Math 71 Spring 2008 Name:

TEST 1 @ 120 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

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1. Solve the following equations:

a) $0.4\left(0.2n-0.3\right)=0.01$

b) $\frac{4}{5}\left(x+2\right)=\frac{1}{2}+\frac{5}{6}\left(x+3\right)$

c) $\left|2x+5\right|-10=13$

d) $A=\frac{1}{2}h\left(b+B\right)$; solve for *B*.

e) $\left|x-\frac{1}{2}\right|=\left|2x+1\right|$

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2. Solve the following inequalities. Graph the solution set. Write the solution set in interval notation.

a) $\frac{1}{2}x-3>2x+3\left(x-\frac{1}{3}\right)$

b) $\left|5+2x\right|\le-\frac{1}{2}$

c) $\left|2x+1\right|+1\le5$

d) $\left|1-2x\right|-5>4$

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3. Let $2x+\frac{1}{3}y=1$ a linear equation in two variables.

a) Graph the equation by the intercepts method. Clearly label the axes and the intercepts.
b) Find the slope of the line.
c) Find an equation for the line perpendicular to the given line and passing through $\left(1,-2\right)$.

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4. Four functions are given:

$$l\left(x\right)=\frac{3x+1}{2x+7};\qquad g\left(x\right)=\sqrt{1-3x};\qquad h\left(x\right)=3x^2+5x-1;\qquad f\left(x\right)=4x-6$$

Answer the following:

a) Find the domain of each function.

b) Find $g\left(2a\right),h\left(x+1\right),\left(h+f\right)\left(x\right),\left(h-f\right)\left(-1\right)$.

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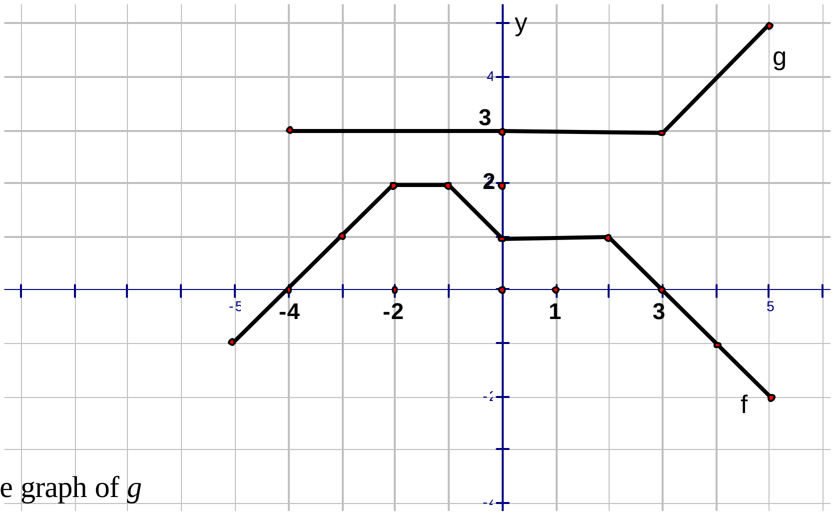
5. Let $f\left(x\right)=\begin{cases}x^2+1, & \text{if } x\le0\\ 2, & \text{if } 0<x\le5\\ 1-6x, & \text{if } x>5\end{cases}$ be a piece – wise defined function. Answer the following:

a) What is the domain of the function?

b) Find $f\left(-2\right), f\left(0\right), f\left(\frac{1}{3}\right), f\left(10\right)$.

6.

Use the graphs of *f* and *g* to answer the following:



a) Are *f* and *g* functions? Why?
b) State the domain and range of *f*.
c) State the domain and range of *g*.
d) $(f+g)(-4)$
e) $(fg)(3)$
f) $\left(\frac{f}{g}\right)(1)$
g) Solve $f(x)=0$. What do the solutions of this equation represent for the graph of *f*?
h) Find $g(0)$. What does this represent for the graph of *g*

7.

a) Graph the solution set of the following system of inequalities. Show clearly how you graph the lines and what test points you're using. Clearly label the axes, the lines, and the points used.

b) Find the coordinates of all vertices of the solution set.

$$\begin{cases} x \geq -2 \\ y \leq 4 \\ 3x+y \leq 6 \\ 2x-y \leq -1 \end{cases}$$

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8. Solve the following system of equation:

$$\begin{cases} 2x+y=2 \\ x+y-z=4 \\ 3x+2y+z=0 \end{cases}$$

9.

<u>**Choose THREE**</u> of the following word problems. Show clearly what your variables represent. Show clearly the equation(s) you use to solve each problem. You may solve **one problem for extra credit.**

A. A person invested \$6700 for one year, part at 8%, part at 10%, and the remainder at 12%. The total annual income from these investments was \$716. The amount of money invested at 12% was \$300 more than the amount invested at 8% and 10% combined. Find the amount invested at each rate.

B. At a price of *p* dollars per ticket, the number of tickets to a rock concert that can be sold is given by the demand model $D=-25p+7800$. At a price of *p* dollars per ticket, the number of tickets that the concert's promoters are willing to make available is given by the supply model $S=5p+6000$.

a) How many tickets can be sold and supplied for \$50 per ticket?

b) Find the ticket price at which the supply and demand are equal. At this price, how many tickets will be supplied and sold?

C. The function $W(x)=0.05x+3.2$ models the number of women, $W(x)$, in millions, enrolled in U.S. colleges x years after 1980.

The function $M(x)=0.02x+3.6$ models the number of men, $M(x)$, in millions, enrolled in U.S. colleges x years after 1980. Use these functions to answer the following questions:

a) Find and interpret $W(11)$.

b) Find and interpret $M(13)$

c) Find and interpret $W(10)-M(10)$.

d) Was there a time after 1980 when the number of women enrolled in U.S. colleges was equal to the number of men enrolled in U.S. colleges?

D. The cost in dollars of renting a car for one day from two different rental agencies and driving it *d* miles is given by the following equations: $C_1=50+0.10d$ and $C_2=30+0.20d$

a) Explain the meaning of each equation.

b) Graph both equations on the same coordinate system. Label the axes and all points used.

c) Which agency is cheaper?

E. An equation for the concentration of toxic chemicals is $C(t)=285-15t$, where C is the concentration in part per milliliter (ppm), and *t* is the number of years from now.

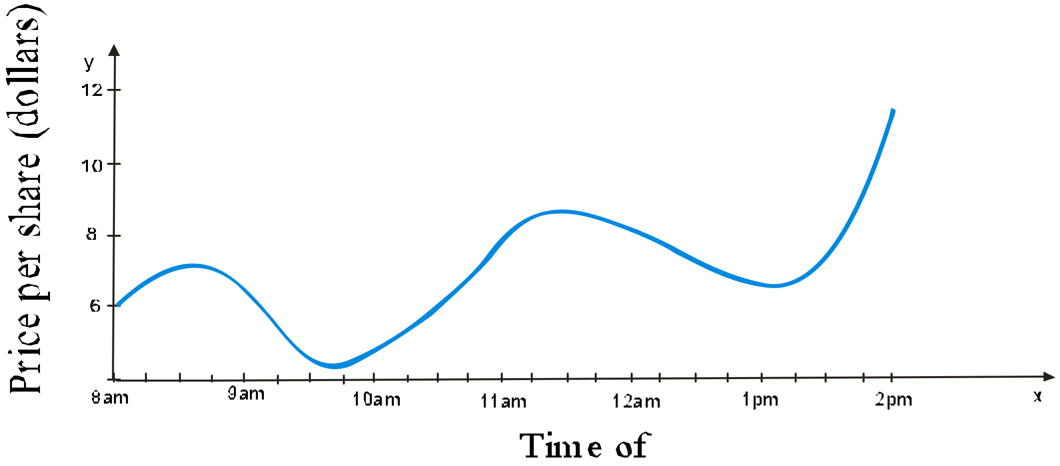
a) Find the intercepts of the graph and graph the equation using the intercepts.

b) What is the significance of the intercepts ?

c) Use function notation to express the concentration of the chemicals three years from now. What will the concentration be then?

d) Use function notation to express the question, "When will the concentration of chemicals be 180ppm?" In what year will the concentration of chemicals be 180 ppm?

F. The value of a stock varies during the course of any trading day. The price per share "*P*" of a certain stock is shown on the graph below for a particular trading day. Note "*t*" represents any time between 8 am and 2 pm.



a) Is "*P*" a function of "*t*"? Explain using the definition of function.

Using the graph, estimate the answers to the following questions (Use correct units).

b) What is the domain? What is the range?

c) For what value(s) of "*t*" does *P*(t)=8 and what does it mean in practical terms?

d) What is *P*(11) and what does it mean in practical terms?

e) For what value(s) of "*t*" is *P*(t) >5.50?

MATH 71 TEST 1- SOLUTIONS

①ⓐ $0.4(0.2n-0.3)=0.01$

$0.08n-0.12=0.01$

$0.08n=0.01+0.12$

$0.08n=0.13$

$n=\frac{0.13}{0.08}$ $\boxed{n=\frac{13}{8}}$

OR

$0.4(0.2n-0.3)=0.01 \quad /\cdot 100$

$10\cdot 10(0.4)(0.2n-0.3)=100(0.01)$

$4(2n-3)=1$

$8n-12=1$

$8n=13 \quad n=\frac{13}{8}$

(b) $\overset{6/}{\frac{4}{5}}(x+2)=\overset{15/}{\frac{1}{2}}+\overset{5/}{\frac{5}{6}}(x+3)$

LCD = 30

$24(x+2)=15+25(x+3)$

$24x+48=15+25x+75$

$24x+48=25x+90$

$48-90=25x-24x$

$-42=x$

$\boxed{x=-42}$

ⓒ $|2x+5|-10=13$

$|2x+5|=23$

$2x+5=23$ OR $2x+5=-23$

$2x=23-5 \qquad 2x=-23-5$

$2x=18 \qquad 2x=-28$

$x=9 \qquad x=-14$

$\boxed{x\in\{9,-14\}}$

(d) $A=\frac{1}{2}h(b+B)$ solve for B

$2A=h(b+B) \quad /\div h$

$b+B=\frac{2A}{h}$

$\boxed{B=\frac{2A}{h}-b}$ or $B=\frac{2A-bh}{h}$

ⓔ $|x-\frac{1}{2}|=|2x+1|$

$x-\frac{1}{2}=2x+1$ OR $x-\frac{1}{2}=-(2x+1)$

$-\frac{1}{2}-1=2x-x \qquad x-\frac{1}{2}=-2x-1$

$-\frac{3}{2}=x \qquad x+2x=\frac{1}{2}-1$

$x=\frac{-3}{2} \qquad 3x=\frac{-1}{2} \quad /\cdot\frac{1}{3}$

$x=\frac{-1}{6}$

$\boxed{x\in\{-\frac{3}{2},\frac{-1}{6}\}}$

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② ⓐ $\frac{1}{2}x-3>2x+3(x-\frac{1}{3})$

$\frac{1}{2}x-3>2x+3x-1$

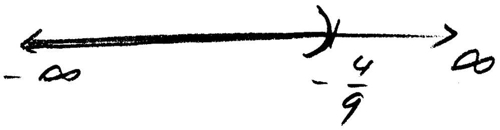
$-3+1>5x-\frac{1}{2}x$

$-2>\frac{10x-x}{2}$

$-2>\frac{9x}{2} \quad /\cdot\frac{2}{9}$

$-\frac{4}{9}>x \iff \boxed{x<-\frac{4}{9}}$

graph



interval notation

$x\in(-\infty,-\frac{4}{9})$

(b) $|5+2x| \le -\frac{1}{2}$

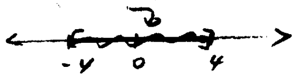
not possible because $|a| \ge 0$ for any $a \in \mathbb{R}$

Therefore, $\boxed{x \in \emptyset}$

(no solutions)

(c) $|2x+1|+1 \le 5$

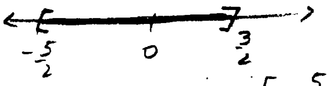
$|2x+1| \le 4$



$-4 \le 2x+1 \le 4 \quad /-1$

$-5 \le 2x \le 3 \quad /\div 2$

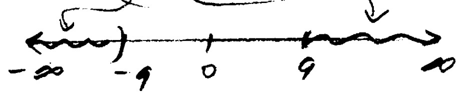
$\boxed{-\frac{5}{2} \le x \le \frac{3}{2}}$

graph: 

interval notation: $x \in [-\frac{5}{2}, \frac{3}{2}]$

(d) $|1-2x|-5 > 4$

$|1-2x| > 9$

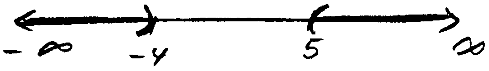


$1-2x < -9$ OR $1-2x > 9$

$-2x < -10$ $\quad$ $-2x > 8$

$x > 5$ $\quad$ $x < -4$

$\boxed{x > 5 \text{ OR } x < -4}$

graph: 

Interval notation

$x \in (-\infty, -4) \cup (5, \infty)$

③ $2x + \frac{1}{3}y = 1$

(a)

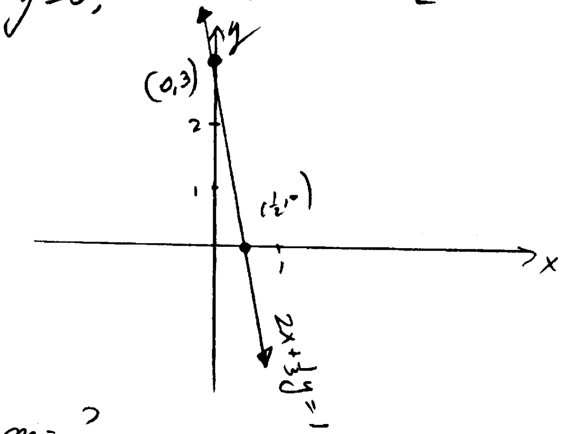
x	y
0	3
$\frac{1}{2}$	0

(0,3) y-∩

$(\frac{1}{2}, 0)$ x-∩

if $x=0$, $\frac{1}{3}y = 1$, $y = 3$

if $y=0$, $2x = 1$, $x = \frac{1}{2}$



(b) $m = ?$

$2x + \frac{1}{3}y = 1$

$\frac{1}{3}y = -2x + 1 \quad /\cdot 3$

$y = -6x + 3$

$\boxed{m = -6}$

OR

$m = \frac{\Delta y}{\Delta x}$

$m = \frac{3-0}{0-\frac{1}{2}}$

$m = \frac{3}{-\frac{1}{2}} = -6$

(c) line ⊥ to $2x + \frac{1}{3}y = 1$ through $(1,-2)$

$m_{\perp} = \frac{1}{6}$

Use $(1,-2)$ and $m = \frac{1}{6}$

$y - y_1 = m(x - x_1)$

$y - (-2) = \frac{1}{6}(x-1)$

$\boxed{y + 2 = \frac{1}{6}(x-1)}$

OR

$y = \frac{1}{6}x - \frac{13}{6}$

(4) (a) $\ell(x) = \frac{3x+1}{2x+7}$

condition: $2x+7 \neq 0$

$x \neq \frac{-7}{2}$

$\boxed{\text{Domain} = \mathbb{R} \setminus \{\frac{-7}{2}\}}$

$g(x) = \sqrt{1-3x}$

condition: $1-3x \geq 0$

$1 \geq 3x$

$\frac{1}{3} \geq x$ or $x \leq \frac{1}{3}$

$\boxed{\text{Domain} = (-\infty, \frac{1}{3}]}$

$h(x) = 3x^2+5x-1$

$\boxed{\text{Domain} = \mathbb{R}}$ (there are no restrictions)

$f(x) = 4x-6$ (there are no restrictions)

$\boxed{\text{Domain} = \mathbb{R}}$

(b) $g(2a) = \sqrt{1-3(2a)}$

$\boxed{g(2a) = \sqrt{1-6a}}$

$h(x+1) = 3(x+1)^2+5(x+1)-1$

$= 3(x^2+2x+1)+5x+5-1$

$= 3x^2+6x+3+5x+4$

$\boxed{h(x+1) = 3x^2+11x+7}$

$(h+f)(x) = h(x)+f(x)$

$= (3x^2+5x-1)+(4x-6)$

$= 3x^2+9x-7$

$\boxed{(h+f)(x) = 3x^2+9x-7}$

$(h-f)(-1) = h(-1) - f(-1)$

$= (3(-1)^2+5(-1)-1) - (4(-1)-6)$

$= (3-5-1) - (-4-6)$

$= -3-(-10) = -3+10 = 7$

$\boxed{(h-f)(-1) = 7}$

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(5) (a) The function is defined for any $x \in (-\infty, \infty)$

Therefore, $\boxed{\text{Domain} = \mathbb{R}}$

(b) $f(-2) = (-2)^2+1 = 5$

because $x = -2 \leq 0$

$f(0) = 0^2+1 = 1$

because $0 \leq 0$

$f(\frac{1}{3}) = 2$ because $x = \frac{1}{2} \in (0, 5]$

$f(10) = 1-6(10) = -59$

because $x = 10 > 5$

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(6) (a) They are both functions because their graphs pass the vertical line test — that is, any vertical line has at most one common point with the graph

(b) $D_f = [-5, 5]$

$R_f = [-2, 2]$

$D_g = [-4, 5]$

$R_g = [3, 5]$

(d) $(f+g)(-4) = f(-4) + g(-4)$

$= 0 + 3 = 3$

(e) $(fg)(3) = f(3)\,g(3)$

$= 0 \cdot (3) = 0$

(f) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1}{3}$

(g) $f(x) = 0$ iff $x = -4$ or $x = 3$
The solutions represent the x-intercepts of the graph of f. $(-4,0)$ and $(3,0)$

(h) $g(0) = 3$
$(0,3)$ represents the y-intercept of the graph of g

(7) (a) $\boxed{x \geq -2}$

Boundary line: $x = -2$
(vertical line)

$\boxed{y \leq 4}$

Boundary line: $y = 4$
(horizontal line)

$\boxed{3x + y \leq 6}$

Boundary line: $3x + y = 6$

x	y
0	6
2	0

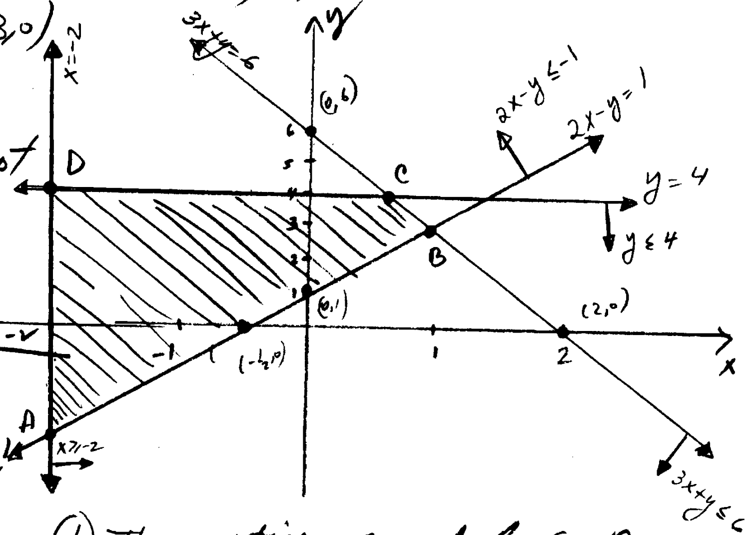
Test point $(0,0) \notin 3x + y = 6$
$3(0) + 0 \leq 6$ (true)
so $(0,0) =$ solution

$\boxed{2x - y \leq -1}$

Boundary line: $2x - y = -1$

x	y
0	1
$-\frac{1}{2}$	0

Test point $(0,0) \notin 2x - y = -1$
$2(0) - 0 \leq -1$ (false)
so $(0,0) \neq$ solution



(b) The vertices are A, B, C, D
A is the intersection of

$$\left\{\begin{array}{l} x = -2 \\ \text{and} \\ 2x - y = -1 \end{array}\right.$$

$2(-2) - y = -1$
$-4 - y = -1$
$-3 = +y$
$y = -3$

$\boxed{A(-2,-3)}$

D is the intersection of

$x=-2$ and $y=4$

$\boxed{D(-2,4)}$ /

C is the intersection of

$\begin{cases} y=4 \text{ and} \\ 3x+y=6 \end{cases}$

$3x+4=6$

$3x=2$

$x=\frac{2}{3}$

$\boxed{C(\frac{2}{3},4)}$ /

B is the intersection of

$\begin{cases} 2x-y=-1 \\ \text{and} \\ 3x+y=6 \end{cases}$

(+) $5x=5$, $x=1$

$3x+y=6$

$3(1)+y=6$, $y=3$

$\boxed{B(1,3)}$ /

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(8) $\begin{cases} 2x+y=2 & ① \\ x+y-z=4 & ② \\ 3x+2y+z=0 & ③ \end{cases}$

② $\begin{cases} x+y-z=4 \end{cases}$
③ $\begin{cases} 3x+2y+z=0 \end{cases}$

(+) $4x+3y=4$ (4)

Use ① $\begin{cases} 2x+y=2 \\ 4x+3y=4 \end{cases}$ / -2
(4)

$\begin{cases} -4x-2y=-4 \\ 4x+3y=4 \end{cases}$

$y=0$

(1) $2x+y=2$

$2x=2$, $x=1$

(2) $x+y-z=4$

$1+0-z=4$

$-z=3$

$z=-3$

The solution is $\boxed{(1,0,-3)}$ /

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(94)

\$ 6700 { I account at 8%, II account at 10%, III account at 12%

Total annual income = \$716

Let x = amount invested in I account

y = amount invested in II

z = amount invested in III

$\begin{cases} x+y+z=6700 \\ 8\%x+10\%y+12\%z=716 \\ z=300+(x+y) \end{cases}$

$\begin{cases} x+y+z=6700 \\ \frac{8}{100}x+\frac{10}{100}y+\frac{12}{100}z=716 \\ z=300+x+y \end{cases}$ / $\cdot 100$

$\begin{cases} x+y+z=6700 \\ 8x+10y+12z=71600 \\ z=300+x+y \end{cases}$ / $\div 2$

$$\begin{cases} x+y+z = 6700 & (1) \\ 4x+5y+6z = 35800 & (2) \\ z = 300 + x+y & (3) \end{cases}$$

Use substitution method

Substitute z in equation (1) and (2)

$$\begin{cases} x+y+(300+x+y) = 6700 \\ 4x+5y+6(300+x+y) = 35,800 \end{cases}$$

$$\begin{cases} 2x+2y = 6400 \quad /\div 2 \\ 4x+5y+1800+6x+6y = 35,800 \end{cases}$$

$$\begin{cases} 2x+2y = 6400 \\ 10x+11y = 34000 \end{cases} \quad /\cdot 5$$

$$\begin{cases} -10x-10y = -32,000 \\ 10x+11y = 34,000 \end{cases}$$

$(+)$ $\boxed{y = 2,000\ \$}$ invested at 10%

$2x+2y = 6400 \quad /\div 2$

$x+y = 3200$

$x+2000 = 3200$

$\boxed{x = 1200\ \$}$ invested at 8%

(3) $z = 300+x+y$

$z = 300+1200+2000$

$\boxed{z = 3500\ \$}$ invested at 12%

(9B) $D = -25p + 7800$

p = price per ticket

D = # tickets that can be sold (demand)

$S = 5p + 6000$

p = price per ticket

S = # tickets made available (supply)

(a) If $p = 50$ $/ticket

$D = -25(50)+7800$

$\boxed{D = 6550}$ tickets can be sold

$S = 5(50)+6000$

$\boxed{S = 6250}$ tickets can be supplied

(b) $p = ?$ such that $D = S$

$-25p + 7800 = 5p + 6000$

$7800 - 6000 = 5p + 25p$

$1800 = 30p$

$\boxed{p = 60\ \$/\text{ticket}}$ in order for supply = demand

Then, $D = S = 5(60)+6000$

$\boxed{= 6300 \text{ tickets}}$

If a ticket is 60$, then the supply and demand are both equal to 6300 tickets

(9c) $W(x) = 0.05x + 3.2$
$M(x) = 0.02x + 3.6$

x = # years after 1980
$W(x)$ = # women enrolled in colleges
$M(x)$ = # men — " —

(a) $W(11) = 0.05(11) + 3.2$
$W(11) = 3.75$ million women enrolled 11 years after 1980, that is, in 1991

(b) $M(13) = 0.02(13) + 3.6$
$M(13) = 3.86$ million men enrolled 13 years after 1980 - that is, in 1993

(c) $W(10) - M(10) =$
$= (0.05(10) + 3.2) - (0.02(10) + 3.6)$
$= 3.7 - 3.8$
$W(10) - M(10) = -0.1$ million
less women than men enrolled 10 years after 1980, that is, in 1990.

(d) $x = ?$ such that
$W(x) = M(x)$

$0.05x + 3.2 = 0.02x + 3.6$
$0.05x - 0.02x = 3.6 - 3.2$
$0.03x = 0.4$
$x = \frac{0.4}{0.03}$
$x \approx 13.3$ years after 1980 (1993)

(9D) $C_1 = 50 + 0.10d$
$C_2 = 30 + 0.20d$

d = # miles driven
C_1 = cost of renting a car from agency I
C_2 = cost of renting a car from agency II

(a) $C_1 = 50 + 0.10d$
It costs 50 $/day plus 0.10 $ per mile driven to rent a car from agency I

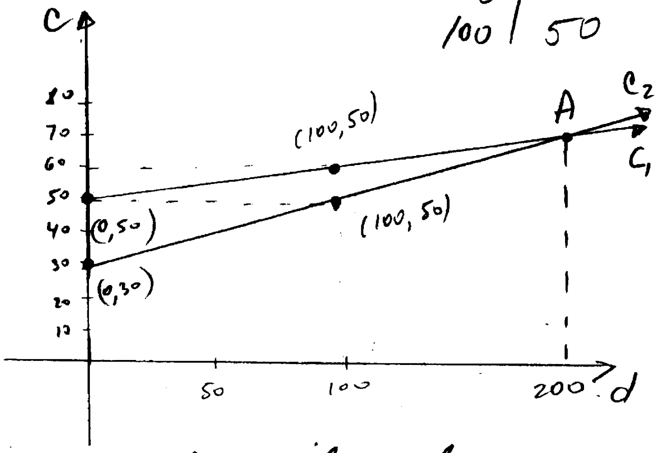
$C_2 = 30 + 0.20d$
It costs 30 $/day plus 0.20 $ per mile driven to rent a car from agency II

(b) $C_1 = 50 + 0.10d$

d	C
0	50
100	60

$C_2 = 30 + 0.20d$

d	C
0	30
100	50



find A - intersection between C_1 and C_2

$C_1 = 50 + 0.10 d$

$C_2 = 30 + 0.20 d$

$50 + 0.10 d = 30 + 0.20 d$

$20 = 0.10 d$

$d = \frac{20}{0.10}$ $d = 200$ miles

Therefore, if d (# miles driven) is such that $0 \le d < 200$

$C_2 < C_1$

so agency II is cheaper

if $d = 200$,

$C_1 = C_2$

both agency cost the same

if $d > 200$, $C_1 < C_2$

so agency I is cheaper

(E) $C(t) = 285 - 15t$

t = # years from now

$C(t)$ = concentration of chemical

(a) t = independent variable

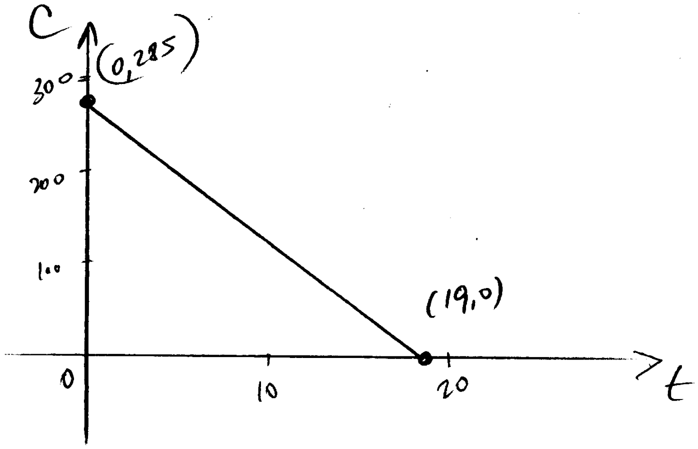
$C(t)$ = dependent

t	C
0	285
19	0

$t=0$, $C = 285$

$C=0$, $285 = 15t$

$t=19$



(b) t-n: (19,0)

when $t=19$, $C=0$

therefore, it will take 19 years from now for the concentration to be zero

C-n: (0,285)

when $t=0$, $C=285$

Today's concentration is 285 ppm.

(c) $t=3$, $C=?$

$C(3) = 285 - 15(3)$

$C(3) = 240$ ppm

(d) $t=?$ if $C = 180$ ppm

solve $C(t) = 180$

$285 - 15t = 180$

$285 - 180 = 15t$

$105 = 15t$

$t = \frac{105}{15}$

$t = 7$ years

7 years from now the concentration will be 180 ppm.

(9F) (a) P is a function of t because for every input (independent variable), there is only one output (dependent variable)

(b) Domain = [8am, 2pm]

Range ≈ [1\$, 12\$]

(c) $P(t) = 8$ iff $t \approx$ 11 am or $t =$ 12:15 pm or
$t \approx$ 1:30 pm

At 11 am, 12:15 pm and 1:30 pm the price per share was 8 \$.

(d) $P(11) = 8$ dollars

At 11 am, the price per share was 8 \$

(e) $P(t) > 5.50$ iff $t \in$ [8am, 9:15am) or
$t \in$ (10.15 am, 2pm]