Math 150 Spring 2008 Name:

QUIZ #1 @ 85 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given

1. Solve the following equations:

a) $2x^2 = 10x$

b) $(3t+1)^2 = 16$

c) $2x^2 + x = 1$

d) $x^3 + 4x^2 - 9x - 36 = 0$

2. Find the remaining sides of a $30^\circ - 60^\circ - 90^\circ$ if the shortest side is 1.

3. Draw an angle of 135° in standard position.

a) Find a point on the terminal side of the angle.
b) Find the distance from the origin to that point.
c) Find two other angles that are coterminal with the given angle, one positive and one negative. Mark them on the drawing.
d) What is the equation of the line containing the terminal side of the angle?

4. Find the remaining functions of q if $\cos q = \frac{3}{4}$ and q terminates in quadrant IV.

5. Make a drawing of the angle q and indicate the quadrants in which the terminal side of q must lie in order that

a) $\cos q > 0$

b) $\sin q < 0$

c) $\tan q < 0$

6. Simplify the following expressions:

a) $\frac{\sin x}{\cos x} + \frac{1}{\sin x}$

b) $(1 - \sin a)(1 + \sin a)$

Quiz 1 - SOLUTIONS

(1) a) $2x^2 = 10x$

$2x^2 - 10x = 0$

$2x(x-5) = 0$

$x = 0$ OR $x - 5 = 0$

$x = 5$

$\boxed{x \in \{0, 5\}}$

b) $(3t+1)^2 = 16 \quad | \sqrt{}$

$\sqrt{(3t+1)^2} = \sqrt{16}$

$3t + 1 = \pm 4$

$3t = -1 \pm 4$

$t = \frac{-1 \pm 4}{3}$ $\begin{cases} t = \frac{-1+4}{3} = 1 \\ t = \frac{-1-4}{3} = -\frac{5}{3} \end{cases}$

$\boxed{t \in \{1, -\frac{5}{3}\}}$

c) $2x^2 + x = 1$

$2x^2 + x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\begin{cases} a = 2 \\ b = 1 \\ c = -1 \end{cases}$

$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$

$x = \frac{-1+3}{4} = \frac{2}{4} = \frac{1}{2}$ OR

$x = \frac{-1-3}{4} = -1$

$\boxed{x \in \{\frac{1}{2}, -1\}}$

d) $x^3 + 4x^2 - 9x - 36 = 0$

$x^2(x+4) - 9(x+4) = 0$

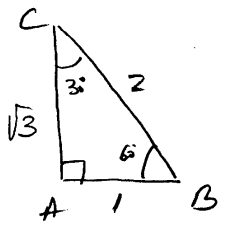
$(x+4)(x^2 - 9) = 0$

$(x+4)(x-3)(x+3) = 0$

$x = -4$ OR $x = 3$ OR $x = -3$

$\boxed{x \in \{-4, 3, -3\}}$

(2)



$\triangle ABC$

$C = 30°$

$B = 60°$

then AB = shortest side $\Rightarrow AB = 1$

$AB = \frac{BC}{2} \Rightarrow BC = 2AB$

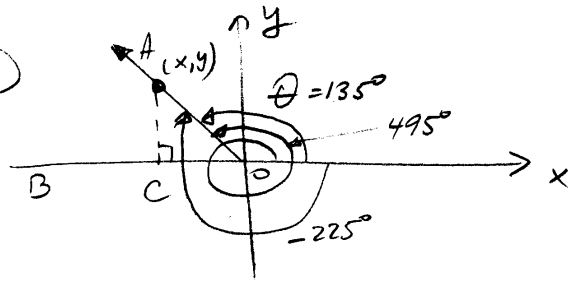
$BC = 2$

$AC^2 + AB^2 = BC^2$

$AC^2 = BC^2 - AB^2 = 2^2 - 1^2$

$AC^2 = 3, \quad AC = \sqrt{3}$

(3)



$\theta = 135° \Rightarrow \angle AOC = 45°$

$\Rightarrow \triangle AOC$ - isosceles, $AC = OC$

$A(x,y) \in$ III

a) $A(-1,1)$

or, in general $P(-a,a)$ where $a > 0$

b) $AO = ?$

$AO^2 = x^2 + y^2 = 2$

$AO = \sqrt{2}$

or, in general $PO^2 = x^2 + y^2$

$PO^2 = a^2 + a^2$

$PO^2 = 2a^2$

$PO = a\sqrt{2}, \; a > 0$

d) $y = -x$

e) coterminal angles:

$360° + 135° = 495°$

$-(360° - 135°) = -225°$

(4) Method I

$\cos\theta = \frac{3}{4}$, $\theta \in$ IV

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = 1 - \left(\frac{3}{4}\right)^2$

$\sin^2\theta = 1 - \frac{9}{16}$

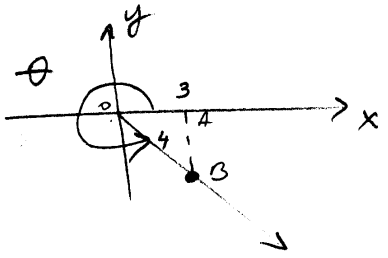
$\sin^2\theta = \frac{7}{16}$

$\sin\theta = \pm \frac{\sqrt{7}}{4}$

$\theta \in$ IV $\Rightarrow \sin\theta < 0 \Rightarrow$

$\boxed{\sin\theta = -\frac{\sqrt{7}}{4}}$

Method II



$\cos\theta = \frac{3}{4}$

$\cos\theta = \frac{x}{r}$

Let $x = 3$, $r = 4$

Then, $\triangle AOB$: $AB^2 = OB^2 - OA^2$

$AB^2 = 16 - 9 = 7$

$AB = \pm\sqrt{7}$

$\theta \in$ IV $\Rightarrow y < 0 \Rightarrow y = -\sqrt{7}$

$\sin\theta = \frac{y}{r}$

$\boxed{\sin\theta = \frac{-\sqrt{7}}{4}}$

Then $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{7}}{3}$

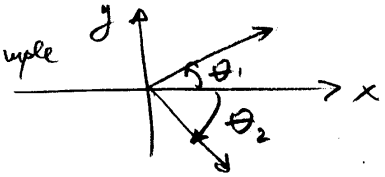
$\cot\theta = \frac{1}{\tan\theta} = -\frac{3}{\sqrt{7}} = \frac{-3\sqrt{7}}{7}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{4}{3}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{-4}{\sqrt{7}} = \frac{-4\sqrt{7}}{7}$

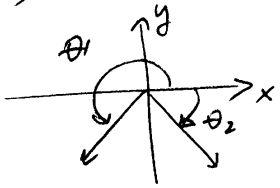
(5) a) $\cos\theta > 0$ iff $\theta \in$ I or IV

for example



(b/c $\cos\theta = \frac{x}{r}$, $\cos\theta > 0$ iff $x > 0$)

b) $\sin\theta < 0$ iff $\theta \in$ III or IV

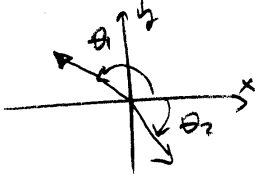


(b/c $\sin\theta = \frac{y}{r}$

$\sin\theta < 0$ iff $y < 0$)

c) $\tan\theta < 0$ iff $\theta \in$ II or IV

for example,



(b/c $\tan\theta = \frac{y}{x}$

$\tan\theta < 0$ iff

x, y have opposite signs)

(6) (a) $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos x}{\sin x \cos x}$

LCD $= \sin x \cos x$

(b) $(1 - \sin a)(1 + \sin a) =$

$= 1 - \sin^2 a$

$= \boxed{\cos^2 a}$