Math 160 Spring 2008 Name:

QUIZ #2 @ 100 points

Write neatly. Show all work. **Write all responses on separate paper.** **<u>Clearly label the exercises.</u>**

1. Find the exact value of each of the other five trigonometric functions of θ if $\sin\theta = -\frac{2}{3}$ and θ is in the third quadrant.

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2. Evaluate the following expressions. Give exact values whenever possible.

a) $\sin\frac{5\pi}{4}$

b) $\cos^{-1}\left(-\frac{1}{2}\right)$

c) $\tan\frac{\pi}{6}$

d) $\cos\left(\sin^{-1}\frac{1}{5}\right)$

e) $\ln\left|\sec x + \tan x\right| + \ln\left|\sec x - \tan x\right|$

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3. a) FIND a formula for $\sin x$ in terms of $\cos 2x$.

b) FIND a formula for $\cos a \cos b$ as a sum of sine and cosine functions.

c) Find a formula for $\cos 3\theta$ in terms of $\cos\theta$.

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4. Graph the following function between -2π and π. Show the exact x-and y-intercepts.

$f(x) = e^x \cos x$

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5. Prove the following identities:

a) $\ln\cot x = -\ln\tan x$

b) $\frac{\cos t}{1-\sin t} = \sec t + \tan t$

c) $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{\left|\cos\theta\right|}{1+\sin\theta}$

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6. Solve the following equations:

a) Solve in $[0, 2\pi)$: $\cos\left(2x - \frac{\pi}{4}\right) = 0$

b) Find ALL solutions: $\cos(\ln x) = 0$

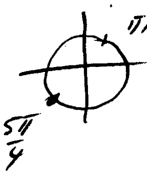
c) Solve in $[-2\pi, 2\pi]$: $\frac{1}{2} + \cos x = 0$

d) Solve in $[0, 2\pi)$: $2\sin^2 u = 1 - \sin u$

e) Find ALL solutions: $\cos x = 0.2$

f) Solve in $[0, 2\pi)$: $\cos x - \sin x = 1$

Math 160 Quiz #2 - Solutions



① $\sin\theta = -\frac{2}{3}, \quad \theta \in \underline{III}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{2}{3}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{4}{9}, \quad \cos^2\theta = \frac{5}{9}$

$\cos\theta = \pm\frac{\sqrt{5}}{3}$

$\theta \in \underline{III}$, so $\cos\theta < 0$ $\Rightarrow$

$\cos\theta = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{2}{3}}{-\frac{\sqrt{5}}{3}}$

$\tan\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{\sqrt{5}}{2}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{-3}{\sqrt{5}} = \frac{-3\sqrt{5}}{5}$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{3}{2}$

So,
$$\begin{cases} \sin\theta = \frac{-2}{3} \text{ (given)} \\ \cos\theta = \frac{-\sqrt{5}}{3} \\ \tan\theta = \frac{2\sqrt{5}}{5} \\ \cot\theta = \frac{\sqrt{5}}{2} \\ \sec\theta = \frac{-3\sqrt{5}}{5} \\ \csc\theta = \frac{-3}{2} \end{cases}$$

② (a) $\sin\frac{5\pi}{4} = \frac{-\sqrt{2}}{2}$

(b) $\cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$ because

$\cos\frac{2\pi}{3} = \frac{-1}{2}$ and $\frac{2\pi}{3} \in [0, \pi]$

(c) $\tan\frac{\pi}{6} = \frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

(d) $\cos\left(\sin^{-1}\frac{1}{5}\right) = ?$

let $\sin^{-1}\frac{1}{5} = u$, $u \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

then $\sin u = \frac{1}{5}$

$\sin^2 u + \cos^2 u = 1$

$\left(\frac{1}{5}\right)^2 + \cos^2 u = 1$

$\cos^2 u = 1 - \frac{1}{25}$

$\cos u = \pm\frac{\sqrt{24}}{5}$

$u \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so $\cos u > 0$

So. $\cos\left(\sin^{-1}\frac{1}{5}\right) = \frac{\sqrt{24}}{5} = \frac{2\sqrt{6}}{5}$

(e) $\ln|\sec x + \tan x| + \ln|\sec x - \tan x|$

$= \ln\left(|\sec x + \tan x| \cdot |\sec x - \tan x|\right)$

$= \ln|(\sec x + \tan x)(\sec x - \tan x)|$

$= \ln|\sec^2 x - \tan^2 x| = 0.$

(Recall that $\sin^2 x + \cos^2 x = 1$

so $\tan^2 x + 1 = \sec^2 x$

so $\sec^2 x - \tan^2 x = 1$)

so
$\therefore = \ln|1| = \ln 1 = 0$

(3) (a) $\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = 1 - 2\sin^2 x$
$2\sin^2 x = 1 - \cos 2x$
$\sin^2 x = \dfrac{1-\cos 2x}{2}$
$\sin x = \pm\sqrt{\dfrac{1-\cos 2x}{2}}$

(b) $\cos(a+b) = \cos a\cos b - \sin a\sin b$
$\cos(a-b) = \cos a\cos b + \sin a\sin b$

(+) $\cos(a+b) + \cos(a-b) = $
$= 2\cos a\cos b$
$\Rightarrow$
$\cos a\cos b = \dfrac{\cos(a+b)+\cos(a-b)}{2}$

(c) $\cos 3\theta = \cos(2\theta+\theta)$
$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$
$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\cdot\sin\theta$
$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cdot\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2\cos\theta +$
$+ 2\cos^3\theta$

so
$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

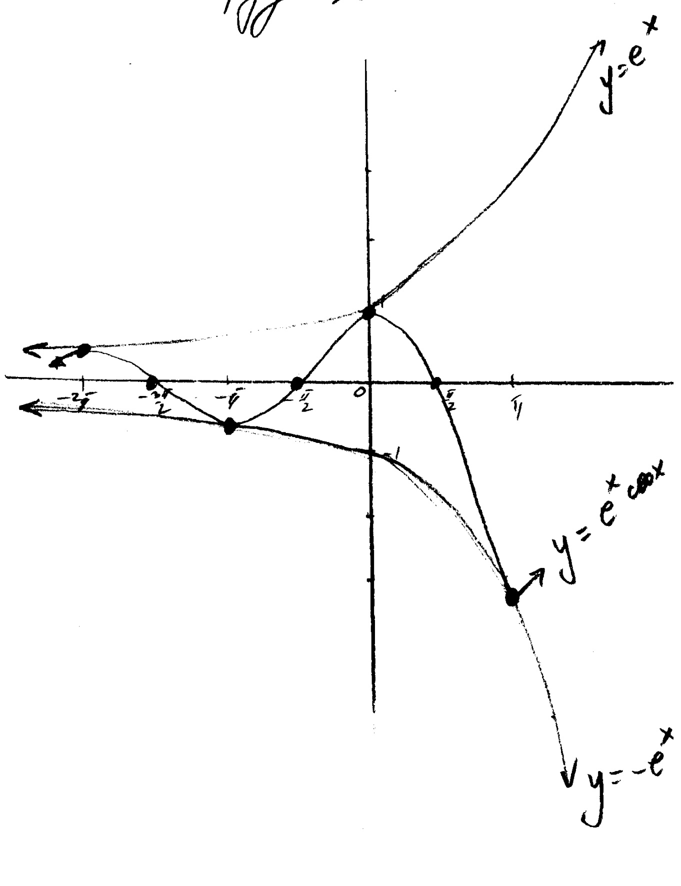
(4) $f(x) = e^x\cos x$
$-1 \le \cos x \le 1 \quad |\cdot e^x > 0$
$-e^x \le e^x\cos x \le e^x$

Therefore, the graph of $f(x)$ lies between the graphs of $y = e^x$ and $y = -e^x$

x-n. $e^x\cos x = 0$ iff
$\cos x = 0$ iff
$x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$

$f(x) = e^x$ iff $\cos x = 1$
iff $x = 0, \pm 2\pi$

$f(x) = -e^x$ iff $\cos x = -1$
iff $x = \pm\pi$



(5) (a) $\ln \cot x = -\ln \tan x$

Proof

$LHS = \ln \cot x$

$= \ln \frac{1}{\tan x}$

$= \ln 1 - \ln \tan x$

$= 0 - \ln \tan x$

$= -\ln \tan x = RHS$

thus, the given equation is an identity

(b) $\frac{\cos t}{1-\sin t} = \sec t + \tan t$

Proof

$RHS = \sec t + \tan t$

$= \frac{1}{\cos t} + \frac{\sin t}{\cos t}$

$= \frac{1+\sin t}{\cos t}$

$= \frac{(1+\sin t)(1-\sin t)}{\cos t(1-\sin t)}$

$= \frac{1-\sin^2 t}{\cos t(1-\sin t)}$

$= \frac{\cos^2 t}{\cos t(1-\sin t)}$

$= \frac{\cos t}{1-\sin t} = LHS$

so, the given equation is an identity

(c) $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{|\cos\theta|}{1+\sin\theta}$

Proof

$LHS = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{(1-\sin\theta)(1+\sin\theta)}{(1+\sin\theta)^2}}$

$= \frac{\sqrt{1-\sin^2\theta}}{\sqrt{(1+\sin\theta)^2}} = \frac{\sqrt{\cos^2\theta}}{\sqrt{(1+\sin\theta)^2}} = *$

note that $1+\sin\theta \geq 0$

so $\sqrt{(1+\sin\theta)^2} = 1+\sin\theta$

so $* = \frac{|\cos\theta|}{1+\sin\theta} = RHS$

Therefore, the given eq. is an identity.

(6) (a) $\cos(2x - \frac{\pi}{4}) = 0$

$2x - \frac{\pi}{4} = \frac{\pi}{2} + \pi k, \quad k \in \mathbb{Z}$

$2x = \frac{\pi}{2} + \frac{\pi}{4} + \pi k$

$2x = \frac{3\pi}{4} + \pi k \quad \Big/ \div 2$

$x = \frac{3\pi}{8} + \frac{\pi k}{2}$

$k=0, \quad x = \frac{3\pi}{8}$

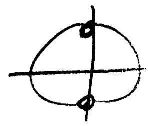
$k=1, \quad x = \frac{3\pi}{8} + \frac{\pi}{2} = \frac{7\pi}{8}$

$k=2, \quad x = \frac{3\pi}{8} + \pi = \frac{11\pi}{8}$

$k=3$. $x = \frac{3\pi}{8} + \frac{3\pi}{2} = \frac{15\pi}{8}$

$x \in \left\{ \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8} \right\}$

(b) $\cos(\ln x) = 0$



Condition: $x > 0$

$\ln x = \frac{\pi}{2} + \pi k$, $k \in \mathbb{Z}$

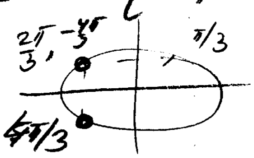
$x = e^{\frac{\pi}{2}+\pi k}$

Note that $x > 0$ for any k

$x \in \left\{ e^{\frac{\pi}{2}+\pi k} \mid k \in \mathbb{Z} \right\}$

(c) $\frac{1}{2} + \cos x = 0$ in $[-2\pi, 2\pi]$

$\cos x = \frac{-1}{2}$



$x = \frac{2\pi}{3}$ OR $x = -\frac{2\pi}{3}$ OR

$x = \frac{4\pi}{3}$ OR $x = -\frac{4\pi}{3}$

$x \in \left\{ \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3} \right\}$

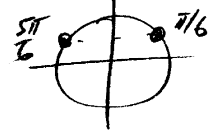
(d) $2\sin^2 u = 1 - \sin u$ in $[0, 2\pi)$

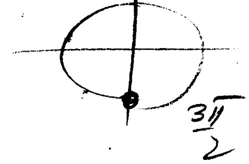
$2\sin^2 u + \sin u - 1 = 0$

$(2\sin u - 1)(\sin u + 1) = 0$

$2\sin u - 1 = 0$ OR $\sin u = -1$

$\sin u = \frac{1}{2}$ OR $\sin u = -1$



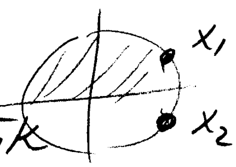


$u = \frac{5\pi}{6}$ OR $u = \frac{3\pi}{2}$

$u = \frac{\pi}{6}$

$u \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$

(e) $\cos x = 0.2$



$\begin{cases} x_1 = \cos^{-1} 0.2 + 2\pi k \\ \text{OR} \\ x_2 = 2\pi - x_1 + 2\pi k \end{cases}$

$\begin{cases} x \approx 1.37 + 2\pi k, \quad k \in \mathbb{Z} \\ \text{OR} \\ x \approx 4.91 + 2\pi k \end{cases}$

(f) Method I $\begin{cases} \cos x - \sin x = 1 & (1) \\ \cos^2 x + \sin^2 x = 1 & (2) \end{cases}$

(1) $\Rightarrow \cos x = 1 + \sin x$

(2) $\Rightarrow (1 + \sin x)^2 + \sin^2 x = 1$

$1 + 2\sin x + 2\sin^2 x = 1$

$2\sin^2 x + 2\sin x = 0$

$2\sin x(\sin x + 1) = 0$

$\sin x = 0$ OR $\sin x = -1$

$x = 0$ OR $x = \frac{3\pi}{2}$

$\cancel{x = \pi}$ doesn't check

$x \in \left\{ 0, \frac{3\pi}{2} \right\}$

Method II

$$\cos x - \sin x = 1 \quad)^2$$

$$(\cos x - \sin x)^2 = 1$$

$$\cos^2 x - 2\sin x \cos x + \sin^2 x = 1$$

$$1 - 2\sin x \cos x = 1$$

$$2\sin x \cos x = 0$$

$\sin x = 0$ OR $\cos x = 0$

$x = 0$ OR $x = \frac{\pi}{2}$ OR

$x = \pi$ $x = \frac{3\pi}{2}$

check: Only $x = 0$ and $x = \frac{3\pi}{2}$ satisfy the given equation

$$x \in \{0, \frac{3\pi}{2}\}$$