Math 160 Spring 2008 Name:

QUIZ #1 @ 100 points

Write neatly. Show all work. **Write all responses on separate paper.** **Clearly label the exercises.**

1. Let $y = 2x + 5$. Answer the following:
   a) Does this equation represent a function? Why?
   b) Graph the equation showing the x- and y-intercepts.
   c) What is the domain and the range of the function?
   d) Find and simplify $\dfrac{f(x+h)-f(x)}{h}$ (if $h \neq 0$).

2. Let $x^2 + y^2 + 4x - 2y + 5 = 9$ be the equation of a circle.

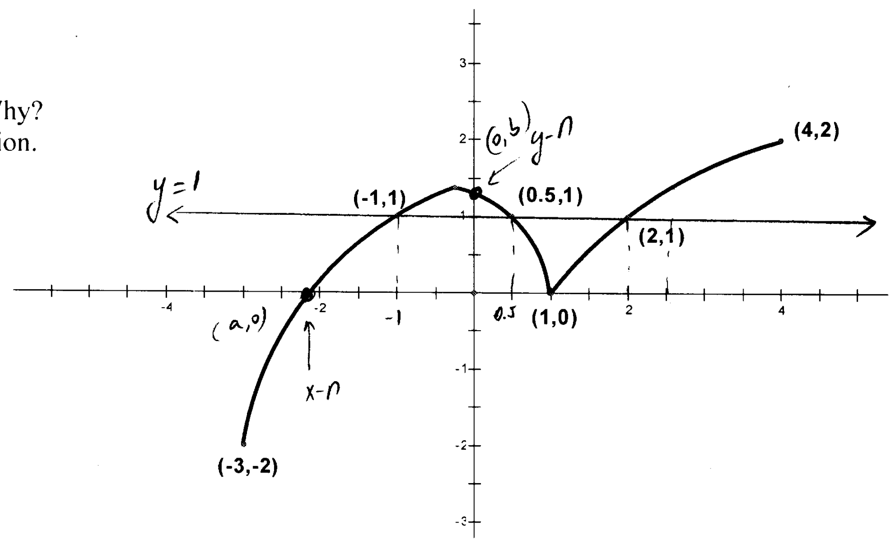
   a) Find the center and radius and graph the circle.
   b) Find the equations of the upper and lower half.
   c) Find the exact x- and y-intercepts (if any).

3. Newborn blue whales are approximately 24 feet long and weigh 3 tons. Young whales are nursed for 7 months, and by the time of weaning they often are 53 feet long and weigh 23 tons. Let *L* and *W* denote the length (in feet) and the weight (in tons), respectively, of a whale that is *t* months of age.

   a) If *W* and *t* are linearly related, express *W* in terms of t.
   b) What is the daily increase in the weight of a young whale? (use 1 month = 30 days.)

4. A graph is given. Answer all the questions:

   a) Does the graph represent a function? Why?
   b) Find the domain and range of the function.
   c) Find $f(1)$.
   d) Identify the intercepts on the graph.
   e) Find all x such that $f(x) = 1$.
   f) Find all x such that $f(x) > 1$.



5. Let $x^2 + y^2 = 25$ be a circle. Answer all the questions:

   a) Show that the point $(3,4)$ is on the circle.
   b) Find the equation of the line tangent to the circle at the point $(3,4)$.
   Note: The tangent to the circle is perpendicular to the radius of the circle at the point of tangency.

M160 Quiz 1 - Solutions

① $y = 2x+5$

(a) yes. because for every x there is only one y. Therefore, y is a function of x

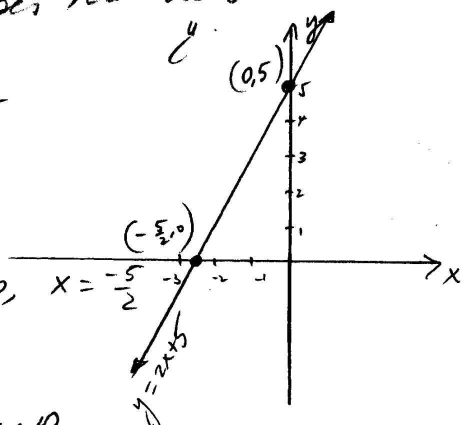
OR

The equation represents an ascending line, therefore its graph passes the vertical-line test.

(b) $y = 2x+5$

x	y
0	5
$-\frac{5}{2}$	0

$y=0,\ 2x+5=0,\ x=-\frac{5}{2}$



(c) Domain: $x \in \mathbb{R}$
Range: $y \in \mathbb{R}$

(d) $\dfrac{f(x+h)-f(x)}{h} =$

$= \dfrac{\left(2(x+h)+5\right)-(2x+5)}{h}$

$= \dfrac{\cancel{2x}+2h\cancel{+5}\cancel{-2x}\cancel{-5}}{h} = \dfrac{2h}{h} = 2$

$\dfrac{f(x+h)-f(x)}{h} = 2$

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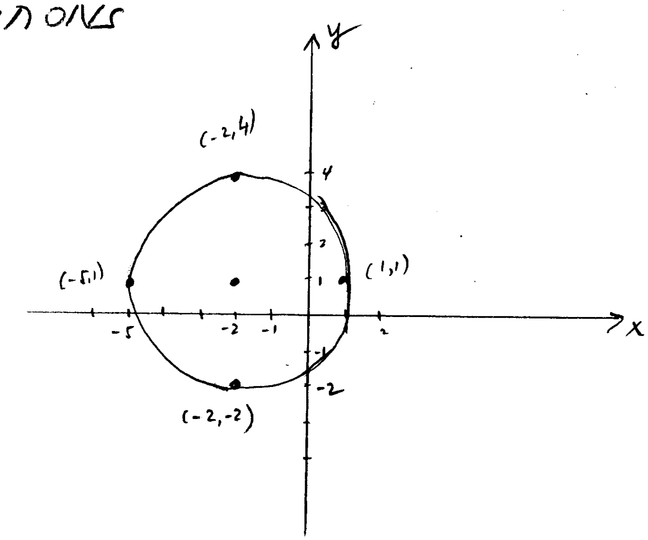
② ⓐ $x^2+y^2+4x-2y+5=9$

$x^2+4x+4+y^2-2y+1 = 9-5+4+1$

$(x+2)^2+(y-1)^2 = 9$

center $(-2,1)$
radius $\sqrt{9} = 3$



(b) $(x+2)^2+(y-1)^2 = 9$

$(y-1)^2 = 9-(x+2)^2 \quad /\sqrt{\ }$

$y-1 = \pm\sqrt{9-(x+2)^2}$

$y = 1 \pm \sqrt{9-(x+2)^2}$

upper half: $y = 1+\sqrt{9-(x+2)^2}$
lower half: $y = 1-\sqrt{9-(x+2)^2}$

ⓒ x-int: set $y=0$

$(x+2)^2+(-1)^2 = 9$

$(x+2)^2+1 = 9$

$(x+2)^2 = 8 \quad /\sqrt{\ }$

$x+2 = \pm\sqrt{8}$

$x = -2 \pm 2\sqrt{2}$

x-int: $(-2\pm2\sqrt{2},\ 0)$

y-int: set $x=0$

$2^2+(y-1)^2 = 9$

$(y-1)^2 = 5 \quad /\sqrt{\ }$

$y-1 = \pm\sqrt{5}$

$y = 1\pm\sqrt{5}$

y-int: $(0,\ 1\pm\sqrt{5})$

(3) W = weight (in tons)

(a) t = time (in months)

t	W
0	3
7	23

$m = \frac{\Delta W}{\Delta t} = \frac{23-3}{7-0} = \frac{20}{7}$ t/mo

$W = \frac{20}{7} t + 3 \quad (y = mx+b)$

(b) $m = \frac{20 \text{ tons}}{7 \text{ months}} = \frac{20 \text{ tons}}{7 \cdot 30 \text{ days}}$

$m = \frac{20}{210} = \frac{2}{21}$ tons/day

- the daily increase in weight

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(4) (a) yes, because it passes the vertical line test.

(b) Domain: $x \in [-3,4]$

Range: $y \in [-2,2]$

(c) $f(1) = ?$

when $x=1$, $y=?$

So, $f(1) = 0$

(d) There are two x-intercepts $(1,0)$ and $(a,0)$ where $a \in (-3,-2)$

There is one y-intercept $(0,b)$, where $b \in (1,1.5)$

(e) $x = ?$ so $f(x) = 1$

$x = ?$ when $y = 1$

So, $x = -1$ or $x = 0.5$

(f) $x = ?$ so $f(x) > 1$

$f(x) > 1$ iff

$x \in (-1, 0.5) \cup (2,4]$

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(5) $x^2 + y^2 = 25$

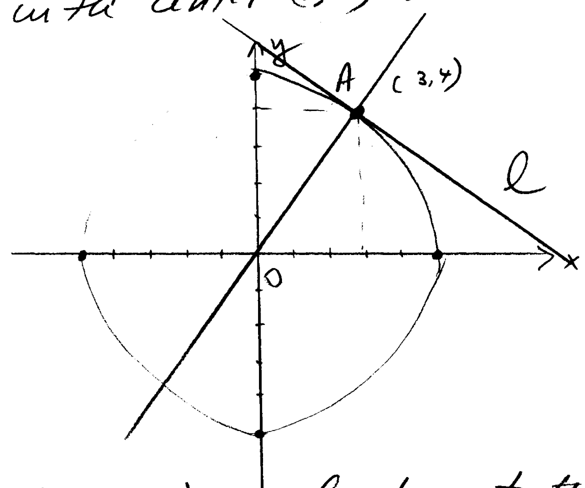
(a) $(3,4) \in$ circle iff it satisfies the equation

$x=3$, $y=4$

and $3^2 + 4^2 = 25$ true

Therefore, $(3,4) \in$ circle.

(b) $x^2+y^2=25$ is a circle with center $(0,0)$ and radius 5



Let $A(3,4)$, l = tangent to the circle at A

Then, $l \perp OA$

First, find slope of OA

$m_{OA} = \frac{\Delta y}{\Delta x} = \frac{4-0}{3-0} = \frac{4}{3}$

Then, $m_l = \frac{-3}{4}$

l: $y - y_1 = m(x - x_1)$

$\boxed{y - 4 = -\frac{3}{4}(x-3)}$

or $y = -\frac{3}{4}x + \frac{25}{4}$