Math 61 Summer 2008 Name:

TEST #1 @ 180 points

Write in a neat and organized fashion. Use a pencil. Use a straightedge and compass for your drawings. Write all the answers and proofs on separate paper.

1)

a) Write the inverse, converse, and contrapositive of the following statement:

"*A baby sneezes when it gets pepper in its nose.*"

b) - Write the inverse, converse, and contrapositive of the following statement;
- Classify each statement (the given, inverse, converse, and contrapositive) as true or false.
If true, state the definition, postulate, or theorem your conclusion is based on.
If false, say why or draw a counterexample.

" *If D is the midpoint of* $\overline{AB}$, *then* $\overline{AD} \cong \overline{DB}$."

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2) Study each argument carefully to decide whether or not it is valid.

a) If you walk under a coconut tree, you will probably be hit on the head.
If you visit Hawaii, then you will walk under coconut trees.
Therefore, if you visit Hawaii, you will probably be hit on the head.

b) If you are using this book, then you must be able to read.
If you are a geometry student, you must be able to read.
Therefore, if you are using this book, you are a geometry students.

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3) a) Write the negation of $P \vee Q$; that is, complete the statement: $\sim(P \vee Q) \equiv$ __________

b) Then prove the above law using a truth table. Explain in words why the table shows that the two statements are equivalent.

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4) Draw a figure and write the hypothesis and conclusion for each of the following. Make sure that you write the hypothesis and conclusion using math notation pertinent to your drawing. DO NOT PROVE.

a) The median to the base of an isosceles triangle bisects the vertex angle.

b) Supplements of equal angles are equal.

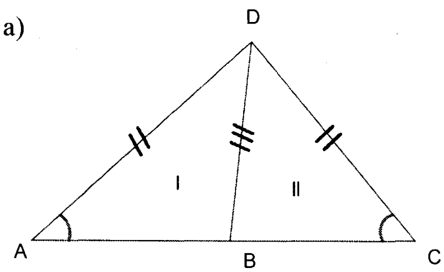
c) Vertical angles are congruent.

d) Two angles are congruent if they are both right angles.

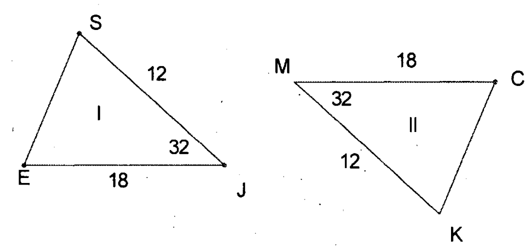
6) - Write the congruences given by the indicated measures or marks – that is, what sides and angles are congruent from the given figure.

- State whether from the given congruences only you may conclude that triangles I and II are congruent. If congruent, write what case of congruency applies.

a)



b)



7) Answer true or false:

a) The hypotenuse is the side opposite one of the acute angles in a right triangle.

b) A right isosceles triangle has two right angles.

c) If three angles of one triangle are congruent with three angles of a second triangle, then the two triangles are congruent.

d) Triangles can be proved congruent using SSA.

e) Corresponding parts of congruent triangles are congruent.

f) An exterior angle of a triangle is the supplement of one of the interior angles of the triangle.

8) Complete the following Postulates or Theorems. DO NOT PROVE.

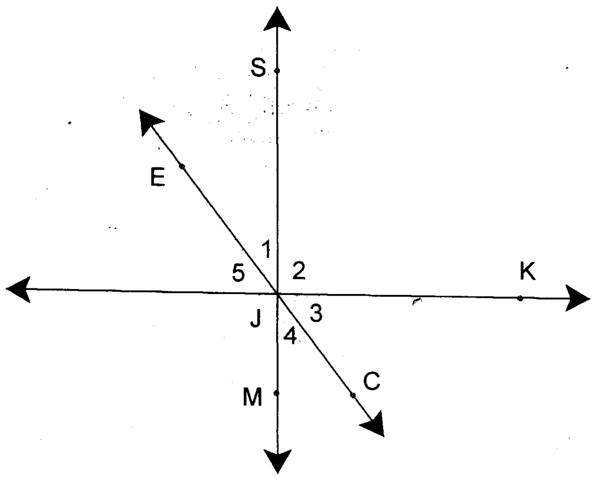
a) Two points determine __________.

b) Given two points in a plane, the line containing these points________________.

c) Segment – Addition Postulate (you may need to make a drawing first)

____

9)



Given $\overleftrightarrow{JK} \perp \overleftrightarrow{SM}$
$m\angle EJK = 105^\circ$

Find angles 1 through 5. Informal proof, But justify your answers.

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10) Prove the following theorem. Make sure you state the hypothesis and conclusion of the theorem and make a drawing. Use math notation pertinent to your drawing.

Two equal supplementary angles are right angles.

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11) Draw a figure and write the hypothesis and conclusion using math notation pertinent to your drawing. Mark the figure and write a formal proof.

If the bisector of an angle of a triangle is perpendicular to the opposite side, then the triangle is isosceles.

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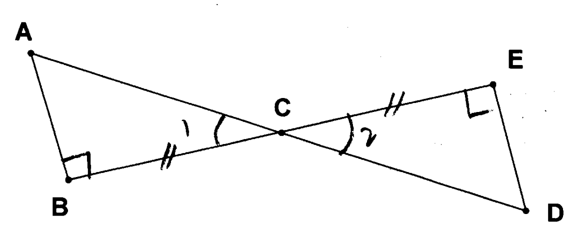
12) Given: $\angle B$, $\angle E$ right angles
$\overline{AD}$ bisects $\overline{BE}$
Prove: $\overline{AB} \cong \overline{DE}$ (formal proof)



Extra credit on next page ☺

Extra Credit

1) 2 points
Does any conclusion follow from the following pair of premises?

> If you can play the banjo while standing on your head, then you have practiced the banjo very hard.
> If you can't play "Oh Susannah," then you haven't practiced the banjo very hard.

2) 3 points
Can you rearrange the following statements in logical order? If so, what theorem do they prove?

1. If I have trouble with a proof, it is not easy.
2. If I study a proof without getting dizzy, it is one I understand.
3. If a proof is not arranged in a logical order, I can't understand it.
4. A proof is giving me trouble if I get dizzy while studying it.
5. This proof is not arranged in a logical order.

3) 2 points
What can you say about

a) the supplement of an acute angle?
b) the complement fo an acute angle?
c) the complement of an obtuse angle?
d) the supplement of a right angle?

4) 3 points
Make drawings of three coplanar rays having the same end-point such that they form:

a) three acute angles; mark the angles.
b) two acute angles and one obtuse angle; mark the angles..
c) one acute angle and two obtuse angles; mark the angles..
d) three obtuse angles; mark the angles..
e) only two angles, one of which is acute and the other obtuse; mark the angles.

TEST 1 – SOLUTIONS

(1a) if a baby gets pepper in its nose, then it sneezes. $(P \to Q)$

INVERSE $(\sim P \to \sim Q)$: if a baby doesn't get pepper in its nose, then it doesn't sneeze.

CONVERSE $(Q \to P)$: if a baby sneezes, then it gets pepper in the nose.

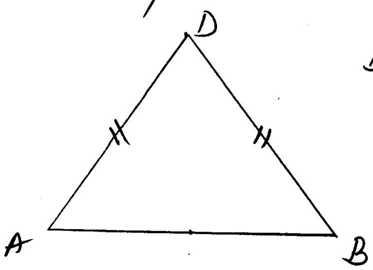
CONTRAPOSITIVE $(\sim Q \to \sim P)$: if a baby doesn't sneeze, then it doesn't get pepper in the nose.

(1b) Given statement = TRUE (definition of midpoint)

INVERSE = FALSE

" if D not the midpoint of $\overline{AB}$, then $\overline{AD} \not\cong \overline{DB}$."

For example, $\triangle DAB$ isosceles



$D \neq$ midpoint $\overline{AB}$

but

$\overline{AD} \cong \overline{BD}$

CONVERSE = FALSE

" if $\overline{AD} \cong \overline{DB}$, then D is the midpoint of $\overline{AB}$"

See previous example: $\triangle ADB$ isosceles with $\overline{AD} \cong \overline{DB}$, but D not the midpoint of $\overline{AB}$.

CONTRAPOSITIVE = TRUE

" if $\overline{AD} \not\cong \overline{DB}$, then D not the midpoint of $\overline{AB}$" (definition of midpoint)

(2) (a) VALID

(b) INVALID

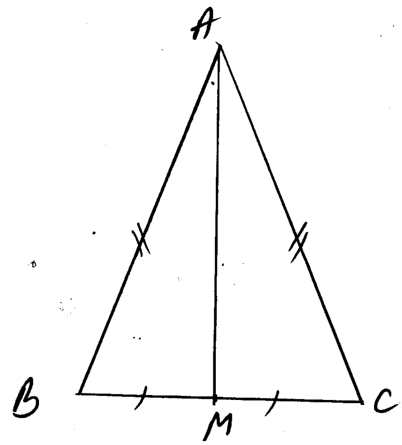
(3) $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

P	Q	$\sim$	$(P \vee Q)$	$\sim P$	$\wedge$	$\sim Q$
T	T	F	T	F	F	F
T	F	F	T	F	F	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T

The two statements are equivalent because they have identical truth values.

(4)

(a)



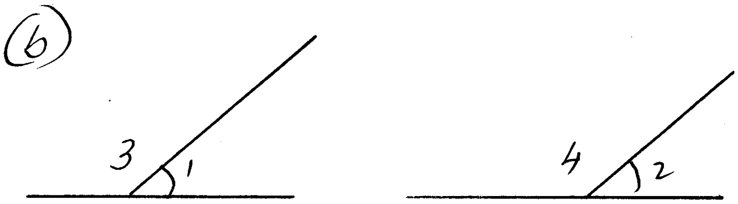
Given (hypothesis):

$\left\{ \begin{array}{l} \Delta ABC \text{ - isosceles with base } \overline{BC} \\ \overline{AM} \text{ - median }, M \in \overline{BC} \end{array} \right.$

Prove (conclusion):

$\overline{AM}$ - bisector $\measuredangle A$

(b)



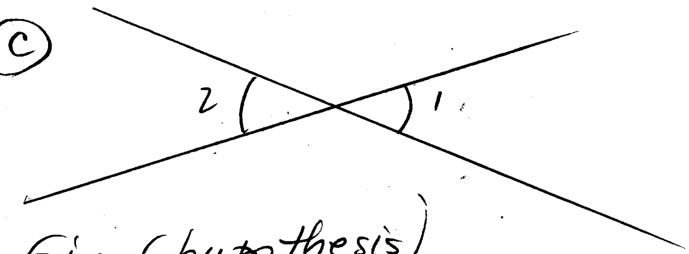
Given (hypothesis):

$\left\{ \begin{array}{l} m\angle 1 = m\angle 2 \\ \angle 1, \angle 3 = \text{supplementary} \\ \angle 2, \angle 4 = \text{supplementary} \end{array} \right.$

Prove (conclusion)

$m\angle 3 = m\angle 4$

(c)



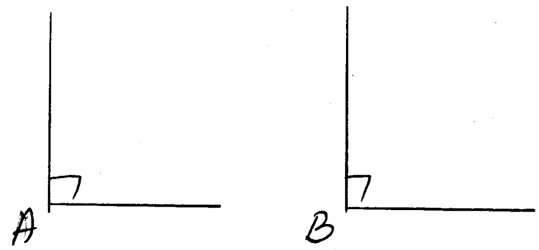
Given (hypothesis)

$\angle 1$ and $\angle 2$ = vertical angles

Prove (conclusion)

$\angle 1 \cong \angle 2$

(d)



Given (hypothesis)

$\angle A, \angle B$ = right angles

Prove (conclusion)

$\angle A \cong \angle B$

(6) (a) $\overline{AD} \cong \overline{DC}$

$\angle A \cong \angle C$

$\overline{BD} \cong \overline{BD}$

not congruent

(b) $\overline{EJ} \cong \overline{MC}$

$\overline{SJ} \cong \overline{MK}$

$\angle J \cong \angle M$

congruent, SAS

(7) a) False

b) False

c) False

d) False

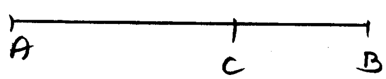
e) True

f) True

(8) a) Two points determine a line.

b) Given two points in a plane, the line containing these points is also in the plane.

c)



if $\overline{AB}$ is a line segment and C is a point on $\overline{AB}$, then $AC + CB = AB$.

(9) $m\angle 2 = 90°$ ($\overleftrightarrow{JK} \perp \overleftrightarrow{SM}$)

$m\angle 1 = m\angle EJK - m\angle 2$

$= 105° - 90°$

$= 15°$

$m\angle 4 = m\angle 1$ ($\angle 1$ and $\angle 4$ are vertical angles)

$= 15°$

$m\angle 5 = 90° - m\angle 1$

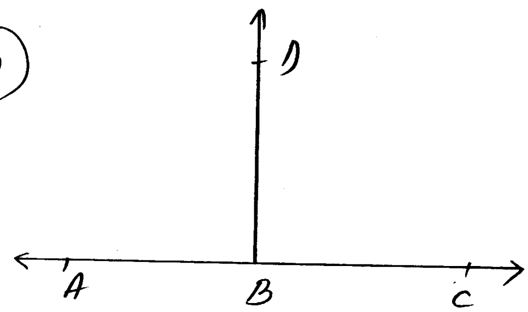
$= 90° - 15°$

$= 75°$ ($\angle 5$ and $\angle 1$ are complementary)

$m\angle 3 = m\angle 5$ ($\angle 3$ and $\angle 5$ are vertical angles)

$= 75°$

(10)



Given: $\angle ABD$, $\angle DBC$ = supplementary

$m\angle ABD = m\angle DBC$

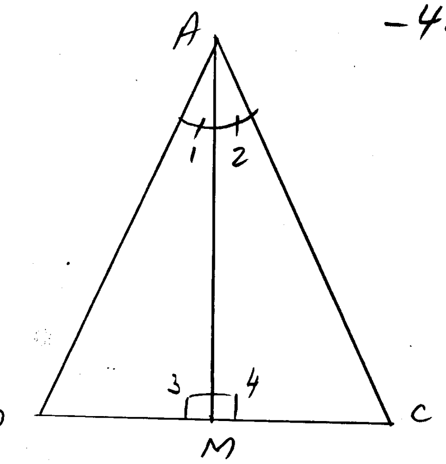
Prove: $\angle ABD$, $\angle DBC$ – right $\angle$'s

Proof

Statements	Reasons
1. $\angle ABD$, $\angle DBC$ = supplm.	1. given
2. $m\angle ABD + m\angle DBC = 180°$	2. def. supplm. $\angle$'s
3. $m\angle ABD = m\angle DBC$	3. given
4. (2,3) $m\angle ABD + m\angle ABD = 180°$	4. substitution
5. $2\,m\angle ABD = 180°$	5. simplifying
6. $m\angle ABD = 90°$	6. $\cdot/\div$ prop. of $=$
7. (3,6) $m\angle DBC = 90°$	7. substitution
8. (6,7) $\angle ABD$ and $\angle DBC$ are right $\angle$'s	8. def. right $\angle$'s

Q.E.D.

(11)



Given: $\triangle ABC$
$\overline{AM}$ - bisector $\angle A$
$\overline{AM} \perp \overline{BC}$, $M \in \overline{BC}$

Prove: $\triangle ABC$ = isosceles

Plan: We'll show that $\overline{AB} \cong \overline{AC}$ by showing $\triangle ABM \cong \triangle ACM$ (ASA)

Proof

Statements	Reasons
1. $\triangle ABC$	1. given
2. $\overline{AM} \perp \overline{BC}$	2. given
3. $\angle 3 \cong \angle 4$	3. def. $\perp$ lines ($\perp$ iff adjacent $\angle$'s $\cong$)
4. $\overline{AM}$ - bisector $\angle A$	4. given
5. $\angle 1 \cong \angle 2$	5. def. angle bisector
6. $\triangle ABM$, $\triangle ACM$ $\left\{ \angle 1 \cong \angle 2,\ \overline{AM} \cong \overline{AM},\ \angle 3 \cong \angle 4 \right.$	6. (5) above; reflexive prop. of $\cong$; (3) above
7. $\triangle ABM \cong \triangle ACM$	7. ASA
8. $\overline{AB} \cong \overline{AC}$	8. CPCTC
9. $\triangle ABC$ - isosceles	9. def. isosc. $\triangle$

(12) Proof

Statements	Reasons
1. $\angle B$, $\angle E$ - right	1. given
2. $\angle B \cong \angle E$	2. all right $\angle$'s $\cong$
3. $\overline{AD}$ bisects $\overline{BE}$	3. given
4. $\overline{BC} \cong \overline{CE}$	4. def. segment bisector
5. $\triangle ACB$, $\triangle DCE$ $\left\{ \angle B \cong \angle E,\ \overline{BC} \cong \overline{CE},\ \angle ACB \cong \angle DCE \right.$	5. (2) above; (4) above; vertical $\angle$'s
6. $\triangle ACB \cong \triangle DCE$	6. ASA
7. $\overline{AB} \cong \overline{DE}$	7. corresponding parts of $\cong$ $\triangle$'s (CPCTC)

Q.E.D.

EXTRA CREDIT

(1) let

P: "You can play banjo while standing on your head"

Q: "you have practiced the banjo very hard."

R: "you can't play "Oh Susanna""

the given statements are:

$\left\{ \begin{array}{l} P \to Q \\ R \to \sim Q \end{array} \right.$ which is equivalent to

$Q \to \sim R$

So, the conclusion is $P \to \sim R$

Therefore, if you can play banjo

while standing on your head,
then you can play
"Oh Susannah".

(2)

#1. "If I have trouble with a proof,
it is not easy."

#2. is equivalent to
"If I don't understand a proof,
then I don't study without
getting dizzy."

#3. "If a proof is not arranged
in a logical order, I can't
understand it."

#4. "If I get dizzy while
studying a proof, then the proof
is giving me trouble"

#5. "this proof is not arranged
in a logical order."

The logical order is:

#5, #3, #2, #4, #1.

The theorem proved is

"A proof not arranged in a
logical order is not easy."

(3) (a) obtuse

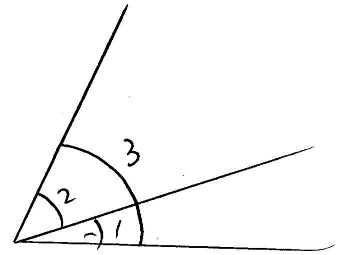
(b) acute

(c) not possible

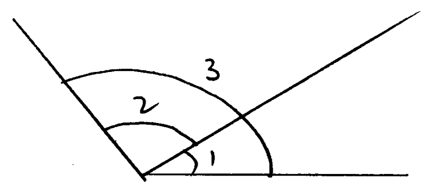
(d) right

(4)

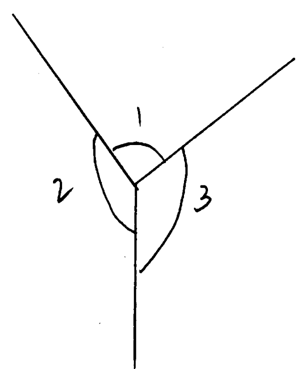
(a)



(b)



(c)



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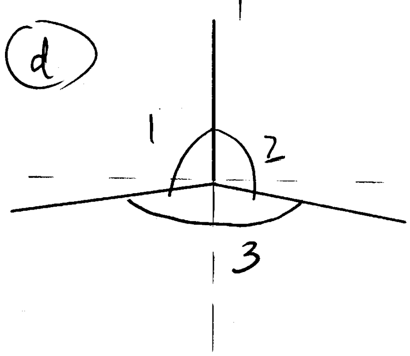

(e)