frac{\ln a}{b-k}}$

(7) (a) $f(x) = 3^x + 1$

Domain: $x \in \mathbb{R}$)

$\left\{ \begin{array}{l} 1.\ \text{let } y = 3^x \\ 2\ \ y = 3^x + 1 \\ \text{shift } 1 \text{ up} \end{array} \right.$



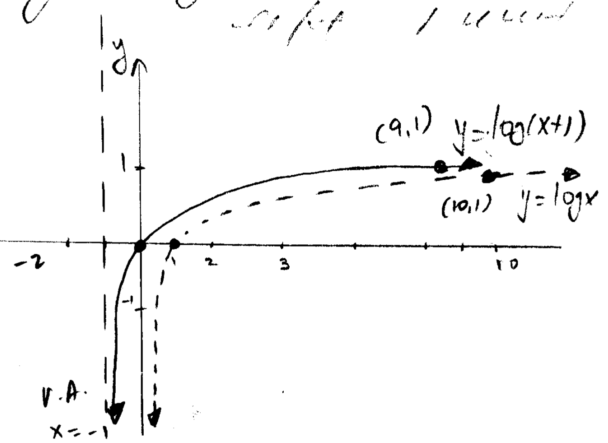
(b) $g(x) = \log(x+1)$

Domain: $x+1 > 0$

$x > -1$

$\boxed{x \in (-1, \infty)}$

1. let $y = \log x$
2. $y = \log(x+1)$ shift left 1 unit



(8) (a) $f(x) = \dfrac{8x^2+2x-10}{2x^2-3x-5}$

$f(x) = \dfrac{2(4x^2+x-5)}{(2x-5)(x+1)}$

$f(x) = \dfrac{2(4x+5)(x-1)}{(2x-5)(x+1)}$

V.A $\quad x = \frac{5}{2}$, $x = -1$

H.A $\quad y = 4$ $\quad$ $\left(y = \frac{8}{2} = 4\right)$ same degree

(b) $g(x) = \dfrac{2x^2+5}{x-3}$

V.A. $x = 3$

O.A $y = 2x+6$

$$\begin{array}{c|ccc} & 2 & 0 & 5 \\ \hline 3 & 2 & 6 & 21 \end{array} \; R$$

(9) (Example 6 in 4.6 textbook)

$f(t) = T_0 + Ce^{-kt}$

t = time (in hours)

T_0 = room temperature

$f(t)$ = body temperature

C = difference between the body and environment

(a) need to find C and k

when $t = 0$, $T_0 = 20$

and $f(0) = 100$

so $100 = 20 + C \cdot e^0$

so $C = 100 - 20$, $\underline{\underline{C = 80}}$

when $t = 1$, $f(1) = 60$

so $60 = 20 + 80e^{-k \cdot 1}$

$40 = 80e^{-k}$

$\frac{1}{2} = e^{-k}$ $\quad$ // ln

$\ln\frac{1}{2} = \ln(e^{-k})$

$-k = \ln\frac{1}{2}$, $k = -\ln\frac{1}{2}$

$\underline{k \approx 0.693}$

Then for,

$\boxed{f(t) = 20 + 80e^{-0.693t}}$

(b) $t = \frac{1}{2}$

$f(\frac{1}{2}) = 20 + 80 \cdot e^{-0.693(0.5)}$

$\boxed{f(\frac{1}{2}) \approx 76.6^\circ C}$

(c) $t = ?$ when $f(t) = 50$

$50 = 20 + 80e^{-0.693t}$

$30 = 80e^{-0.693t}$

$\frac{3}{8} = e^{-0.693t}$ // ln

$\ln(\frac{3}{8}) = \ln e^{-0.693t}$

$-0.693t = \ln(\frac{3}{8})$

$t = \frac{\ln(\frac{3}{8})}{-0.693} \approx 1.415 \text{ hr}$

$\boxed{t \approx 1.415 \text{ hr}}$

(10) (Handout 4.3-4.6)

$P = 500e^{0.5t}$

t = time (hours)

$P = P(t)$ = number of bacteria

(a) $P(2) = 500e^{0.5(2)}$

$\boxed{P(2) \approx 1359 \text{ bacterial}}$

(b) $t = ?$ when $P(t) = 2{,}500$

$2500 = 500e^{0.5t}$

$\frac{2500}{500} = e^{0.5t}$

$e^{0.5t} = 5$ // ln

$\ln e^{0.5t} = \ln 5$

$0.5t = \ln 5 \Rightarrow t = \frac{\ln 5}{0.5}$

$\boxed{t \approx 3.2 \text{ hrs}}$

(c) $P_0 = 500$ bacteria (when $t=0$)

$2P_0 = 1000$ bacteria

find t

$1000 = 500e^{0.5t}$

$2 = e^{0.5t}$ // ln

$\ln 2 = \ln(e^{0.5t})$

$0.5t = \ln 2 \Rightarrow t = \frac{\ln 2}{0.5}$

$\boxed{t \approx 1.4 \text{ hrs}}$ doubling time