Math 130 Spring 2009 Name:

QUIZ #2 @ 90 points

Write neatly. Show all work. **Write all responses on separate paper.** Clearly label the exercises.

1. Let $f(x) = x^4 + 2x^3 - 7x^2 - 20x - 12$. Answer the following questions:

   a) Use synthetic division to divide $f(x)$ by $x+5$ and relate dividend, divisor, quotient and remainder in an equation

   b) What is the maximum number of real zeros?

   c) Using Descartes' rule of signs, determine the possible number of positive real zeros and negative real zeros for the polynomial.

   d) Explain why the Rational Zeros Theorem can be applied; use the theorem to list all possible rational zeros.

   e) Find all the real zeros of the polynomial.

   f) Factor the polynomial completely into linear factors.

   g) Describe the end-behavior of the polynomial; that is, what happens as $x \to \infty$ and $x \to -\infty$ (say why, do not just write an answer).

   h) What are the intercepts of the graph of $f(x)$? Write each intercept as an ordered pair.

   i) Sketch a graph of $f(x)$ showing how it passes through its intercepts. Clearly label all the points.

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2. Find a polynomial function of least degree having only rational coefficients with zeros as given.

$$1+\sqrt{3},\ 2-i,\ 5,\ -\frac{1}{2}$$

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3. Let $f(x) = \dfrac{x^2 - 2x - 3}{2x^2 - x - 10}$. Graph the function showing the following:

   a) Domain.

   b) Asymptotes.

   c) Intercepts; write each intercept as an ordered pair.

   d) Intersection of the function with the horizontal or oblique asymptote.

   e) Test points (when necessary).

MATH 130 Quiz # 2

① (a)

	1	2	-7	-20	-12
-5	1	-3	8	-60	288

$f(x) = (x+5)(x^3-3x^2+8x-60) + 288$

(b) at most 4 real zeros

(c) f(x) has one variation in sign => 1 positive zero

$f(-x) = \underbrace{x^4 - 2x^3}_{} - \underbrace{7x^2 + 20x}_{2} \underbrace{-12}_{3}$

f(-x) has three variations in sign => 3 or 1 negative zeros

(d) all coefficients are integers
constant term ≠ 0
therefore the Rational zeros theorem can be applied

$\frac{p}{q} = \frac{\text{factor of } 12}{\text{factor of } 1}$

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1}$

so $\frac{p}{q} \in \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$ possible rational zeros

(e) Note $f(1) \neq 0$

	1	2	-7	-20	-12
-1	1	1	-8	-12	0

$f(x) = (x+1)(x^3+x^2-8x-12)$

	1	1	-8	-12
-2	1	-1	-6	0

$f(x) = (x+1)(x+2)(x^2-x-6)$

$f(x) = (x+1)(x+2)(x-3)(x+2)$

so $f(x) = (x+1)(x-3)(x+2)^2$

All the zeros of f(x) are:

$x = -1$, $x = 3$ } multiplicity = 1

$x = -2$ multiplicity 2

(f) $f(x) = (x+1)(x-3)(x+2)^2$

(g) The end behavior is given by the leading term x^4

when $x \to \infty, y \to \infty$

$x \to -\infty, y \to +\infty$

(h) x-int: $(-1, 0)$

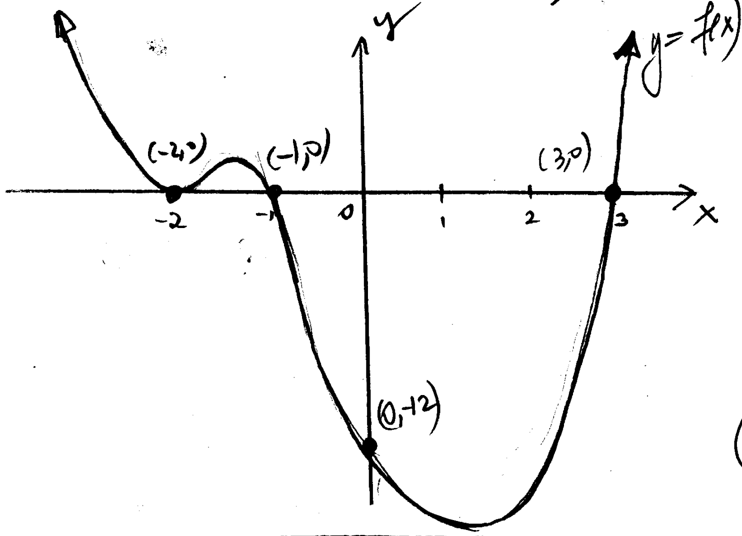
$(3, 0)$

$(-2, 0)$

y-int: $(0, -12)$

(i)

x	$-\infty$	-2	-1	0	3	∞
$f(x)$	∞	0	0	-12	0	∞
		$m=2$	$m=1$		$m=1$	



(2) $\begin{cases} x = 1+\sqrt{3} \\ x = 2-i \\ x = 5 \\ x = \frac{-1}{2} \end{cases}$

Polynomial has rational coefficients $\Rightarrow$

$\begin{cases} x = 1-\sqrt{3} \\ x = 2+i \end{cases}$ also zeros

$$f(x) = \left(x-(1+\sqrt{3})\right)\left(x-(1-\sqrt{3})\right)\left(x-(2+i)\right)\left(x-(2-i)\right)(x-5)\left(x+\tfrac{1}{2}\right)$$

$$f(x) = (x-1-\sqrt{3})(x-1+\sqrt{3})(x-2-i)(x-2+i)(x-5)\left(x+\tfrac{1}{2}\right)$$

$$f(x) = \left((x-1)^2-3\right)\left((x-2)^2-i^2\right)(x-5)\left(x+\tfrac{1}{2}\right)$$

$$f(x) = (x^2-2x-2)(x^2-4x+5)(x-5)\left(x+\tfrac{1}{2}\right)$$

(3) $f(x) = \dfrac{(x-3)(x+1)}{(2x-5)(x+2)}$

(a) $x \in \mathbb{R} \setminus \{\frac{5}{2}, -2\}$

(b) V.A. $x = \frac{5}{2}$, $x = -2$

H.A. $y = \frac{1}{2}$

(c) x-n: $y=0$ when $x=3$, $x=-1$

$(3,0)$ and $(-1,0)$

y-n: $x=0$, $y = \frac{3}{10}$

$(0, \frac{3}{10})$

(d) $f(x) \stackrel{?}{=} \frac{1}{2}$

$\dfrac{x^2-2x-3}{2x^2-x-10} \stackrel{?}{=} \dfrac{1}{2}$ iff

$2(x^2-2x-3) = 2x^2-x-10$

$\cancel{2x^2}-4x-6 = \cancel{2x^2}-x-10$

$4 = 3x$, so $x = \frac{4}{3}$

common point $(\frac{4}{3}, \frac{1}{2})$