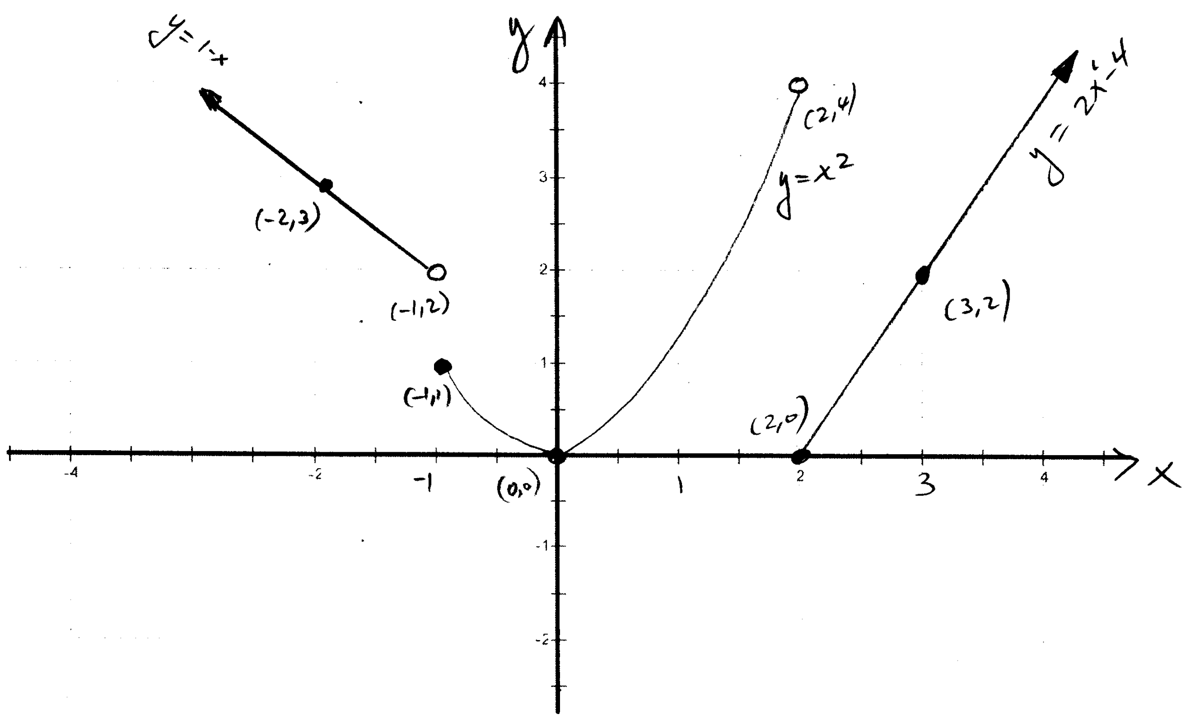
Math 130 Spring 2009 Name:

TEST #1 @ 160 points

Write neatly. Show all work. Write all responses on separate paper. Clearly label the exercises.

1. A piecewise-defined function is given.

$$f(x)=\begin{cases}1-x & if\ \ x<-1\\ x^2 & if\ -1\le x<2\\ 2x-4 & if\ \ x\ge 2\end{cases}$$



You may use the above grid to graph. Write all the answers and show ALL your work on separate paper.

a) Sketch a graph for the function. Clearly show how you obtain the points you are using for the graph. Label the axes and all points used.

b) State its domain and range in interval notation.

c) On what interval(s) is the function increasing, decreasing, constant?

d) Find $f\left(f(-2)\right)$ and $(f\circ f)(2)$.

2. For each function

$$f(x) = 5x^4 - 3x^2 + 1$$
$$g(x) = 2x^3 + 5x + 3$$
$$h(x) = 6x^3 - x$$

a) Determine whether each function is even, odd, or neither. Show all work.

b) Without graphing, identify which graph (if any) is symmetric about the x-axis, about the y-axis, and about the origin.

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3. Let

$f(x) = x^2 - x + 1$, $g(x) = 3x + 1$, $l(x) = \dfrac{x+2}{x^2 - 4}$, $F(x) = \sqrt{1 - 3x}$ be four functions. Do the following.

a) Find the domain of each function.

b) Find $(f \circ g)(x)$ and its domain.

c) Find $\dfrac{g(x+h) - g(x)}{h}$

d) Find $(g \circ F)(x)$ and its domain.

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4. Let $2x - 3y = 5$ be a linear equation in two variables. Do the following:

a) Graph the equation using the intercepts method. Clearly label the axes and the intercepts.

b) Find the slope of the line.

c) Find an equation for the line that is perpendicular to the given line and passes through $(-1, 4)$.

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5. Let $f(x) = -2x^2 - 12x + 5$. Find the domain and the range of the function.

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6. If air resistance is neglected, the height s (in feet) of an object propelled directly upward from an initial height s_0 feet with initial velocity v_0 feet per second is

$$s(t) = -16t^2 + v_0 t + s_0,$$

where t is the number of seconds after the object is propelled. A ball is thrown directly upward from the top of a building 100 ft tall with an initial velocity of 80 ft per sec.

a) Give the function that describes the height of the ball in terms of time t.

b) Determine the time at which the ball reaches its maximum height, and the maximum height in feet.

c) After how many seconds will it hit the ground?

d) For what interval will the ball be more than 160 feet above the ground level?

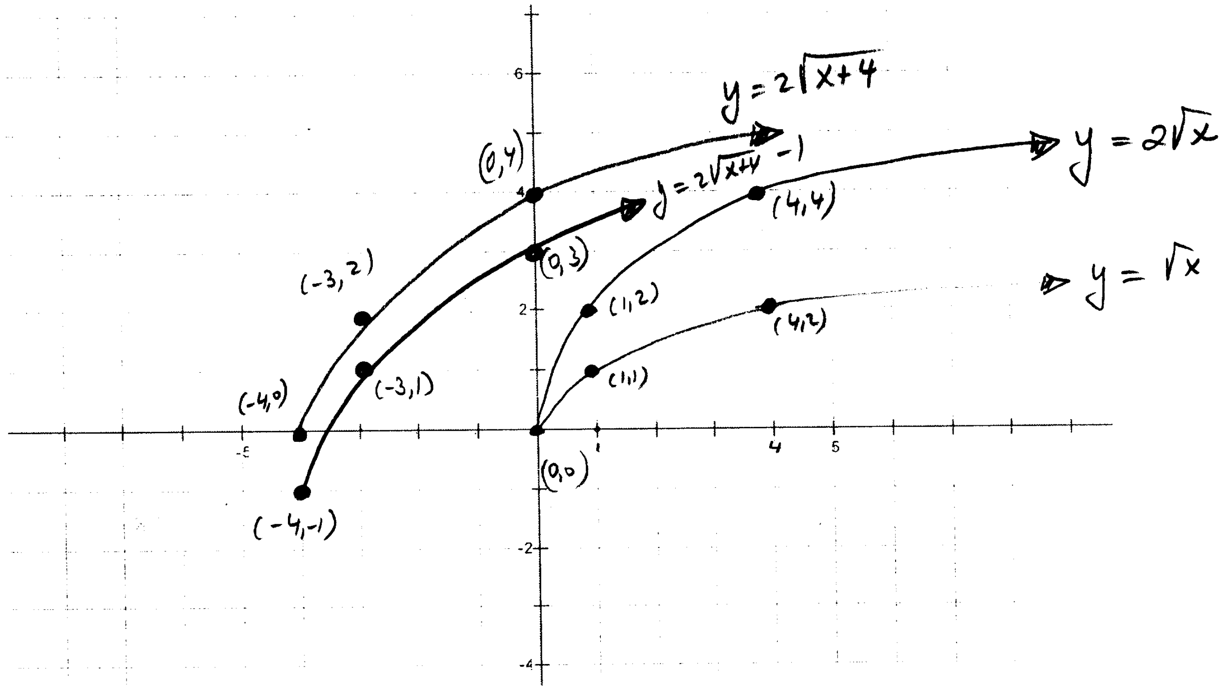
e) Sketch the path of the ball and identify the points that represent the answers to parts b, c, and d on the graph.

7. Let $f(x) = 2\sqrt{x+4} - 1$. Answer the following questions:

a) Graph the function using transformations. You may use the grid to graph. Clearly show all the steps: the equations and their meaning on separate paper. Graph all steps.

b) Find the domain and the range.

c) Find the intercepts.



8. Using the graph $y = f(x)$ shown, answer the following:

a) Is y a function of x? Explain.

b) Find the domain and range of f.

c) List the x- and y- intercepts (as ordered pairs).

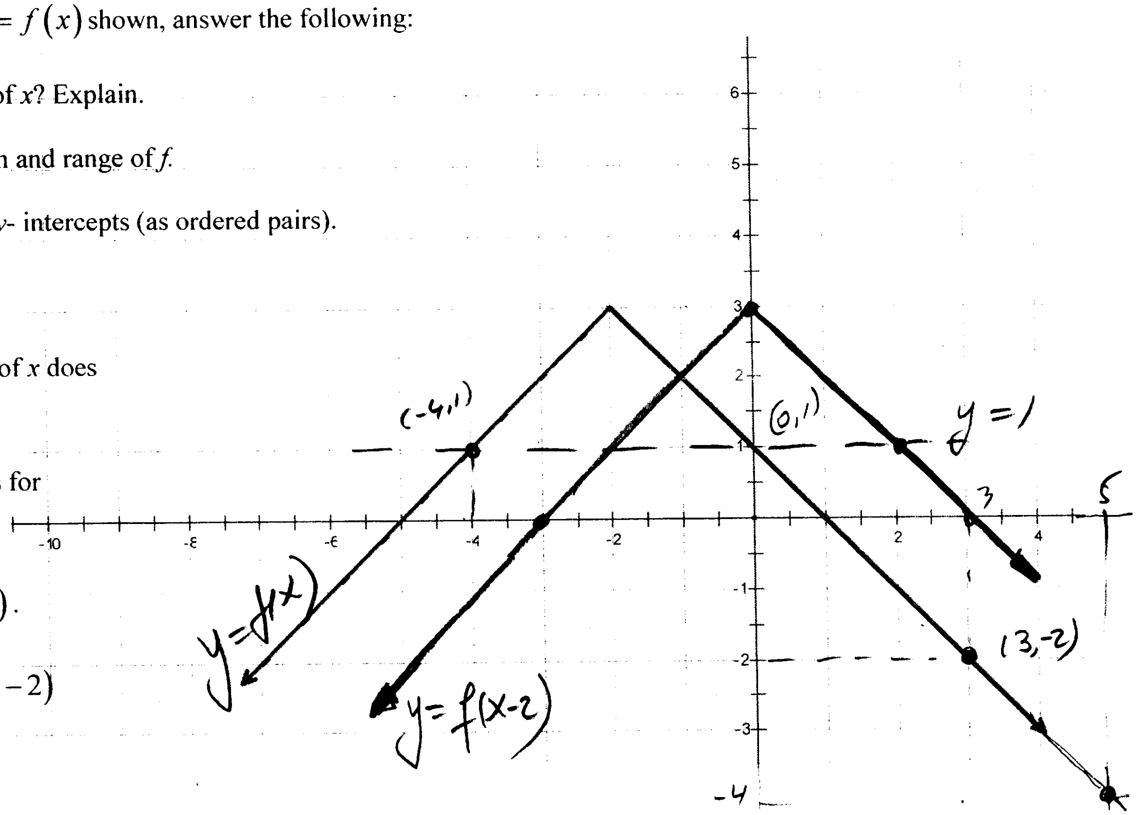
d) Find $f(3)$.

e) For what values of x does $f(x) = 1$

~~e~~ f) Find the values for which $f(x) > 1$.

g) Find $(f \circ f)(5)$.

h) Graph $y = f(x-2)$



EXTRA CREDIT

Problem 1 @ 4 points

At a local jazz club, the cost of an evening is based on a cover charge of \$5 plus a beverage charge of \$3 per drink.

a) If x is the number of drinks consumed and $t(x)$ is the total cost, write a formula for $t(x)$.

b) If the price of the cover charge is raised ~~to~~ by \$1, express the new total cost function $n(x)$ as a transformation of $t(x)$.

c) The management decides to increase the cover charge to \$10, leave the price of a drink at \$3, but include the first two drinks for free. Write a function for $x \geq 2$ that gives the new total cost $p(x)$. For $x \geq 2$, express $p(x)$ as a transformation of $t(x)$.

Problem 2 @ 4 points

Research facility on the Isle of Shoals has a limited quantity (800 gal) of fresh water which it must conserve over a two-month period.

a) If there are 7 members of the research team, and each is allowed 2 gallons of water per day, find a formula for $f(t)$, the amount of fresh water left on the island after t days has elapsed.

b) Evaluate and interpret the following expressions:

i) $f(0)$

ii) t if $f(t) = \frac{1}{2} f(0)$

M130

TEST 1 - SOLUTIONS

(1) (a) $y = 1 - x$ if $x < -1$
(part of a line)

x	y	
-1	2	(-1,2) open
-2	3	(-2,3)

$y = x^2$ if $-1 \le x < 2$
(part of a parabola)

x	y	
-1	1	(-1,1) closed
0	0	
2	4	(2,4) open

$y = 2x - 4$ if $x \ge 2$
(part of a line)

x	y	
2	0	(2,0) closed
3	2	(3,2)

(b) Domain: $x \in \mathbb{R}$
Range: $y \in [0, \infty)$

(c)
f is decreasing on $(-\infty, -1)$ and on $[-1, 0]$

f is increasing on $(0, 2)$ and on $[2, \infty)$

(d) $f(f(-2)) = f(3)$
$= 2$

$(f \circ f)(2) = f(f(2))$
$= f(0)$
$= 0$

(2) $f(x) = 5x^4 - 3x^2 + 1$
$f(-x) = 5(-x)^4 - 3(-x)^2 + 1$
$= 5x^4 - 3x^2 + 1$
$= f(x)$

Therefore, f is even and its graph is symmetric about the y-axis

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$g(x) = 2x^3 + 5x + 3$
$g(-x) = 2(-x)^3 + 5(-x) + 3$
$= -2x^3 - 5x + 3$
$g(-x) \ne g(x)$
$g(-x) \ne -g(x)$
so g is neither even nor odd.
its graph has no symmetry

$h(x) = 6x^3 - x$
$h(-x) = 6(-x)^3 - (-x)$
$= -6x^3 + x$
$= -(6x^3 - x)$
$= -h(x)$

Therefore, h is odd and its graph is symmetric about the origin.

(3) (a) $f(x) = x^2 - x + 1$

Domain: $x \in \mathbb{R}$

$g(x) = 3x + 1$

Domain: $x \in \mathbb{R}$

$$\ell(x) = \frac{x+2}{x^2-4}$$

$$\ell(x) = \frac{x+2}{(x+2)(x-2)}$$

Condition: $\begin{cases} x+2 \neq 0, & x \neq -2 \\ x-2 \neq 0, & x \neq 2 \end{cases}$

Domain: $x \in \mathbb{R} \setminus \{2, -2\}$

$F(x) = \sqrt{1-3x}$

Condition: $1 - 3x \geq 0$

$1 \geq 3x$

$x \leq \frac{1}{3}$

Domain: $x \in (-\infty, \frac{1}{3}]$

(b) $(f \circ g)(x) = f(g(x))$

$= f(3x+1)$

$= (3x+1)^2 - (3x+1) + 1$

$(f \circ g)(x) = 9x^2 + 6x + 1 - 3x - 1 + 1$

$(f \circ g)(x) = 9x^2 + 3x + 1$

Domain$(f \circ g)$: $x \in \mathbb{R}$

(Domain(f) $\cap$ Domain(g))

(c) $$\frac{g(x+h) - g(x)}{h} =$$

$$= \frac{(3(x+h)+1) - (3x+1)}{h}$$

$$= \frac{\cancel{3x} + 3h + \cancel{1} - \cancel{3x} - \cancel{1}}{h}$$

$$= \frac{3h}{h} = 3$$

(d) $(g \circ F)(x) = g(F(x))$

$= g(\sqrt{1-3x})$

$= 3\sqrt{1-3x} + 1$

$(g \circ F)(x) = 3\sqrt{1-3x} + 1$

Domain $(g \circ F)(x)$:

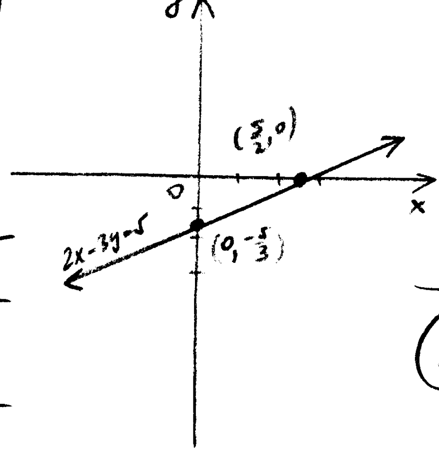
$\begin{cases} x \in \text{Domain}(F) \\ \text{and} \\ 1 - 3x \geq 0 \end{cases}$

therefore, $x \in (-\infty, \frac{1}{3}]$

(4) $2x - 3y = 5$

(a)

x	y
0	$-5/3$
$5/2$	0



$x = 0, \quad -3y = 5$

$y = \frac{-5}{3}$

$y = 0, \quad 2x = 5$

$x = 5/2$

(b) $2x - 3y = 5$

$2x - 5 = 3y \quad) \div 3$

$y = \frac{2}{3}x - \frac{5}{3}$

$m = \frac{2}{3}$

(c) $m_{\perp} = -\frac{3}{2}$

Use $(-1, 4)$ and $m = -\frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{3}{2}(x + 1)$

(5) $f(x) = -2x^2 - 12x + 5$

parabola opening down

$V(x_v, y_v)$ vertex

$x_v = \frac{-b}{2a} = \frac{12}{2(-2)} = -3$

$y_v = -2(9) - 12(-3) + 5$

$y_v = 23$

So $V(-3, 23)$

Therefore,

Domain: $x \in \mathbb{R}$

Range: $y \in (-\infty, 23]$

(6) $s(t) = -16t^2 + v_0 t + s_0$

s_0 = initial height

v_0 = initial velocity

t = time

$s(t)$ = height

(a) $s_0 = 100$ ft

$v_0 = 80$ ft/sec

$s(t) = -16t^2 + 80t + 100$

(b) The equation represents a parabola that opens downward, so its maximum occurs at the vertex $V(t_v, s_v)$

$t_v = \frac{-b}{2a} = \frac{-80}{2(-16)} = 2.5$ sec

$S_v = S_{max} = -16(2.5)^2 + 80(2.5) + 100$

$S_{max} = 200$ ft

Maximum height is 200 ft and it gets there after 2.5 seconds

(c) $t = ?$ when $s(t) = 0$

$-16t^2 + 80t + 100 = 0$

$16t^2 - 80t - 100 = 0 \quad) \div 4$

$4t^2 - 20t - 25 = 0$

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$t = \frac{20 \pm \sqrt{400-4(4)(-25)}}{2(4)}$$

$$t = \frac{20 \pm \sqrt{800}}{8} \begin{cases} t_1 < 0 \\ t_2 \approx 6.03 \text{ sec} \end{cases}$$

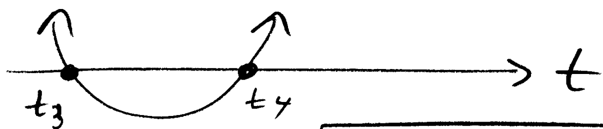
It will hit the ground after about 6.03 seconds.

(d) $t = ?$ when $s(t) > 160$

$-16t^2 + 80t + 100 > 160$

$16t^2 - 80t + 60 < 0$

$4t^2 - 20t + 15 < 0$

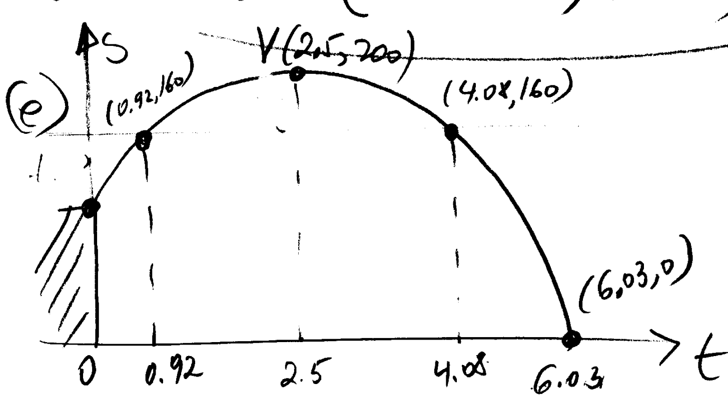


$$t_{3,4} = \frac{20 \pm \sqrt{400 - 4(4)(15)}}{2(4)}$$

$$t_{3,4} = \frac{20 \pm \sqrt{160}}{8} \begin{cases} t_4 \approx 4.08 \\ t_3 \approx 0.92 \end{cases}$$

It will be above 160 ft when $t \in (0.92 \text{ sec}, 4.08 \text{ sec})$

(e)



(7) $f(x) = 2\sqrt{x+4} - 1$

(a) 1st $y = \sqrt{x}$

2nd $y = 2\sqrt{x}$ vertical stretch by 2

3rd $y = 2\sqrt{x+4}$ left 4

4th $y = 2\sqrt{x+4} - 1$ down 1

(b) Domain: $x \in [-4, \infty)$

Range: $y \in [-1, \infty)$

(c) y-n: $(0,3)$

x-n: $2\sqrt{x+4} - 1 = 0$

$2\sqrt{x+4} = 1$ $/^2$

$4(x+4) = 1$

$x + 4 = \frac{1}{4}$

$x = \frac{1}{4} - 4 = -\frac{15}{4}$

x-n: $\left(-\frac{15}{4}, 0\right)$

(8) (a) Yes, the graph passes the vertical line test

(b) Domain: $x \in \mathbb{R}$
Range: $y \in (-\infty, 3]$

(c) x-int: $(-5,0)$ and $(1,0)$
y-int: $(0,1)$

(d) $f(3) = -2$

(e) $f(x) = 1$ when $x = -4$ and $x = 0$

(f) $f(x) > 1$ when $x \in (-4, 0)$

(g) $(f \circ f)(5) = f(f(5))$
$= f(-4)$
$= 1$

(h) $y = f(x-2)$
shift right 2

EXTRA CREDIT

(1)
(a) $t(x) = 5 + 3x$

(b) $n(x) = t(x) + 1$

(c) $p(x) = 10 + 3(x-2)$
$p(x) = t(x-2) + 5$

(2) (a) $f(t) = 800 - 14t$

(b) (i) $f(0) = 800$ gallons
the initial quantity of fresh water

(ii) $f(t) = \frac{1}{2} f(0)$
t will represent when the quantity left is half of what it originally was

$f(t) = \frac{1}{2}(800)$
$800 - 14t = 400$
$400 = 14t$
$t \approx 28.6$ days

After about 28.6 days there were 400 gallons of water left.