Math 130 Spring 2009 Name:

QUIZ #3 @ 90 points

Write neatly. Show all work. **Write all responses on separate paper.** Clearly label the exercises.

1) Solve the following system using matrices: Gaussian elimination or Gauss – Jordan method.

$$\begin{cases} 2x+y-z+3w=0 \\ 3x-2y+z-4w=-24 \\ x+y-z+w=2 \\ x-y+2z-5w=-16 \end{cases}$$

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2) Let

$$A=\begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} \quad B=\begin{pmatrix} 1 & 6 & -7 \\ -2 & 1 & 3 \end{pmatrix} \quad C=\begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix} \quad D=\begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix}$$

Do the following operations. If not defined, say so and explain why.

a) $3A+B$ b) AB . c) AC d) AD

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3) Graph the solution set of the following system of inequalities. Then find the coordinates of the vertices (if any).

$$\begin{cases} y-\log x \le 0 \\ x+y<4 \\ y\ge -2 \end{cases}$$

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4) Let $18, 6, 2, \ldots$ be a sequence. Answer the following:

a) Is this an arithmetic sequence or a geometric one? If arithmetic, find the common difference. If geometric, find the common ratio.

b) Find a formula for the general term a_n .

c) Write the sum of the first 50 terms using sigma notation and find the sum.

d) Find the infinite sum $18+6+2+\ldots$.

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5) Find the sum of the first 1000 natural numbers.

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6) Find a fraction representation for the rational number $1.\overline{243}$.

MATH 130

Quiz 3 - Solutions

(1)
$$\begin{pmatrix} 2 & 1 & -1 & 3 & 0 \\ 3 & -2 & 1 & -4 & -24 \\ 1 & 1 & -1 & 1 & 2 \\ 1 & -1 & 2 & -5 & -16 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3}$$

$$\begin{pmatrix} 1 & 1 & -1 & 1 & 2 \\ \boxed{3} & -2 & 1 & -4 & -24 \\ \boxed{2} & 1 & -1 & 3 & 0 \\ \boxed{1} & -1 & 2 & -5 & -16 \end{pmatrix} \xrightarrow[R_4 \to -R_1+R_4]{R_2 \to -3R_1+R_2,\; R_3 \to -2R_1+R_3}$$

$$\begin{pmatrix} 1 & 1 & -1 & 1 & 2 \\ 0 & -5 & 4 & -7 & -30 \\ 0 & \boxed{-1} & 1 & 1 & -4 \\ 0 & \boxed{-2} & 3 & -6 & -18 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\begin{pmatrix} 1 & 1 & -1 & 1 & 2 \\ 0 & -1 & 1 & 1 & -4 \\ 0 & \boxed{-5} & 4 & -7 & -30 \\ 0 & \boxed{-2} & 3 & -6 & -18 \end{pmatrix} \xrightarrow[R_4 \to -2R_2+R_4]{R_3 \to -5R_2+R_3}$$

$$\begin{pmatrix} 1 & 1 & -1 & 1 & 2 \\ 0 & -1 & 1 & 1 & -4 \\ 0 & 0 & -1 & -12 & -10 \\ 0 & 0 & \boxed{1} & -8 & -10 \end{pmatrix} \xrightarrow{R_4 \to R_3+R_4}$$

$$\begin{pmatrix} 1 & 1 & -1 & 1 & 2 \\ 0 & -1 & 1 & 1 & -4 \\ 0 & 0 & -1 & -12 & -10 \\ 0 & 0 & 0 & -20 & -20 \end{pmatrix}$$

4th row $\Rightarrow -20w = -20 \Rightarrow w = 1$

3rd row $\Rightarrow -z - 12w = -10$

$-z - 12 = -10 \Rightarrow z = -2$

2nd row $\Rightarrow -y + z + w = -4$

$-y - 2 + 1 = -4 \Rightarrow y = 3$

1st row $\Rightarrow x + y - z + w = 2$

$x + 3 + 2 + 1 = 2 \Rightarrow x = -4$

The solution is $\boxed{(-4, 3, -2, 1)}$.

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(2) dim $A = 3 \times 3$

dim $B = 2 \times 3$

dim $C = 3 \times 3$

dim $D = 3 \times 2$

(a) $3A + B$ - not defined (dim $A \neq$ dim B)

(b) AB - not defined number of columns $A \neq$ number of rows B

(c) $AC = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix}$

$AC = \begin{pmatrix} 0 & 5 & 1 \\ -8 & 5 & -10 \\ -11 & 15 & -24 \end{pmatrix}$

(d) $AD = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -13 & -7 \\ -8 & -2 \end{pmatrix}$

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(3) $\begin{cases} y - \log x \le 0 & (1°) \\ x + y < 4 & (2°) \\ y \geq -2 & (3°) \end{cases}$

(1°) $y - \log x \le 0$

$y \le \log x$

Boundary curve is $y = \log x$

$x > 0$

x	y
$\frac{1}{10}$	-1
1	0
10	1

Solution set = set of all points below the boundary curve

(2°) $x + y < 4$

$y < -x + 4$

Boundary line is $y = -x + 4$

Solution set = set of all points below the boundary line

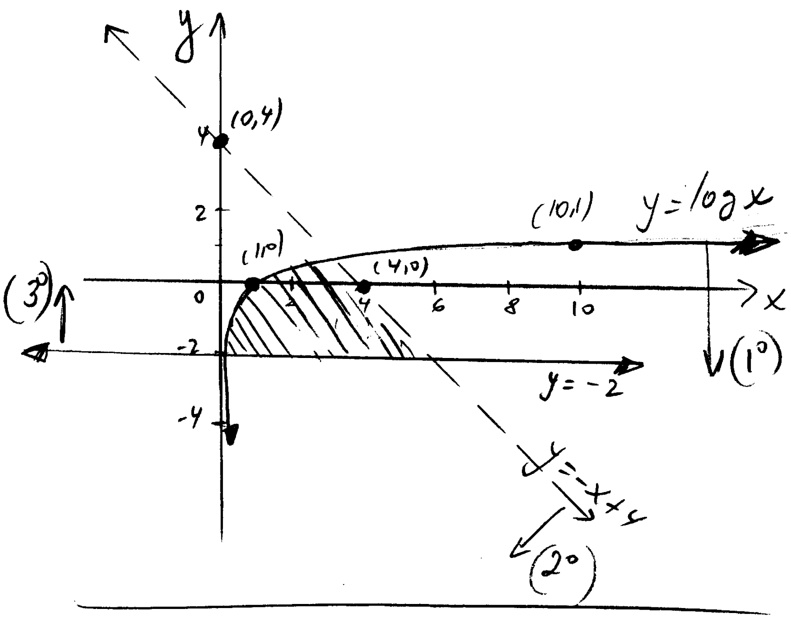
x	y
0	4
4	0

(3°) $y \geq -2$

Boundary line is $y = -2$

Solution set is the set of all points above the boundary line

Therefore, the solution of the system is the set of all points in the shaded region



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(4) $18, 6, 2, \ldots$

(a) Note that

$\frac{a_2}{a_1} = \frac{6}{18} = \frac{1}{3}$

$\frac{a_3}{a_2} = \frac{2}{6} = \frac{1}{3}$

Therefore, it is a geometric sequence with $\boxed{a_1 = 18,\ r = \frac{1}{3}}$

(b) $a_n = a_1 r^{n-1}$

$a_n = 18 \cdot \left(\frac{1}{3}\right)^{n-1}$

$\boxed{a_n = \frac{18}{3^{n-1}}}$

(c) $S_{50} = \sum_{n=1}^{50} 18\left(\frac{1}{3}\right)^{n-1}$

$$\boxed{S_{50} = 18 \sum_{n=1}^{50} \frac{1}{3^{n-1}}}$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{50} = \frac{18\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$$

$$S_{50} = \frac{18\left(1-\frac{1}{3^{50}}\right)}{\frac{2}{3}}$$

$$\boxed{S_{50} = 27\left(1-\frac{1}{3^{50}}\right)}$$

(d) $S_\infty = 18 + 6 + 2 + \cdots$

$$= \sum_{n=1}^{\infty} 18\left(\frac{1}{3}\right)^{n-1}$$

Because $r = \frac{1}{3} \in (-1,1)$,

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{18}{1-\frac{1}{3}}$$

$$= \frac{18}{\frac{2}{3}}$$

$$\boxed{S_\infty = 27}$$

(5) $1+2+\cdots+1000 = \sum_{n=1}^{1000} n$

This is an arithmetic series with $a_1 = 1$ and $d = 1$

so $S_n = \frac{(a_1+a_n)n}{2}$

Therefore,

$$S_{1000} = \sum_{n=1}^{1000} n = \frac{(1+1000)\,1000}{2}$$

$$S_{1000} = \frac{1001 \cdot 1000}{2}$$

$$\boxed{S_{1000} = 500{,}500}$$

(6) $1.2\overline{43} = 1 + \frac{2}{10} + \underbrace{\frac{43}{10^3} + \frac{43}{10^5} + \cdots}_{S_\infty}$

$S_\infty = \frac{43}{10^3} + \frac{43}{10^5} + \cdots$

is an infinite geometric series with $a_1 = \frac{43}{10^3}$

and $r = \frac{1}{100}$

therefore $S_\infty = \frac{a_1}{1-r} = \frac{\frac{43}{1000}}{1-\frac{1}{100}}$

$$S_\infty = \frac{\frac{43}{1000}}{\frac{99}{100}} = \frac{43}{990}$$

$$1.2\overline{43} = 1 + \frac{2}{10} + \frac{43}{990}$$

$$= \frac{990 + 2\cdot 99 + 43}{990}$$

$$\boxed{1.2\overline{43} = \frac{1231}{990}}$$