REVIEW TEST 1 - Chapters 2, 3, and 4

Chapter 2 – Functions and Graphs
After completing this section, you should be able to:
- find the distance between two points in the plane
- find the coordinates of the midpoint of a segment
- read a graph (identify the variables, domain, range, x- and y-intercepts, find whether the graph has symmetry with respect to the x-axis, y-axis, or the origin, solve equations or inequalities using the graph, identify intervals on which the graph is increasing or decreasing)
- graph a linear equation and identify its slope
- know the definition of a function; give examples of functions;
- know the definitions of increasing, decreasing, and constant functions
- graph transformations of functions
- graph a piece-wise defined function
- graph a quadratic function
- find minimum or maximum of a quadratic function
- do operations with functions

Chapter 3 – Polynomial and Rational Functions
After completing this section, you should be able to:
- know and apply the Intermediate Value Theorem
- graph a polynomial function;
- solve polynomial equations and inequalities of polynomial functions
- factor a polynomial function completely over the real number set
- write the equation of a polynomial functions given certain conditions
- graph a rational function
- write an equation for a rational function given certain conditions

Chapter 4 – Inverse, Exponential, and Logarithmic Functions
After completing this section, you should be able to:
- prove that a function is one to one algebraically
- prove that a function is not one – to – one algebraically
- find the inverse of a one-to-one function
- know the relationship between a function and its inverse (algebraically and graphically)
- graph an exponential and logarithmic function
- solve exponential and logarithmic equations
- know the properties of logarithms

Textbook Applications
Study the following problems (done in class or from the assigned homework):
- 2.3 - # 54, 57, 61, 62, 77
- 2.4 - # 71
- 2.5 - # 65, 79 (by hand, not with a calculator)
- 2.6 - # 35, 36, 40, 41, 45, 51, 55
- 2.7 - # 41, 45,
- 3.1 - #45
- Chapter 3 Review - # 45
- 4.1 - #59, 60
- 4.2 - # 71
- 4.4 - # 65
- 4.6 - #55, 59

1.
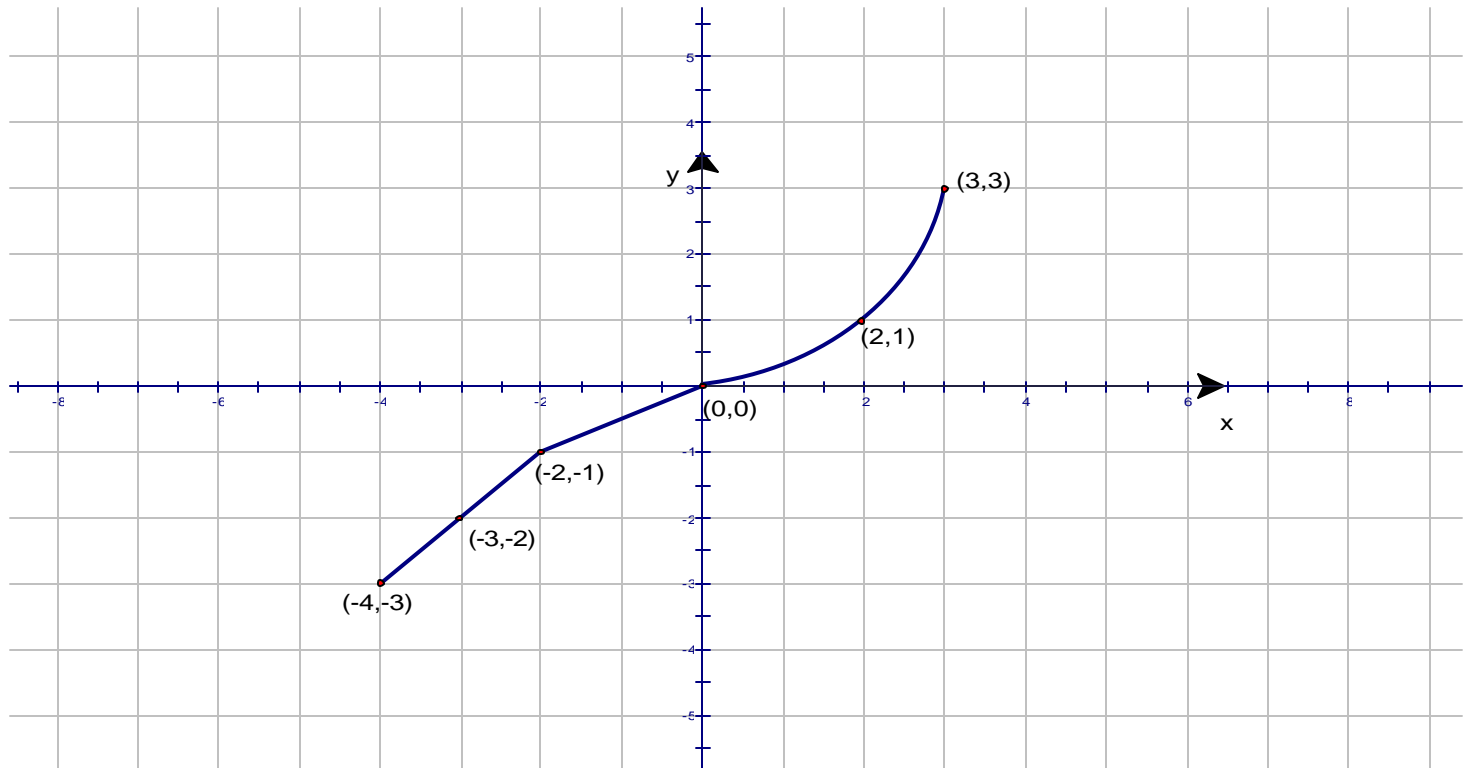
Using the graph $y = f(x)$ shown, answer the following:

a) Is y a function of x? Explain.

b) Find the domain and range of f.

c) List the intercepts (as ordered pairs).

d) Find $f(-2)$.

e) For what values of x does $f(x) = -3$?

f) Solve $f(x) > 0$.

g) Find $(f \circ f)(-3)$

h) Graph $y = f(x-2)$

i) Graph $y = f(2x)$

j) Graph $y = f(-x)$

k) Is f even, odd, or neither?

l) Does f have an inverse? Explain.

m) Find the domain and range of f^{-1}.

n) Graph $y = f^{-1}(x)$

o) Find $f^{-1}(-3)$.

p) $f\left(f^{-1}(1)\right)$

r) Find the average rate of change of f from $x = 2$ to $x = 3$.

2. $f(x) = \begin{cases} 3-x & if \ |x| \geq 3 \\ 2 & if \ |x| < 3 \end{cases}$

a) Sketch a graph for the function

b) State its domain and range in interval notation.

c) On what interval(s) is the function increasing, decreasing, constant?

d) Find $f(-5)$, $f(3)$, $f(0)$, $f(11)$.

e) Locate the intercepts.

f) Find the values of $f\left(f(1)\right)$, $(f \circ f)(3)$, and $(f \circ f)(7)$.

3. Let $g(x) = \dfrac{1}{x+1}$.

a) Sketch a graph of the function (using transformations or by finding asymptotes and plotting points).

b) What are the asymptotes for the graph?

c) State its domain and range.
d) Find the intercepts.
e) Calculate $g(-2)$.
f) Solve $g(x) = -2$.
g) Find points that correspond to parts (d) and (e) on the graph of the function.
h) Calculate the average rate of change of g from $x = 0$ to $x = 2$. That is, $\dfrac{g(2) - g(0)}{2 - 0}$.
i) Does g have an inverse? Explain.
j) Find a formula for the inverse function for g and graph the inverse g^{-1} showing the symmetry through $y = x$.
k) State the domain and range for the inverse function g^{-1}.
l) Find $(g \circ g)(1)$ and $(g^{-1} \circ g)(a)$ $(a \neq -1)$

4. Consider the function $f(x) = \dfrac{100}{1 + x^2}$.
a) Determine whether the function is even, odd, or neither using algebra.
b) What is the domain of this function? What is the range?
c) Find the asymptotes of the graph.
d) What are the intercepts (if any)?
e) Make a table of values including $x = 0$, $x = \pm 1$, $x = \pm 2$. Then use these points along with the previous information (a) – (d) to sketch the graph of this function.
f) Then sketch a graph of $y = f(x - 3)$ (using the points from the table from (e)).

5. Suppose a patient is given 250 mg injection of a therapeutic drug that has the side effect of raising her heart rate. Over time, the patient's body metabolizes the drug, and the level of drug in her bloodstream falls accordingly. Since the patient's heart rate depends on the drug level, and the drug level depends on time, her heart rate depends, via the drug level, on time. A formula for the heart rate as a function of drug level is

$R = f(Q) = 60 + 0.2Q$. A formula for the drug level as a function of time is $Q = g(t) = 250(.8)^t$.

Given these formulas, find the heart rate as a function of time.

6. The formula for the volume of a cube of side s is $V = s^3$. The formula for the surface area of a cube is $A = 6s^2$. Express the volume of a cube, V, as a function of its surface area, A.

7. Resistance is an electrical property that measures the extent to which a given material resists the flow of electric current. The resistance of certain materials varies significantly with temperature. Table 1 gives values of R, the electrical resistance of a certain circuit element (in ohms), for values of T, the temperature (in degrees Celsius).

a) Is R a function of T? Explain.
b) Show that R depends linearly on T.
   Find a formula that gives R in terms of T.
c) If $R = f(T)$ does R have an inverse? Explain.
d) Find a formula for the inverse function f^{-1} and explain its meaning.
e) Find $f(10)$ and its meaning.
f) Find $f^{-1}(100)$ and its meaning.

T, temperature ($^{\circ}C$)	R, resistance (ohms)
-20	50
-10	100
0	150
10	200
20	250
30	300

8. Let $f(t) = 1 + \ln t$.
a) Graph the function.
b) State the domain, range, and vertical asymptote.
c) Find the exact x- and y-intercepts (if any).
d) Does the function have an inverse> Explain. Find $f^{-1}(x)$.

e) Graph the inverse $f^{-1}(x)$ showing the symmetry through $y = x$.

f) State the domain, range, and horizontal asymptote for the inverse function $f^{-1}(x)$.

g) Find the exact x- and y-intercepts of the inverse function $f^{-1}(x)$ (if any).

9. If $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$ is a one-to-one function, find $f^{-1}(x)$.

10. Let $f(x) = 3^{x-1} - 2$.

a) Graph the function.
b) State the domain, range, and horizontal asymptote.
c) Find the exact x- and y-intercepts (if any).

d) Does the function have an inverse? Explain. Find the inverse function $f^{-1}(x)$.

e) Graph the inverse function showing how it can be obtained from the graph of f.

f) Find the exact x- and y-intercepts for $f^{-1}(x)$ (if any).

11. Let $f(x) = \log_x\left(x^2 - 3x + 9\right) - 2$

a) Find the domain of this function.

b) Solve the equation $f(x) = 0$.

12) Consider the following polynomial function $f(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$.
Questions a-g below relate to this polynomial function.

    a) Use the leading term to describe the long-term behavior of this function; that is, what happens as $x \to \pm\infty$.

    b) Use synthetic division to divide $f(x)$ by $x - 1$ and relate dividend, divisor, quotient and remainder in an equation.

    c) Compute and compare the values of $f(1)$ and $f(2)$. What can you conclude using the intermediate value theorem?

    d) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list all possible rational zeros for $f(x)$.

    e) Find all the zeros of the polynomial.

    f) Factor $f(x)$ completely.

    g) What are the x- and y-intercepts of the graph?

    h) Sketch a graph of $f(x)$ showing how it passes through its intercepts.

13) Graph the function $f(x) = \dfrac{x^3 + 6x^2 + 9x}{2x^3 - 2}$.

14) Suppose $f(x) = \sqrt{x-1}$ and $g(x) = \dfrac{1}{x-2}$.

    a) Find $(f \circ g)(x)$ and its domain.

    b) Find $(g \circ f)(x)$ and its domain.

15) Suppose the graph of $y = f(x)$ is symmetric about the y-axis, and g is an odd function.
Is $g \circ f$ an even function, an odd function, or neither?