Math 71 – Spring 2008 Name:

QUIZ #2 @ 50 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. If $f(x)=x+1$, and $g(x)=x^2-2x+3$. Find the following and simplify:

a) $f(a+h)-f(a)$

b) $(f+g)(x)$

c) $(f-g)(x)$

d) $(fg)(-1)$

2. Factor each polynomial completely:

a) $9t^2-15t+4$

b) $x^2-5x-24$

c) m^3+27a^3

d) $2x^4-6x-x^3y+3y$

e) $1-81v^4$

3. Solve each equation by factoring:

a) $x^3=49x$

b) $(x-3)(x+8)=-30$

d) $-3(5x+1)\left(3x-\frac{1}{3}\right)(2x^2+7x)=0$

e) $\frac{1}{4}x^2-\frac{5}{2}x+6=0$

4. Solve or simplify, whichever is appropriate:

a) $\frac{x+2}{x^2-x}-\frac{6}{x^2-1}$

b) $\frac{3y}{y^2-4}+\frac{y}{y^2-y-2}=\frac{2y-1}{y^2+3y+2}$

c) $\frac{3x+1}{x-4}=\frac{6x+5}{2x-7}$

d) $\frac{x^2-49}{x^2-4x-21}\cdot\frac{x+3}{x}$

MATH 71 Quiz #2 – SOLUTIONS

(1) $f(x) = x+1$
$g(x) = x^2 - 2x + 3$

(a) $f(a+h) - f(a) =$
$= (a+h+1) - (a+1)$
$= a + h + 1 - a - 1 = h$

(b) $(f+g)(x) = f(x) + g(x)$
$= (x+1) + (x^2 - 2x + 3)$
$= x + 1 + x^2 - 2x + 3$
$= x^2 - x + 4$

(c) $(f-g)(x) = f(x) - g(x)$
$= (x+1) - (x^2 - 2x + 3)$
$= x + 1 - x^2 + 2x - 3$
$= -x^2 + 3x - 2$

(d) $(fg)(-1) = f(-1)\, g(-1)$
$= (-1+1)((-1)^2 - 2(-1) + 3)$
$= 0(1+2+3)$
$= 0$

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(2) (a) $9t^2 - 15t + 4 =$
(product $= ac = 9(4) = 36$ $\langle$ -12, -3; sum $= b = -15$)
$= 9t^2 - 12t - 3t + 4$
$= 3t(3t-4) - (3t-4)$
$= (3t-4)(3t-1)$

(b) $x^2 - 5x - 24 = (x-8)(x+3)$
product $= c = -24$ $\langle$ $+3$, -8; sum $= b = -5$
$24 = 8 \cdot 3$

(c) $m^3 + 27a^3 = m^3 + (3a)^3$
sum of cubes
$= (m+3a)(m^2 - m(3a) + (3a)^2)$
$= (m+3a)(m^2 - 3ma + 9a^2)$

(d) $2x^4 - 6x - x^3y + 3y =$
$= 2x(x^3 - 3) - y(x^3 - 3)$
$= (x^3 - 3)(2x - y)$

(e) $1 - 81x^4 = 1 - (9x^2)^2$
difference squares
$= (1 - 9x^2)(1 + 9x^2)$
$= (1 - (3x)^2)(1 + 9x^2)$
diff. squares
$= (1 - 3x)(1 + 3x)(1 + 9x^2)$

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(3) (a) $x^3 = 49x$
$x^3 - 49x = 0$
$x(x^2 - 49) = 0$
$x(x-7)(x+7) = 0$
$x = 0$ OR $x - 7 = 0$ OR $x + 7 = 0$
$x = 7$ $x = -7$

$x \in \{0, 7, -7\}$

(b) $(x-3)(x+8) = -30$

$x^2 + 8x - 3x - 24 + 30 = 0$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x+3 = 0$ OR $x+2 = 0$

$x = -3$ $\quad$ $x = -2$

$x \in \{-3, -2\}$

(d) $-3(5x+1)(3x - \frac{1}{3})(2x^2 + 7x) = 0$

$5x+1 = 0 \Rightarrow x = \frac{-1}{5}$

OR

$3x - \frac{1}{3} = 0 \Rightarrow 3x = \frac{1}{3}, \quad x = \frac{1}{9}$

OR

$2x^2 + 7x = 0 \Rightarrow x(2x+7) = 0$

$x = 0$ OR $2x+7 = 0$

$x = \frac{-7}{2}$

Thus, $x \in \{-\frac{1}{5}, \frac{1}{9}, 0, \frac{-7}{2}\}$

(e) $\frac{1}{4}x^2 - \frac{5}{2}x + 6 = 0 \quad / \cdot 4$

$LCD = 4$

$x^2 - 10x + 24 = 0$

$(x-6)(x-4) = 0$

$x - 6 = 0$ OR $x - 4 = 0$

$x = 6$ $\quad$ $x = 4$

$x \in \{6, 4\}$

(4) (a) $\frac{x+2}{x^2 - x} - \frac{6}{x^2 - 1} =$

(algebraic expression)

$= \frac{x+1}{x(x-1)} - \frac{x}{(x+1)(x-1)}$

$LCD = x(x-1)(x+1)$

$= \frac{(x+1)(x+2) - 6x}{x(x-1)(x+1)}$

$= \frac{x^2 + 3x + 2 - 6x}{x(x-1)(x+1)} = \frac{x^2 - 3x + 2}{x(x-1)(x+1)}$

$= \frac{(x-2)(x-1)}{(x-1)(x+1)} = \frac{x-2}{x+1}$

(b) $\frac{3y}{y^2 - 4} + \frac{y}{y^2 - y - 2} = \frac{2y - 1}{y^2 + 3y + 2}$

(equation)

$\frac{y+1}{3y} + \frac{y+2}{y} = \frac{y-2}{2y-1}$

$\frac{3y}{(y+2)(y-2)} + \frac{y+2}{(y-2)(y+1)} = \frac{y-2}{(y+1)(y+2)}$

conditions: $\begin{cases} y+2 \neq 0 & y \neq -2 \\ y-2 \neq 0 & y \neq 2 \\ y+1 \neq 0 & y \neq -1 \end{cases}$

$LCD = (y+2)(y-2)(y+1)$

$3y(y+1) + y(y+2) = (y-2)(2y-1)$

$3y^2 + 3y + y^2 + 2y = 2y^2 - 5y + 2$

$4y^2 + 5y = 2y^2 - 5y + 2$

$4y^2 + 5y - 2y^2 + 5y - 2 = 0$

$2y^2 + 10y - 2 = 0 \quad / \div 2$

$y^2 + 5y - 1 = 0$ (not factorable)

$y = \frac{-6 \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-5 \pm \sqrt{25 - 4(1)(-1)}}{2(1)} = \frac{-5 \pm \sqrt{29}}{2}$

$y \in \left\{ \frac{-5 \pm \sqrt{29}}{2} \right\}$

(d) $\frac{x^2 - 49}{x^2 - 4x - 21} \cdot \frac{x+3}{x} =$

(algebraic expression)

$= \frac{(x-7)(x+7)}{(x-7)(x+3)} \cdot \frac{x+3}{x}$

$= \frac{x+7}{x}$

(c) $\frac{3x+1}{x-4} = \frac{6x+5}{2x-7}$

(equation)

Conditions: $x - 4 \neq 0 \quad x \neq 4$

$2x - 7 \neq 0 \quad x \neq \frac{7}{2}$

cross-product property:

$(3x+1)(2x-7) = (x-4)(6x+5)$

$6x^2 - 21x + 2x - 7 = 6x^2 + 5x - 24x - 20$

$6x^2 - 19x - 7 = 6x^2 - 19x - 20$

$-7 = -20$ contradiction

$x \in \emptyset$