Math 71 Spring 08 Name:

TEST 3 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

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1. Solve each equation in $\mathbb{C}$ (the set of complex numbers) by the indicated method.
   a) $4(x-3)^2+50=0$ by the square root property.
   b) $3y^2-4y+1=0$ by completing the square.
   c) $\frac{t^2}{5}-\frac{t}{3}=\frac{2}{3}$ by the quadratic formula.
   d) $2x^2+xy+y^2=3$ solve for y in terms of x.

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2. Solve the following equations. Give exact answers.
   a) $2x^4-3x^2+1=0$
   b) $\log_5(3x-1)-2=0$
   c) $2^x=10$ Give both exact and approximate answers.
   d) $\log_7(x+4)-\log_7 3=1$

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3. Solve the following inequalities.
   a) $x^2-6x+5\le 0$
   b) $\frac{1}{x-5}<\frac{3}{2-x}$

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4. Let $f(x)=3x-1$ and $g(x)=\frac{2-x}{x+1}$. Answer the following questions:
   a) Find $(g\circ f)(x)$.
   b) $(f\circ g)(1)$
   c) Find $f^{-1}(x)$.
   d) Find $g^{-1}(x)$.

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5. Simplify the following expressions.
   a) $3\ln x-5\ln y+2\ln z$
   b) $\frac{1}{3}(\log_5 x-\log_5 y)+3\log_5(x+2)$
   c) $\log_3 405-\log_3 5+\log 5+\log 2$
   d) $\log_{10}(\log_3(\log_5 125))$

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6. For the equation given below, answer all the questions and graph the function (Be sure to label the axes and all points used). SHOW ALL WORK!

$$y = -2x^2 + 3x + 2$$

a) What type of curve is this?
b) What is the y-intercept?
c) What is the vertex
d) What are the x- intercept(s) (if any)?
e) What is the domain of the function?
f) What is the range of the function?
g) Using the graph above, solve the following inequality: $-2x^2 + 3x + 2 > 0$
h) What is the vertex form of the equation?

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7. a) Graph $f(x) = 3^x$ by plotting at least 3 points . Find its domain, range, and asymptote. Label the axes and all the points

b) Graph $g(x) = \log_2 x$ by plotting at least 3 points . Find its domain, range, and asymptote. Label the axes and all the points.

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8. State whether each statement is TRUE or FALSE. Justify your answer.

a) $\log(a+b) = \log a \cdot \log b$

b) $\log\left(\frac{a}{b}\right) = \frac{\log a}{\log b}$

c) $\log 3x^5 = 5\log 3x$

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9) The number of bacteria present in a culture after t hours is given by the formula $N = 350e^{0.54t}$.

a) How many bactieria where there initially?
b) How many bacteria will be there after ½ hour?
b) How long will it be before there are 50,000 bacteria?

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10) The owners of a small fruit orchard decide to produce gift baskets as a sideline. The cost per basket for producing x baskets is $C = 0.01x^2 - 2x + 120$. How many baskets should they produce in order to minimize the cost per basket? What will their total cost be at that production level?

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11) India is currently one of the world's fastest-growing countries. By 2040, the population of India will be larger than the population of China; by 2050, nearly one-third of the world's population will live in these two countries alone. The exponential function

$$f(x) = 573(1.027)^x$$

models the population of India, $f(x)$, in millions, x years after 1974.

a) What was India's population in 1974?
b) Find $f(25)$ and its meaning.
c) Find India's population, to the nearest million, in the year 2025 as predicted by this function.

MATH 71 TEST 3 - SOLUTIONS

① (a) $4(x-3)^2+50=0$

$4(x-3)^2=-50$

$(x-3)^2=\frac{-50}{4}$

$(x-3)^2=\frac{-25}{2}$ $\quad /\sqrt{\ }$

$\sqrt{(x-3)^2}=\sqrt{\frac{-25}{2}}$

$x-3=\pm\frac{5i}{\sqrt{2}}$

$\boxed{x=3\pm\frac{5i\sqrt{2}}{2}}$

(b) $3y^2-4y+1=0$

1st $\quad 3y^2-4y=-1 \quad /\div 3$

2nd $\quad y^2-\frac{4}{3}y=\frac{-1}{3} \quad /+\frac{4}{9}$

3rd $\quad \left(\frac{1}{2}\text{coef. }y\right)^2_{3)}=\left(\frac{1}{2}\cdot\frac{4}{3}\right)^2=\left(\frac{2}{3}\right)^2=\frac{4}{9}$

$y^2-\frac{4}{3}y+\frac{4}{9}=\frac{-1}{3}+\frac{4}{9}$

$\left(y-\frac{2}{3}\right)^2=\frac{1}{9} \quad /\sqrt{\ }$

$\sqrt{\left(y-\frac{2}{3}\right)^2}=\sqrt{\frac{1}{9}}$

$y-\frac{2}{3}=\pm\frac{1}{3}$

$y=\frac{2}{3}\pm\frac{1}{3}$

$y=\frac{2}{3}+\frac{1}{3}=1$

OR

$y=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$

$\boxed{y\in\left\{1,\frac{1}{3}\right\}}$

(c) $\frac{{}^{3/}t^2}{5}-\frac{{}^{5/}t}{3}=\frac{{}^{5/}2}{3} \quad /\cdot 15$

LCD = 15

$3t^2-5t=10$

$3t^2-5t-10=0$

$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad \begin{cases} a=3 \\ b=-5 \\ c=-10 \end{cases}$

$t=\frac{5\pm\sqrt{25-4(3)(-10)}}{2(3)}$

$t=\frac{5\pm\sqrt{25+120}}{6}=\frac{5\pm\sqrt{145}}{6}$

$\boxed{t\in\left\{\frac{5\pm\sqrt{145}}{6}\right\}}$

(d) $2x^2+xy+y^2=3$

solve for y

This is a quadratic equation in y

$y^2+xy+2x^2-3=0$

$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad \begin{cases} a=1 \\ b=x \\ c=2x^2-3 \end{cases}$

$y=\frac{-x\pm\sqrt{x^2-4(1)(2x^2-3)}}{2}$

$y=\frac{-x\pm\sqrt{x^2-8x^2+12}}{2}$

$\boxed{y=\frac{-x\pm\sqrt{12-7x^2}}{2}}$

(2) (a) $2x^4-3x^2+1=0$

let $x^2=t$

then $(x^2)^2=t^2$, $x^4=t^2$

$2t^2-3t+1=0$

$$t=\frac{3\pm\sqrt{9-4(2)}}{4}=\frac{3\pm 1}{4}\left\langle\begin{matrix}1\\ \frac{1}{2}\end{matrix}\right.$$

$t=1$ OR $t=\frac{1}{2}$

$x^2=1$ $\qquad x^2=\frac{1}{2}$

$x=\pm 1$ $\qquad x=\pm\sqrt{\frac{1}{2}}$

$$x=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2}$$

$$\boxed{x\in\left\{\pm 1,\ \pm\frac{\sqrt{2}}{2}\right\}}$$

(b) $\log_5(3x-1)-2=0$

CONDITION: $3x-1>0$

$3x>1$

$x>\frac{1}{3}$

$\log_5(3x-1)=2$

$5^2=3x-1$

$3x=26$

$x=\frac{26}{3}>\frac{1}{3}$

$$\boxed{x\in\left\{\frac{26}{3}\right\}}$$

(c) $2^x=10$ $\quad | \ln$

$\ln 2^x=\ln 10$

$x\ln 2=\ln 10$

$$\boxed{x=\frac{\ln 10}{\ln 2}} \qquad x\simeq 3.32$$

(d) $\log_7(x+4)-\log_7 3=1$

CONDITION: $x+4>0$

$x>-4$

$\log_7\frac{x+4}{3}=1$

$7^1=\frac{x+4}{3}$

$x+4=21 \Rightarrow x=17>-4$

$$\boxed{x\in\{17\}}$$

(3) (a) $x^2-6x+5\le 0$

Let $y=x^2-6x+5$

Parabola opens up

x-n: $x^2-6x+5=0$

$(x-5)(x-1)=0$

$x=1,\ x=5$

Therefore, $x^2-6x+5\le 0$

iff $\boxed{x\in[1,5]}$

(b) $\frac{1}{x-5} < \frac{3}{2-x}$

$\frac{1}{x-5}\cdot\frac{2-x}{2-x} - \frac{3}{2-x}\cdot\frac{x-5}{x-5} < 0$

$LCD = (x-5)(2-x)$

$\frac{2-x-3(x-5)}{(x-5)(2-x)} < 0$

$\frac{2-x-3x+15}{(x-5)(2-x)} < 0$

$\frac{17-4x}{(x-5)(2-x)} < 0$

x	$-\infty$ 2 $\frac{17}{4}$ 5 ∞
$17-4x$	+ + + + 0 − − − −
$x-5$	− − − − 0 + +
$2-x$	+ + 0 − − − −
$\frac{17-4x}{(x-5)(2-x)}$	− | + 0 − | +

$\frac{1}{x-5} < \frac{3}{2-x}$ iff

$\boxed{x \in (-\infty, 2) \cup (\frac{17}{4}, 5)}$

(5) $f(x) = 3x-1$

$g(x) = \frac{2-x}{x+1}$

(a) $(g\circ f)(x) = g(f(x))$

$= g(3x-1) = \frac{2-(3x-1)}{(3x-1)+1}$

$= \frac{2-3x+1}{3x} = \frac{3-3x}{3x}$

$= \frac{3(1-x)}{3x} = \frac{1-x}{x}$

$\boxed{(g\circ f)(x) = \frac{1-x}{x}}$

(b) $(f\circ g)(1) = f(g(1))$

$g(1) = \frac{2-1}{1+1} = \frac{1}{2}$

so, $(f\circ g)(1) = f(g(1))$

$= f(\frac{1}{2})$

$= 3\cdot\frac{1}{2} - 1$

$= \frac{3}{2} - 1 = \frac{1}{2}$

$\boxed{(f\circ g)(1) = \frac{1}{2}}$

(c) $f(x) = 3x-1$

1st. $y = 3x-1$

2nd solve for x

$3x = y+1$

$x = \frac{y+1}{3}$

3rd $x \longleftrightarrow y$ $\quad y = \frac{x+1}{3}$

$\boxed{f^{-1}(x) = \frac{x+1}{3}}$

(d) $g(x) = \frac{2-x}{x+1}$

1st $y = \frac{2-x}{x+1}$

2nd solve for x

$y(x+1) = 2-x$

$yx + y = 2 - x$

$yx + x = 2 - y$

$x(y+1) = 2-y$

$x = \frac{2-y}{y+1}$

3rd $x \longleftrightarrow y$

$y = \frac{2-x}{x+1}$

$\boxed{g^{-1}(x) = \frac{2-x}{x+1}}$

(5) (a) $3\ln x - 5\ln y + 2\ln z =$

$= \ln x^3 - \ln y^5 + \ln z^2$

$= \ln \frac{x^3}{y^5} + \ln z^2 = \boxed{\ln \frac{x^3 z^2}{y^5}}$

(b) $\frac{1}{3}(\log_5 x - \log_5 y) + 3\log_5 (x+2)$

$= \frac{1}{3}\left(\log_5 \frac{x}{y}\right) + \log_5 (x+2)^3$

$= \log_5 \left(\frac{x}{y}\right)^{\frac{1}{3}} + \log_5 (x+2)^3$

$= \boxed{\log_5 \left(\sqrt[3]{\frac{x}{y}}\ (x+2)^3\right)}$

(c) $\log_3(405) - \log_3 5 + \log 5 + \log 2$

$= \log_3 \frac{405}{5} + \log(5 \cdot 2)$

$= \log_3 81 + \log 10$

$= 4 + 1 = \boxed{5}$

(d) $\log_{10}(\log_3(\log_5 125))$

$= \log_{10}(\log_3 3)$

$= \log_{10} 1 = \boxed{0}$

(6) $y = -2x^2 + 3x + 2$

(a) Parabola opening downword $(a = -2 < 0)$

(b) y-n: $x = 0, y = 2$

$\boxed{\text{y-n: } (0,2)}$

(c) $V(x_v, y_v)$

$x_v = \frac{-b}{2a} = \frac{-3}{2(-2)} = \frac{3}{4}$

$y_v = -2 \cdot \left(\frac{3}{4}\right)^2 + 3 \cdot \frac{3}{4} + 2$

$= -2 \cdot \frac{9}{16} + \frac{9}{4} + 2$

$= -\frac{9}{8} + \frac{{}^{2}9}{4} + {}^{8}2 = \frac{-9 + 18 + 16}{8}$

$= \frac{25}{8}$ $\boxed{V\left(\frac{3}{4}, \frac{25}{8}\right)}$

(d) x-in: $y=0$

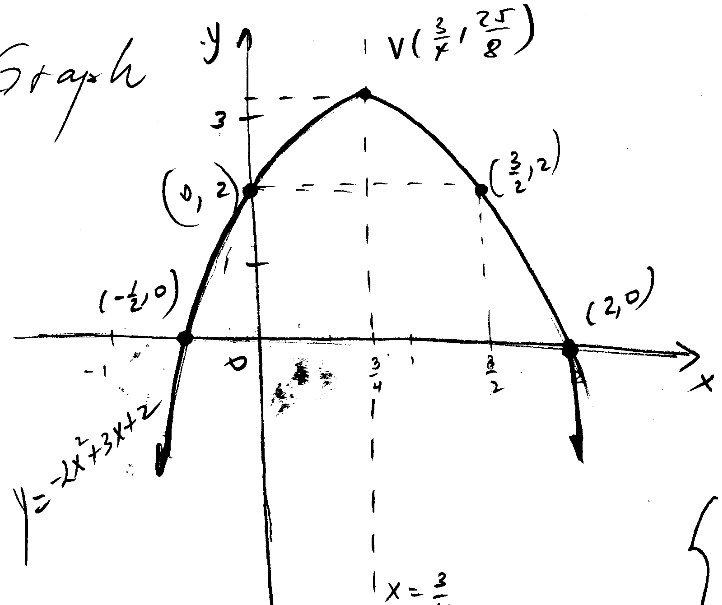
$$-2x^2+3x+2=0$$
$$2x^2-3x-2=0$$
$$(2x+1)(x-2)=0$$

$2x+1=0$ OR $x-2=0$

$x=\frac{-1}{2}$ $\quad$ $x=2$

x-in: $\boxed{\left(\frac{-1}{2},0\right) \text{ and } (2,0)}$

Graph



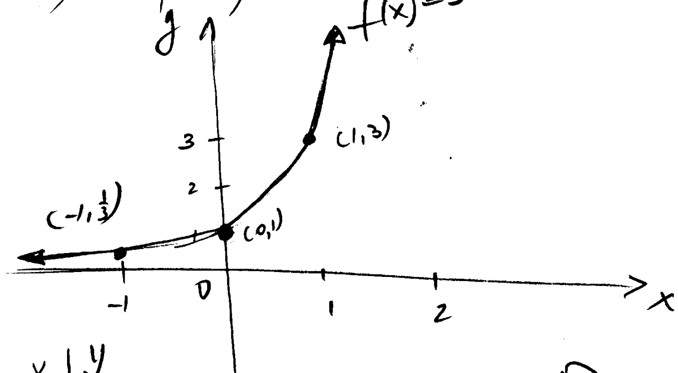
(e) Domain: $x \in \mathbb{R}$

(f) Range: $y \in \left(-\infty, \frac{25}{8}\right]$

(g) $-2x^2+3x+2>0$ iff
$\boxed{x \in \left(\frac{-1}{2}, 2\right)}$

(h) $y=a(x-x_v)^2+y_v$
$\boxed{y=-2\left(x-\frac{3}{4}\right)^2+\frac{25}{8}}$

(7) (a) $f(x)=3^x$



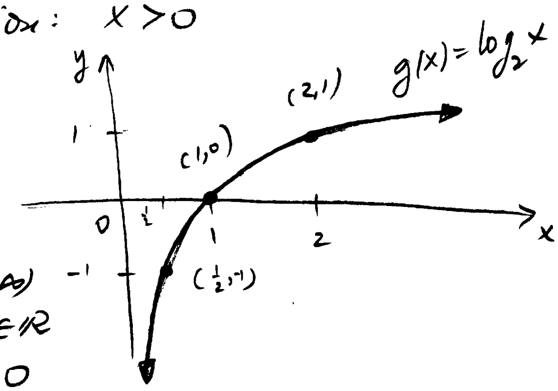
x	y
0	1
1	3
-1	$\frac{1}{3}$

Domain: $x \in \mathbb{R}$
Range $y \in (0,\infty)$
H.A $y=0$

(b) $g(x)=\log_2 x$
Condition: $x>0$

x	y
$\frac{1}{2}$	-1
1	0
2	1

Domain $x \in (0,\infty)$
Range $y \in \mathbb{R}$
V.A. $x=0$



(8) (a) $\log(a+b)=\log a \cdot \log b$
False statement

(b) $\log\left(\frac{a}{b}\right)=\frac{\log a}{\log b}$
False statement
$\log\left(\frac{a}{b}\right)=\log a-\log b$

(c) $\log 3x^5=5\log 3x$
False
$\log 3x^5=\log 3+5\log x$

(9) $N = 350\, e^{0.54t}$

t = # hours

N = # bacteria

(a) $t=0$, $N = 350$ bacteria

(b) $t=0.5$; $N = 350\, e^{0.54(0.5)}$

$N \approx 458$ bacteria

(c) $t = ?$, $N = 50{,}000$

$50{,}000 = 350\, e^{0.54t}$

$e^{0.54t} = \frac{50{,}000}{350}$

$e^{0.54t} = \frac{1000}{7}$ / ln

$\ln e^{0.54t} = \ln \frac{1000}{7}$

$0.54t = \ln \frac{1000}{7}$

$t = \frac{\ln \frac{1000}{7}}{0.54} \approx 9.2$ hours

It will take about 9.2 hours to have 50,000 bacteria

(10) $C = 0.01x^2 - 2x + 120$

x = # baskets

C = cost per basket

The equation represent a parabola opening up, therefore the minimum occurs at the vertex

$V(x_v, C_v)$

$x_v = \frac{-b}{2a} = \frac{-(-2)}{2(0.01)} = 100$ baskets

$C_{min} = 0.01(100)^2 - 2(100) + 120$

$= 20$ \$ / basket

They should produce 100 baskets to minimize the cost per basket.

Total cost =

= 100 baskets · 20 \$/basket

= 2000 \$ total

(11) $f(x) = 573(1.027)^x$

x = # years after 1974

$f(x)$ = population (in millions)

(a) $x=0$, $f(0) = 573$ millions

(b) $f(25) = 573(1.027)^{25}$

$f(25) = 1115.36$ million people

India's population in 1999 was 1115.36 million.

(c) $2025 - 1974 = 51$

$x = 51$

$f(51) = 573(1.027)^{51}$

$f(51) = 2229.7$ million people