Math 71 Spring 08 Name:

TEST 2 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Factor each expression completely:

a) $2x^3 - 6x^2 - 18x + 54$

b) $125x^3 - 8$

c) $6x^2 + 19x + 15$

2. Do the following operations (simplify):

a) $\dfrac{3x}{x^2 - 9x + 20} - \dfrac{5}{2x - 8}$

b) $\dfrac{\dfrac{x}{4} - \dfrac{1}{x}}{1 + \dfrac{x+4}{x}}$

c) $\left(2 - \dfrac{6}{x+1}\right)\left(1 + \dfrac{3}{x-2}\right)$

d) $\left(\dfrac{2x^{\frac{-5}{3}} y^{\frac{2}{4}}}{x^{-\frac{1}{3}}}\right)^{-3}$ and write the final answer using only positive exponents

e) $3\sqrt{75} - 2\sqrt{12} - \left(3\sqrt{5} - \sqrt{2}\right)\left(3\sqrt{5} + \sqrt{2}\right)$

f) $\dfrac{1+2i}{2-3i}$

g) $\dfrac{-14 + \sqrt{-128}}{16}$

h) $\left(\sqrt{x+1} - \sqrt{x+2}\right)^2$

3. If $f(x) = x^2 - 2x + 5$, find $\dfrac{f(a+h) - f(a)}{h}$.

4. If $f(x)=\dfrac{x+2}{x+3}$ and $g(x)=\dfrac{x+1}{x^2+2x-3}$, find all the values of a for which $f(a)=g(a)+1$.

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5. $f(x)=3x^2-2x+1$ Find the following and simplify:

a) $f(2i)$

b) $f\left(1+\sqrt{2}\right)$

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6. Let $f(x)=\sqrt{x-3}$.

a) What is the domain of this function?

b) Sketch the graph of the function by plotting points. Label the axes and all the points used.

c) What is the range of this function.

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7. Solve the following equations:

a) $3x^4-48x^2=0$

b) $(x-3)(x+8)=-30$

c) $S=\dfrac{a}{1-r}$ solve for r.

d) $F=\dfrac{G m_1 m_2}{d^2}$ solve for m_1.

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8. If $f(x)=x+\sqrt{x+5}$, find x such that $f(x)=7$.

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9. Find the perimeter and area of a rectangle whose width is $2\sqrt{20}$ feet and whose length is $\sqrt{125}$ feet. Simplify.

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10. A gymnast dismounts the uneven parallel bars at a height of 8 feet with an initialupward velocity of 8 feet per second. The function $s(t)=-16t^2+8t+8$ describes the height of the gymnast's feet above ground, $s(t)$, in feet, t seconds after dismounting.

a) How long will it take the gymnast to reach the ground?

b) When will the gymnast be 8 feet above the ground?

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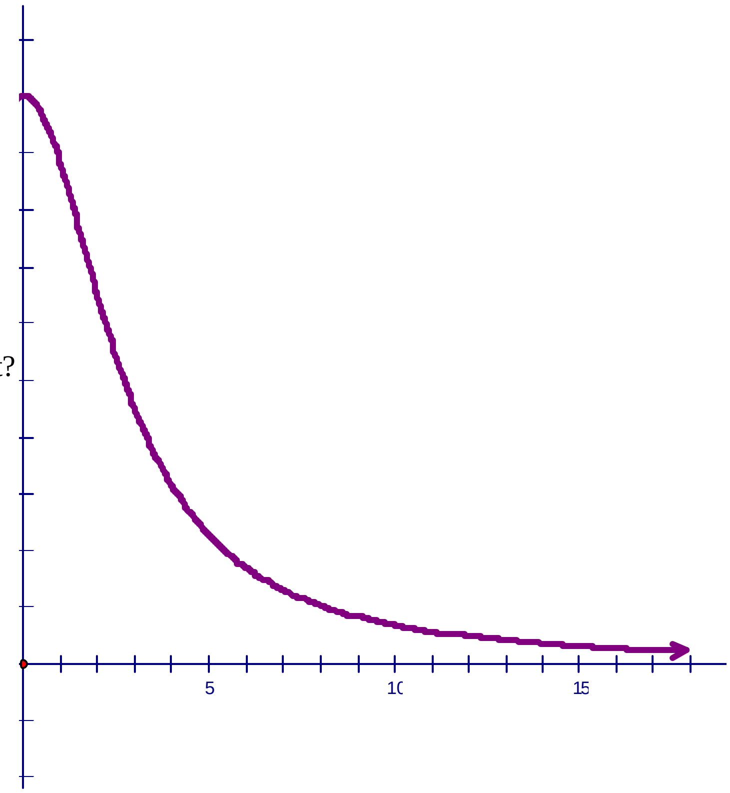
11 The rational function $P(x) = \dfrac{72{,}900}{100x^2 + 729}$ models the percentage of people in the U.S., $P(x)$, with x years of education who are unemployed. The graph of the function is shown below. Answer the following:

a) Identify the independent and dependent variables and label the axes accordingly.

b) What is the domain of the function?
What is the range?

c) Find and interpret $P(10)$. Identify the point on the graph and label it.

d) Describe the end behavior of the graph. Is there an education level that leads to guaranteed employment? How is this indicated by the graph?



MATH 71

TEST 2 - SOLUTIONS

(1) (a) $2x^3 - 6x^2 - 18x + 54 =$

$= 2(\underbrace{x^3 - 3x^2}_{} - \underbrace{9x + 27}_{})$

$= 2\left(x^2(x-3) - 9(x-3)\right)$

$= 2(x-3)(x^2-9)$

$= 2(x-3)(x-3)(x+3)$

$= 2(x-3)^2(x+3)$

(b) $125x^3 - 8 = (5x)^3 - 2^3$

$= (5x-2)\left((5x)^2 + 5x(2) + 2^2\right)$

$= (5x-2)(25x^2 + 10x + 4)$

(c) $6x^2 + 19x + 15 =$

$\left(\begin{array}{l} \text{product} = ac = 6(15) = 90 \left\langle \begin{array}{l} +9 \\ +10 \end{array} \right. \\ \text{sum} = b = 19 \end{array}\right)$

$90 = 9 \cdot 10$

$= \underbrace{6x^2 + 9x}_{} + \underbrace{10x + 15}_{}$

$= 3x(2x+3) + 5(2x+3)$

$= (2x+3)(3x+5)$

(2) (a) $\dfrac{3x}{x^2 - 9x + 20} - \dfrac{5}{2x-8} =$

$= \overset{2/}{\dfrac{3x}{(x-4)(x-5)}} - \overset{x-5/}{\dfrac{5}{2(x-4)}}$

$LCD = 2(x-4)(x-5)$

$= \dfrac{6x - 5(x-5)}{2(x-4)(x-5)} = \dfrac{6x - 5x + 25}{2(x-4)(x-5)}$

$= \dfrac{x+25}{2(x-4)(x-5)}$

(b) $\dfrac{\overset{x/}{\frac{x}{4}} - \overset{4/}{\frac{4}{x}}}{\overset{x/}{1} + \frac{x+4}{x}} = \dfrac{\frac{x^2-4}{4x}}{\frac{x + x + 4}{x}}$

$= \dfrac{x^2-4}{4x} \div \dfrac{2x+4}{x}$

$= \dfrac{(x-2)\cancel{(x+2)}}{4\cancel{x}} \cdot \dfrac{\cancel{x}}{2\cancel{(x+2)}}$

$= \dfrac{x-2}{8}$

(c) $\left(2 - \dfrac{6}{x+1}\right)\left(1 + \dfrac{3}{x-2}\right) =$

$= \dfrac{2(x+1) - 6}{x+1} \cdot \dfrac{x - 2 + 3}{x-2}$

$= \dfrac{2x + 2 - 6}{\cancel{x+1}} \cdot \dfrac{\cancel{x+1}}{x-2}$

$= \dfrac{2x-4}{x-2} = \dfrac{2(x-2)}{x-2} = 2$

(d) $\left(\dfrac{2x^{-\frac{5}{3}} y^{\frac{2}{4}}}{x^{-\frac{1}{3}}}\right)^{-3} = \left(2x^{-\frac{5}{3}+\frac{1}{3}} y^{\frac{1}{2}}\right)^{-3}$

$= \left(2x^{-\frac{4}{3}} y^{\frac{1}{2}}\right)^{-3} = 2^{-3}\left(x^{-\frac{4}{3}}\right)^{-3}\left(y^{\frac{1}{2}}\right)^{-3}$

$= \dfrac{1}{8} x^4 y^{-\frac{3}{2}} = \dfrac{x^4}{8y^{3/2}}$

(e) $3\sqrt{75} - 2\sqrt{12} - (3\sqrt{5}-\sqrt{2})(3\sqrt{5}+\sqrt{2})$

$= 3\sqrt{25\cdot 3} - 2\sqrt{4\cdot 3} - \left((3\sqrt{5})^2 - (\sqrt{2})^2\right)$

$= 3\cdot 5\sqrt{3} - 2\cdot 2\sqrt{3} - (9\cdot 5 - 2)$

$= 15\sqrt{3} - 4\sqrt{3} - 43$

$= 11\sqrt{3} - 43$

(f) $\dfrac{1+2i}{2-3i} = \dfrac{(1+2i)(2+3i)}{(2-3i)(2+3i)}$

$= \dfrac{2+3i+4i+6i^2}{2^2-(3i)^2} = \dfrac{2+7i-6}{4-9i^2}$

$= \dfrac{-4+7i}{4+9} = \dfrac{-4+7i}{13}$

(g) $\dfrac{-14+\sqrt{-128}}{16} = \dfrac{-14+i\sqrt{128}}{16}$

$= \dfrac{-14+i\sqrt{64\cdot 2}}{16} = \dfrac{-14+8i\sqrt{2}}{16}$

$= \dfrac{2(-7+4i\sqrt{2})}{16} = \dfrac{-7+4i\sqrt{2}}{8}$

(h) $\left(\sqrt{x+1} - \sqrt{x+2}\right)^2 =$

$= (\sqrt{x+1})^2 - 2\sqrt{(x+1)(x+2)} + (\sqrt{x+2})^2$

$= x+1 - 2\sqrt{(x+1)(x+2)} + x+2$

$= 2x+3 - 2\sqrt{(x+1)(x+2)}$

(3) $f(x) = x^2 - 2x + 5$

$\dfrac{f(a+h)-f(a)}{h} =$

$= \dfrac{\left((a+h)^2 - 2(a+h)+5\right) - (a^2-2a+5)}{h}$

$= \dfrac{\cancel{a^2}+2ah+h^2-\cancel{2a}-2h+\cancel{5}-\cancel{a^2}+\cancel{2a}-\cancel{5}}{h}$

$= \dfrac{2ah+h^2-2h}{h}$

$= \dfrac{h(2a+h-2)}{h} = 2a+h-2$

(4) $f(x) = \dfrac{x+2}{x+3}$

$g(x) = \dfrac{x+1}{x^2+2x-3}$

$a=?$ $f(a) = g(a)+1$

$\dfrac{a+2}{a+3} = \dfrac{a+1}{a^2+2a-3} + 1$

$\dfrac{a+2}{a+3}^{\,a-1} = \dfrac{a+1}{(a+3)(a-1)} + 1^{\,(a+3)(a-1)}$

Conditions: $\begin{cases} a \neq -3 \\ a \neq 1 \end{cases}$

$LCD = (a+3)(a-1)$

$(a-1)(a+2) = a+1+(a+3)(a-1)$

$\cancel{a^2}+\cancel{2a}-a-2 = a+1+\cancel{a^2}+\cancel{2a}-3$

$-a-\cancel{2} = a-\cancel{2}$

$0 = a+a$

$2a = 0 \Rightarrow a = 0$

so, $a \in \{0\}$

(5) $f(x) = 3x^2 - 2x + 1$

(a) $f(2i) = 3(2i)^2 - 2(2i) + 1$

$= 3(4i^2) - 4i + 1$

$= 12(-1) - 4i + 1$

$f(2i) = -11 - 4i$

(b) $f(1+\sqrt{2}) = 3(1+\sqrt{2})^2 - 2(1+\sqrt{2}) + 1$

$= 3(1 + 2\sqrt{2} + 2) - 2 - 2\sqrt{2} + 1$

$= 3(3 + 2\sqrt{2}) - 1 - 2\sqrt{2}$

$= 9 + 6\sqrt{2} - 1 - 2\sqrt{2}$

$= 8 + 4\sqrt{2}$

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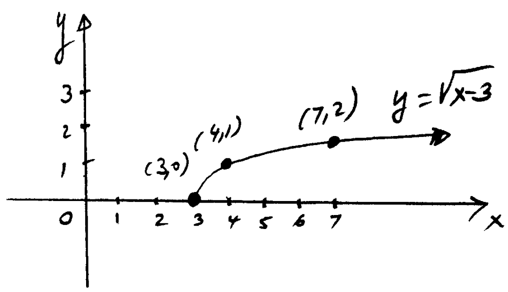
(6) $f(x) = \sqrt{x-3}$

(a) $x - 3 \geq 0$, $x \geq 3$

Domain: $x \in [3, \infty)$

(b)

x	y
3	0
4	1
7	2



(c) Range: $y \in [0, \infty)$

(7) (a) $3x^4 - 48x^2 = 0$

$3x^2(x^2 - 16) = 0$

$3x^2(x-4)(x+4) = 0$

$x = 0$ OR $x = 4$ OR $x = -4$

$x \in \{0, 4, -4\}$

(b) $(x-3)(x+8) = -30$

$x^2 + 5x - 24 = -30$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -2$ OR $x = -3$

$x \in \{-2, -3\}$

(c) $S = \dfrac{a}{1-r}$ for r.

$S(1-r) = a$

$S - Sr = a$

$S - a = Sr \Rightarrow r = \dfrac{S-a}{S}$

(d) $F = \dfrac{G m_1 m_2}{d^2}$ for m_1

$F d^2 = G m_1 m_2$

$m_1 = \dfrac{F d^2}{G m_2}$

(8) $f(x) = x + \sqrt{x+5}$

$x = ?$ $\quad f(x) = 7$

$f(x) = 7.$

$x + \sqrt{x+5} = 7$

$\sqrt{x+5} = 7 - x \ \Big/^2$

$x+5 = (7-x)^2$

$x+5 = 49 - 14x + x^2$

$x^2 - 15x + 44 = 0$

$(x-4)(x-11) = 0$

$x = 4$ $\quad$ OR $\quad$ $x = 11$

check $x=4$: $\quad 4 + \sqrt{4+5} \stackrel{?}{=} 7$

$4 + 3 = 7$ true

check $x = 11$: $\quad 11 + \sqrt{11+5} \stackrel{?}{=} 7$

false

therefore, $x \in \{4\}$

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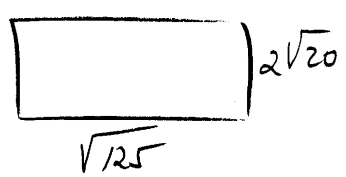
(9)



Perimeter $= 2\sqrt{125} + 2(2\sqrt{20})$

$= 2\sqrt{25 \cdot 5} + 4\sqrt{4 \cdot 5}$

$= 2 \cdot 5\sqrt{5} + 4 \cdot 2\sqrt{5} = 18\sqrt{5}$ ft

Area $= \sqrt{125}\,(2\sqrt{20})$

$= 5\sqrt{5}\,(2 \cdot 2\sqrt{5})$

$= 20(\sqrt{5})^2 = 20(5) = 100$ ft^2

(10) $s(t) = -16t^2 + 8t + 8$

t = time (in seconds)

$s(t)$ = height above ground (in ft)

(a) $t = ?$ $\quad s(t) = 0$

$-16t^2 + 8t + 8 = 0 \quad \Big/ \div(-8)$

$2t^2 - t - 1 = 0$

$(2t+1)(t-1) = 0$

$t = -\frac{1}{2}$ $\quad$ OR $\quad$ $t = 1$

no meaning

so, it takes him 1 second to hit the ground

(b) $t = ?$ $\quad s(t) = 8$

$-16t^2 + 8t + 8 = 8$

$-16t^2 + 8t = 0$

$8t(-2t+1) = 0$

$t = 0$ $\quad$ OR $\quad$ $-2t + 1 = 0$

$2t = 1$

$t = \frac{1}{2}$

The gymnast will be 8 ft above the ground at $t = 0$ seconds (initially) and again, after 0.5 seconds.

(11) $P(x) = \dfrac{72{,}900}{100x^2 + 729}$

x = # years of education

$P(x)$ = % of people in U.S who are unemployed

(a) x = independent variable (on horizontal axis)

$P(x)$ = dependent variable (on the vertical axis)

(b) Domain: $x \in [0, \infty)$

Range: $P(x) \in (0, P_{max}]$

to find P_{max}, let $x = 0$

then $P = P_{max} = \dfrac{72{,}900}{729}$

$= 100$

so, $P(x) \in (0, 100]$

(c) $P(10) = \dfrac{72{,}900}{100(10)^2 + 729}$

$P(10) = \dfrac{72{,}900}{10{,}729} \approx 6.8$

6.8% of people in the U.S with 10 years of education are unemployed.

(d) when $x \to \infty$, $P(x) \to 0$

The graph has a horizontal asymptote $P = 0$

There is no education level that guarantees employment

$P(x) \to 0$, but $P(x) \neq 0$