Math 130 Spring 2008 Name:

TEST #2 @ 150 points

Write neatly. Show all work. **Write all responses on separate paper.** **Clearly label the exercises.**

1. Consider the polynomial function $f(x) = x^5 + x^4 - 5x^3 - 5x^2 + 4x + 4$

Questions $a - g$ below relate to this polynomial function.

a) Describe the long-term behavior of this function; that is, what happens as $|x| \to \infty$.

b) Using Descartes' rule of signs, determine the number of positive real zeros and the number of negative real zeros for $f(x)$.

c) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list possible rational zeros.

d) Find all the real zeros of $f(x)$ and factor $f(x)$.

e) What are the intercepts of the graph of $f(x)$? Write each intercept as an ordered pair.

f) Sketch a graph of $f(x)$ showing how it passes through its intercepts. Plot additional points, as necessary, to get the shape of the graph. Clearly label all the points.

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2. Consider $f(x) = \dfrac{2x^2 - 2x - 4}{x^2 + 2x - 3}$.

Questions $a - e$ below relate to this polynomial function.

a) Factor the numerator and the denominator.

b) What is the domain of the function?

c) What are the vertical asymptotes?

d) What is the horizontal asymptote?

e) What are the intercepts for this function? Write them as ordered pairs.

e) Plot additional points, as necessary, to get the shape of this function and sketch a graph.

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3. Write a function of minimal degree with real coefficients whose zeros are 1, -2, and 1+i.

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4. Do the following:

a) Write the expression as a single logarithm with coefficient 1. Assume all variables are positive real numbers: $2\log_3 x - 3\log_3 y + \log_3 (z+1)$

b) Expand the expression as much as possible. Simplify the result if possible. All variables are positive real numbers: $\log_2 \sqrt[4]{\dfrac{16x^3}{y^5}}$

5. Let $f(x)=2^{x+1}-3$.

a) Graph the function (using table of values or transformations). Clearly show how you're obtaining the graph. If you choose transformations, show all equations and their meaning.

b) State the domain, range, and asymptote.

c) Find the exact x- and y-intercepts (if any).

d) Does the function have an inverse? Explain.

e) Graph the inverse $f^{-1}(x)$ showing the symmetry through $y=x$.

f) State the domain, range, and asymptote for the inverse function $f^{-1}(x)$.

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6. Solve the following equations. Give exact answer(s).

a) $2^{x^2-2x}=8$

b) $5^x=2^{x-1}$

c) $\ln 5x-\ln(2x-1)=\ln 4$

d) Solve for t: $P=P_0e^{kt}$

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7. State whether each statement is TRUE or FALSE. DO NOT prove.

a) $\log(a+b)=\log a+\log b$

b) $\log\left(\frac{a}{b}\right)\neq\frac{\log a}{\log b}$

c) $\log 5a^3=3\log 5a$

d) $\log(xy)=(\log x)(\log y)$

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6. In 2000 India's population reached 1 billion, and in 2025 it is projected to be 1.4 billion.

a) Find values for P_0 and a so that $f(x)=P_0a^{x-2000}$ models the population of India in year x.

b) Estimate India's population in 2009.

c) When will India's population reach 1.6 billion?

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7. Between 1989 and 1997, the percent of households with incomes of \$100,000 or more can be calculated by $f(x)=0.071x^2-0.426x+8.05$, where $x=0$ represents 1989. In what year did the percent of affleutn households reach its minimum?

MATH 130 TEST #2

(1) $f(x) = x^5 + x^4 - 5x^3 - 5x^2 + 4x + 4$

(a) The end-behavior is given by the leading term x^5

when $x \to \infty$, $y \to \infty$ (up)

when $x \to -\infty$, $y \to -\infty$ (down)

(b) There are 2 variations in the sign of $f(x)$ $\Rightarrow$ 2 or 0 positive real zeros

$f(-x) = \underbrace{-x^5 + x^4}_{1} \underbrace{+5x^3 - 5x^2}_{2} \underbrace{-4x + 4}_{3}$

There are 3 variations in the sign of $f(-x)$ $\Rightarrow$ 3 or 1 negative real zeros.

(c) All the coefficients of $f(x)$ are integers and the constant term $\neq 0$, so we can apply the Theorem on rational zeros.

Possible rational zeros:

$\frac{p}{q} = \frac{\text{factor of } 4}{\text{factor of } 1}$

$\frac{p}{q} \in \{\pm 1, \pm 2, \pm 4\}$

(d)

	1	1	-5	-5	4	4
1	1	2	-3	-8	-4	0

$f(x) = (x-1)(x^4 + 2x^3 - 3x^2 - 8x - 4)$

	1	2	-3	-8	-4
-1	1	1	-4	-4	0

$f(x) = (x-1)(x+1)(x^3 + x^2 - 4x - 4)$

$f(x) = (x-1)(x+1)\left(x^2(x+1) - 4(x+1)\right)$

$= (x-1)(x+1)(x+1)(x^2 - 4)$

$f(x) = (x-1)(x+1)^2(x-2)(x+2)$

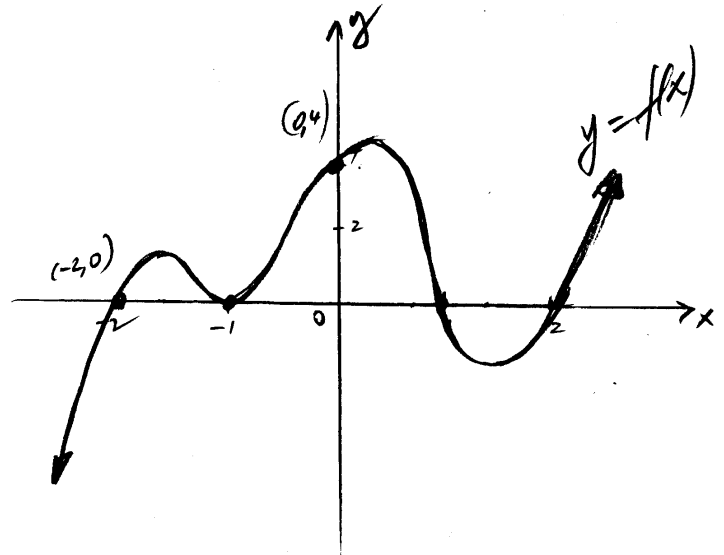
The zeros are:
$$\begin{cases} x = 1 & m = 1 \\ x = -1 & m = 2 \\ x = 2 & m = 1 \\ x = -2 & m = 1 \end{cases}$$

(e) x-n: $(1,0), (-1,0), (2,0), (-2,0)$

y-n: $(0,4)$

(f)

x	$-\infty$	-2	-1	0	1	2	∞
	$-\infty$	0	0	4	0	0	∞
		$m=1$	$m=2$		$m=1$	$m=1$	



(2) $f(x) = \dfrac{2x^2-2x-4}{x^2+2x-3}$

(a) $f(x) = \dfrac{2(x^2-x-2)}{(x+3)(x-1)}$

$f(x) = \dfrac{2(x-2)(x+1)}{(x+3)(x-1)}$

(b) $x \in \mathbb{R} \setminus \{-3, 1\}$

(c) V.A. $x=-3$, $x=1$

(d) H.A. $y=2$

(e) x-n: $y=0$ iff $(x-2)(x+1)=0$
$x=2$ or $x=-1$

x-n: $(2,0)$ and $(-1,0)$

y-n: $x=0$, $y = \frac{-4}{-3} = \frac{4}{3}$

y-n: $(0, \frac{4}{3})$

(f) check the intersection of the graph with H.A. $y=2$

$\dfrac{2x^2-2x-4}{x^2+2x-3} \stackrel{?}{=} 2$

$2x^2-2x-4 = 2(x^2+2x-3)$

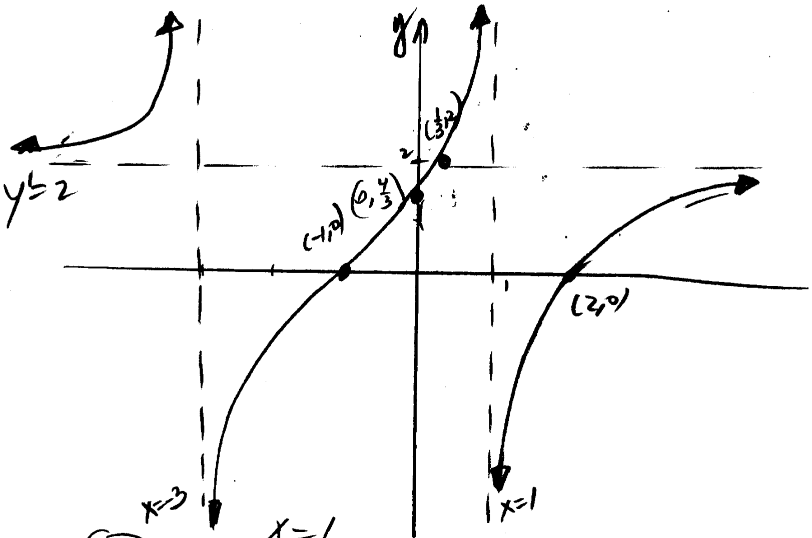
$\cancel{2x^2}-2x-4 = \cancel{2x^2}+4x-6$

$-4+6 = 4x+2x$

$6x=2$

$x=\frac{1}{3}$

$(\frac{1}{3}, 2)$



(3) $x=1$
$x=-2$
$x=1+i$
then $x=1-i$ is also a zero

$f(x) = (x-1)(x+2)(x-(1+i))(x-(1-i))$
$= (x-1)(x+2)(x-1-i)(x-1+i)$
$= (x-1)(x+2)((x-1)^2-i^2)$

$f(x) = (x-1)(x+2)(x^2-2x+2)$

(4) (a)

$2\log_3 x - 3\log_3 y + \log_3(z+1) =$

$= \log_3 x^2 - \log_3 y^3 + \log_3(z+1)$

$= \log_3 \frac{x^2}{y^3} + \log_3(z+1)$

$= \log_3 \dfrac{x^2(z+1)}{y^3}$

TEST POINTS:

$x=-4$, $y = \dfrac{2(-6)(-3)}{(-1)(-5)} > 0$

$x=$

(b) $\log_2 \sqrt[4]{\frac{16x^3}{y^5}} = \log_2\left(\frac{16x^3}{y^5}\right)^{\frac{1}{4}}$

$= \frac{1}{4}\log_2\left(\frac{16x^3}{y^5}\right)$

$= \frac{1}{4}\left(\log_2(16x^3) - \log_2 y^5\right)$

$= \frac{1}{4}\left(\log_2 16 + \log_2 x^3 - 5\log_2 y\right)$

$= \frac{1}{4}\left(4 + 3\log_2 x - 5\log_2 y\right)$

$= 1 + \frac{3}{4}\log_2 x - \frac{5}{4}\log_2 y$

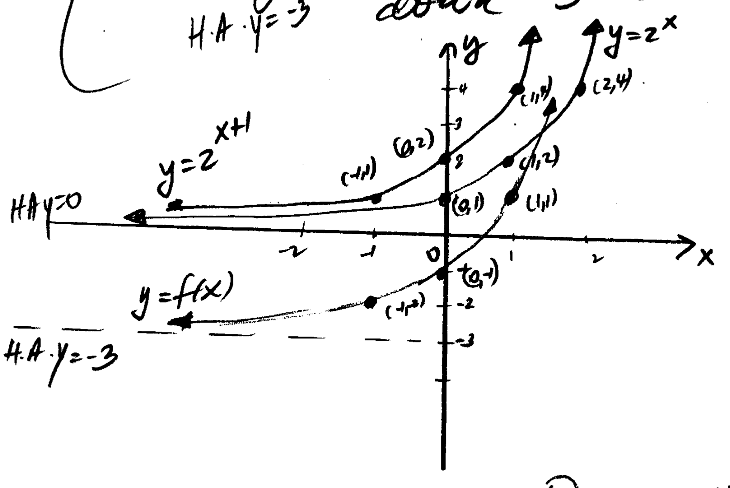
(5) $f(x) = 2^{x+1} - 3$

(a) 1st $y = 2^x$ H.A. $y=0$

2nd $y = 2^{x+1}$ shift $y=2^x$ left 1 unit

3rd $y = 2^{x+1} - 3$ shift $y = 2^{x+1}$ down 3 units

H.A. $y=-3$



(b) Domain: $x \in R$

Range: $y \in (-3, \infty)$

H.A. $y = -3$

(c) x-n: let $y=0$

$2^{x+1} - 3 = 0$

$2^{x+1} = 3$

$\ln 2^{x+1} = \ln 3$

$(x+1)\ln 2 = \ln 3$

$x+1 = \frac{\ln 3}{\ln 2} \Rightarrow x = \frac{\ln 3}{\ln 2} - 1$

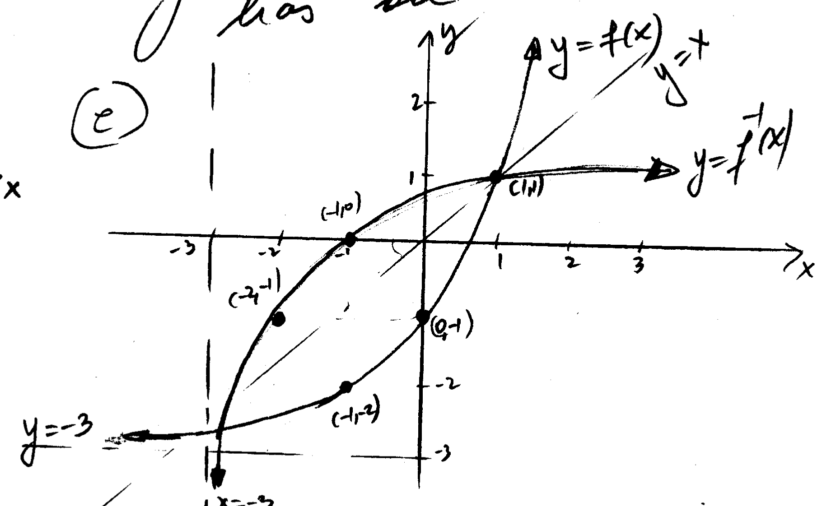
x-n: $\left(\frac{\ln 3}{\ln 2} - 1, 0\right) \approx (0.58, 0)$

y-n: let $x=0$, $y = 2^1 - 3 = -1$

y-n: $(0,-1)$

(d) The graph of $y=f(x)$ passes the Horizontal line test, therefore the function is one-to-one. Any one-to-one function has an inverse.

(e)



(f) Domain: $x \in (-3, \infty)$

Range: $y \in R$

V.A. $x = -3$

⑥ (a) $2^{x^2-2x}=8$

$2^{x^2-2x}=2^3$

The exponential function is one-to-one $\Rightarrow$

$x^2-2x=3$

$x^2-2x-3=0$

$(x-3)(x+1)=0$ $\begin{cases} x=3 \\ x=-1 \end{cases}$

$x \in \{3,-1\}$

(b) $5^x = 2^{x-1}$ $\Big/ \ln$

$\ln 5^x = \ln 2^{x-1}$

$x \ln 5 = (x-1)\ln 2$

$x \ln 5 = x \ln 2 - \ln 2$

$\ln 2 = x \ln 2 - x \ln 5$

$\ln 2 = x(\ln 2 - \ln 5)$

$x = \dfrac{\ln 2}{\ln 2 - \ln 5} = \dfrac{\ln 2}{\ln \frac{2}{5}}$

$x \in \left\{ \dfrac{\ln 2}{\ln\left(\frac{2}{5}\right)} \right\}$

(c) $\ln 5x - \ln(2x-1) = \ln 4$

Conditions

$\begin{cases} 5x>0 \\ \text{and} \\ 2x-1>0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \text{and} \\ x>\frac{1}{2} \end{cases} \Rightarrow x>\frac{1}{2}$

$\ln \dfrac{5x}{2x-1} = \ln 4$

ln function is one-to-one

$\Longrightarrow$

$\dfrac{5x}{2x-1} = 4$

$5x = 4(2x-1)$

$5x = 8x-4$

$4 = 3x,$ $x=\frac{4}{3} > \frac{1}{2}$

$x \in \left\{\frac{4}{3}\right\}$

(d) $P = P_0 e^{kt}$

$\dfrac{P}{P_0} = e^{kt}$ $\Big/ \ln$

$\ln \dfrac{P}{P_0} = \ln e^{kt}$

$\ln \dfrac{P}{P_0} = kt$

$t = \dfrac{\ln \frac{P}{P_0}}{k}$

⑦ (a) $\log(a+b) = \log a + \log b$ is a FALSE statement

(Recall $\log a + \log b = \log(ab)$)

(b) $\log\left(\frac{a}{b}\right) \neq \dfrac{\log a}{\log b}$ is a TRUE statement

(Recall $\log\left(\frac{a}{b}\right) = \log a - \log b$)

(c) $\log 5a^3 = 3 \log 5a$ is a FALSE statement

(Note that $3\log 5a = \log(5a)^3 = \log(125a^3)$)

(d) $\log(xy) = (\log x)(\log y)$
Is a FALSE statement
(recall $\log(xy) = \log x + \log y$)

(8) $f(x) = P_0\, a^{x-2000}$
x = year
$f(x)$ = population (in billions)

(a) when $x = 2000$, $f(x) = 1$ billion
1 billion $= P_0\, a^0$
1 billion $= P_0 \Rightarrow P_0 = 1$ billion

so $f(x) = a^{x-2000}$

when $x = 2025$, $f(x) = 1.4$ billion
1.4 billion $= a^{2025-2000}$
$a^{25} = 1.4$
$\sqrt[25]{a^{25}} = \sqrt[25]{1.4} \Rightarrow a = 1.01$

so $f(x) = (1.01)^{x-2000}$

(b) $x = 2009$, $f(x) = (1.01)^{2009-2000}$
$f(2009) \approx 1.09$ billion

(c) $x = ?$ if $f(x) = 1.6$ billion
$1.6 = (1.01)^{x-2000}$ | ln

$\ln(1.6) = \ln(1.01)^{x-2000}$
$\ln(1.6) = (x-2000)\ln(1.01)$
$x - 2000 = \dfrac{\ln 1.6}{\ln 1.01}$
$x = 2000 + \dfrac{\ln 1.6}{\ln 1.01}$

$x \approx 2047$
India's population will reach 1.6 billion in 2047.

(9) $f(x) = 0.071x^2 - 0.426x + 8.05$
x = # years after 1989
$f(x)$ = % of households

Note that we have a quadratic function whose graph is a parabola that opens up, therefore the minimum occurs at the vertex
$V(x_v, f_{min})$

$x_v = \dfrac{-b}{2a} = \dfrac{-(-0.426)}{2(0.071)} = 3$

The % of households with incomes of \$100,000 or more reached its min. in 1989+3, that is, in 1992.