Math 150 Spring 08 Name:

QUIZ #3 @ 85 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given.

1. Solve the triangle ABC knowing that $A = 40^\circ$, $B = 60^\circ$, and $a = 12cm$.

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2. Solve the triangle ABC knowing that $A = 58^\circ$, $a = 16$ ft, and $b = 18$ ft.

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3. Solve the triangle ABC knowing that $a = 48yd$, $b = 75yd$, and $c = 63yd$.

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4. A ship is anchored off a long straight shoreline that runs north and south. From two observation points 18 miles apart on shore, the bearings of the ship are $N31^\circ E$ and $S53^\circ E$. What is the distance from the ship to each of the observation points?

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5. A ship headed due east is moving through the water at a constant speed of 12 miles per hour. However, the true course of the ship is 60°. If the currents are a constant 6 miles per hour, what is the ground speed of the ship?

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6. a) Draw the vector $\vec{v}$ that goes from the origin to the point $(-3,4)$.

   b) Write the vector $\vec{v}$ in component form $< a,b >$.

   c) Write the vector $\vec{v}$ in terms of the unit vectors $\vec{i}$ and $\vec{j}$.

   d) Find the magnitude of the vector.

   e) Find the angle θ, $0^\circ \le \theta < 360^\circ$ that the vector makes with the positive x-axis.

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7. a) Find the dot product of the following two vectors: $\vec{u} = 2\vec{i} - 4\vec{j}$ and $\vec{v} = -3\vec{i} + 5\vec{j}$.

   b) Find the angle between the given two vectors.

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8. Solve the following trigonometric equations:

   a) $2\sin^2 x - 3\sin x = -1$. Solve in $[0, 2\pi)$.

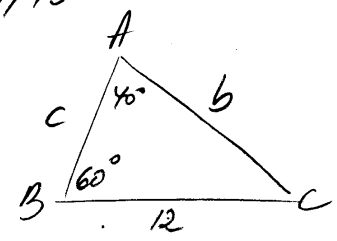
   b) $\sin x \cos x - \frac{1}{3}\sin x = 0$. Find all real solutions (in radians).

   c) $2\cos a - \sin 2a = 0$. Find all real solutions (in radians).

   d) $\sin 2x = \frac{\sqrt{2}}{2}$. Solve in $[0, 2\pi)$.

MATH 150 Quiz #3- Solutions

(1)



$A = 40°$, $B = 60°$, $a = 12$ | $b = ?$, $c = ?$, $C = ?$

Solution

$C = 180° - 40° - 60°$ $\boxed{C = 80°}$

$b = ?$ Law of Sines

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 60°} = \frac{12}{\sin 40°} \Rightarrow$$

$$b = \frac{12 \sin 60°}{\sin 40°} = \frac{12 \cdot \frac{\sqrt{3}}{2}}{\sin 40°} = \frac{6\sqrt{3}}{\sin 40°}$$

$\boxed{b \approx 16 \text{ cm}}$

$c = ?$ Law of Sines

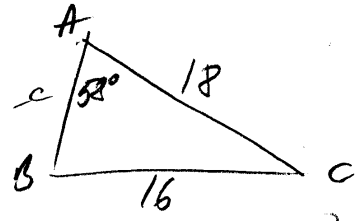
$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 80°} = \frac{12}{\sin 40°}$$

$$c = \frac{12 \sin 80°}{\sin 40°} \approx 18 \text{ cm}$$

$\boxed{c = 18 \text{ cm}}$

(2)



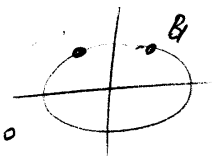
$A = 58°$, $a = 16$ ft, $b = 18$ ft | $B = ?$, $C = ?$, $c = ?$

$B = ?$ Law of Sines

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{b \sin A}{a} = \frac{18 \sin 58°}{16} \approx 0.954$$

$$\begin{cases} B_1 = \sin^{-1}(0.954) \approx 72.6° \\ \text{or} \\ B_2 = 180° - 72.6° = 107.4° \end{cases}$$



Case 1 then $\begin{cases} A = 58° \\ B_1 = 72.6° \\ C_1 = 180° - 58° - 72.6° = 49.4° \end{cases}$

$c = ?$ $\frac{c}{\sin C_1} = \frac{a}{\sin A} \Rightarrow c = \frac{a \sin C_1}{\sin A}$

$$c = \frac{16 \sin(49.4°)}{\sin 58°}$$

$c = 14.33$ ft

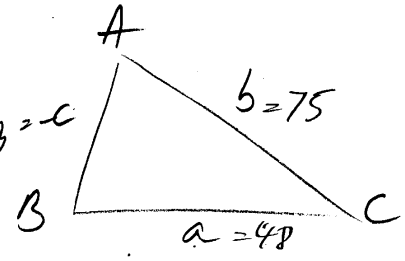
Case 2 then $\begin{cases} A = 58° \\ B_2 = 107.4° \\ C_2 = 180° - 58° - 107.4° = 14.6° \end{cases}$

$c = ?$ $\frac{c}{\sin C_2} = \frac{a}{\sin A} \Rightarrow c = \frac{a \sin C_2}{\sin A}$

$$c = \frac{16 \sin 14.6°}{\sin 58°}$$

$c \approx 4.76$ ft

(3)



$\begin{cases} a = 48 \text{ yd} \\ b = 75 \text{ yd} \\ c = 63 \text{ yd} \end{cases}$ $\quad A = ?$ $\quad B = ?$ $\quad C = ?$

Solution

$A = ?$ Law of cosines

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{75^2 + 63^2 - 48^2}{2(75)(63)} \approx 0.7714$$

$$A = \cos^{-1}(0.7714) = 39.5^\circ = A$$

$B = ?$ Law of cosines

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

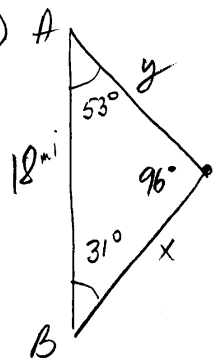
$$\cos B = \frac{48^2 + 63^2 - 75^2}{2(48)(63)} \approx 0.1071$$

$$B = \cos^{-1}(0.1071) = 83.9^\circ = B$$

Then $C = 180^\circ - 39.5^\circ - 83.9^\circ$

$$C = 56.6^\circ$$

(4)



Let x, y be the distances from the ship to the lighthouses.

$A = 53^\circ$, $B = 31^\circ$ $\Rightarrow$ $S = 180^\circ - 53^\circ - 31^\circ$

$S = 96^\circ$

$x = ?$ Law of Sines

$$\frac{x}{\sin A} = \frac{18}{\sin S}$$

$$x = \frac{18 \sin 53^\circ}{\sin 96^\circ} \qquad x \approx 14.5 \text{ mi}$$

$y = ?$ Law of Sines

$$\frac{y}{\sin B} = \frac{18}{\sin S}$$

$$y = \frac{18 \sin 31^\circ}{\sin 96^\circ} \qquad y \approx 9.3 \text{ mi}$$

(8) (a) $2\sin^2 x - 3\sin x = -1$ in $[0, 2\pi)$

$2\sin^2 x - 3\sin x + 1 = 0$

quadratic eq. in $\sin x$

$\sin x = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4}$

$\sin x = 1$ OR $\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\boxed{x \in \left\{\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}}$

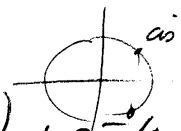
(b) $\sin x \cos x - \frac{1}{3}\sin x = 0$

$\sin x \left(\cos x - \frac{1}{3}\right) = 0$

$\sin x = 0$ OR $\cos x = \frac{1}{3}$

$x = k\pi$

$$\begin{cases} x = \cos^{-1}\left(\frac{1}{3}\right) + 2\pi k \\ \text{OR} \\ x = 2\pi - \cos^{-1}\left(\frac{1}{3}\right) + 2\pi k \end{cases}$$



Therefore,

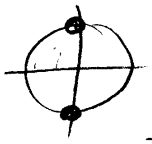
$$\begin{cases} x = k\pi \\ \text{OR} \\ x = 1.23 + 2\pi k \\ \text{OR} \\ x = 5.05 + 2\pi k \end{cases} \quad k \in \mathbb{Z}$$

(c) $2\cos a - \sin 2a = 0$

$2\cos a - 2\sin a \cos a = 0$

$2\cos a (1 - \sin a) = 0$

$\cos a = 0$ OR $\sin a = 1$

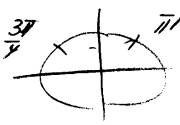


$a = \frac{\pi}{2} + 2\pi k$

$a = \frac{\pi}{2} + k\pi$

$\boxed{a \in \left\{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\right\}}$

(d) $\sin 2x = \frac{\sqrt{2}}{2}$ in $[0, 4\pi)$



$$\begin{cases} 2x = \frac{\pi}{4} + 2\pi k \\ \text{OR} \\ 2x = \frac{3\pi}{4} + 2\pi k \end{cases} \quad k \in \mathbb{Z}$$

$$\begin{cases} x = \frac{\pi}{8} + \pi k \\ \text{OR} \\ x = \frac{3\pi}{8} + \pi k \end{cases} \quad k \in \mathbb{Z}$$

$k = 0, \quad x = \frac{\pi}{8}, \quad x = \frac{3\pi}{8}$

$k = 1 \quad x = \frac{9\pi}{8}, \quad x = \frac{11\pi}{8}$

$\boxed{x \in \left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}\right\}}$