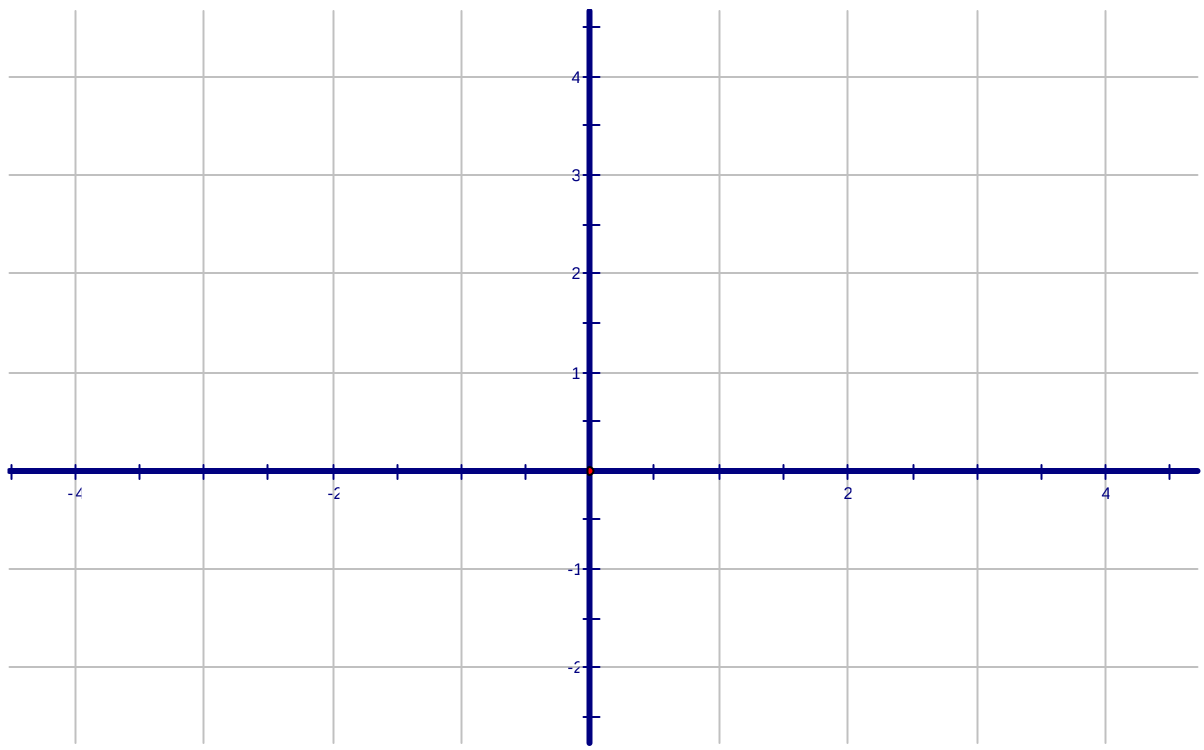
TEST #1 @ 150 points

Write neatly. Show all work. Write all responses on separate paper. Clearly label the exercises.

1. A piecewise-defined function is given.

$$f(x) = \begin{cases} 1 - x & if \quad x < -2 \\ 3 & if \quad -2 \le x < 1 \\ \sqrt{x} & if \quad x \ge 1 \end{cases}$$



You may use the above grid to graph. Write all the answers and show ALL your work on separate paper.

a) Sketch a graph for the function. Clearly show how you obtain the points you are using for the graph. Label the axes and all points used.

b) State its domain and range in interval notation.

c) On what interval(s) is the function increasing, decreasing, constant?

d) Find $f(-3)$, $f(0)$, and $f(3)$.

e) Locate the x – and y – intercepts (if any). Write each intercept as an ordered pair.

f) Find the values of $f(f(1))$ and $(f \circ f)(-3)$.

2. Let $A(2,-1)$ and $B(-3,1)$ be two points in a plane.

a) Find an equation of the circle with diameter *AB* (note that the diameter is twice the radius). Show how you obtain the equation.

b) Does the equation from (a) represent *y* as a function of *x*? Explain.

c) Find the exact *x*- and *y*-intercepts (if any).

d) Find the equation of the line that passes through the two given points.

e) Does the equation from (d) represent *y* as a function of *x*? Explain.

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3. Solve the following equations in the set of complex numbers:

a) $\left(2t-\frac{1}{3}\right)^2-\frac{1}{4}=0$

b) $\frac{1}{3}x^2-1=-\frac{1}{2}x$

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4. Let $f(x)=4x^2+2x+8$, $g(x)=2x+5$, $F(x)=\sqrt{4-3x}$, and $l(x)=\dfrac{2x+1}{3x-1}$ be four functions. Do the following.

a) Find the domain of each function.

b) Find $g(-2x)$

c) Find $\dfrac{g(x+h)-g(x)}{h}$

d) Find $f(x+1)$.

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5. Let $\frac{3}{2}x+\frac{1}{5}y=3$ be a linear equation in two variables. Do the following:

a) Graph the equation using the intercepts method. Clearly label the axes and the intercepts.

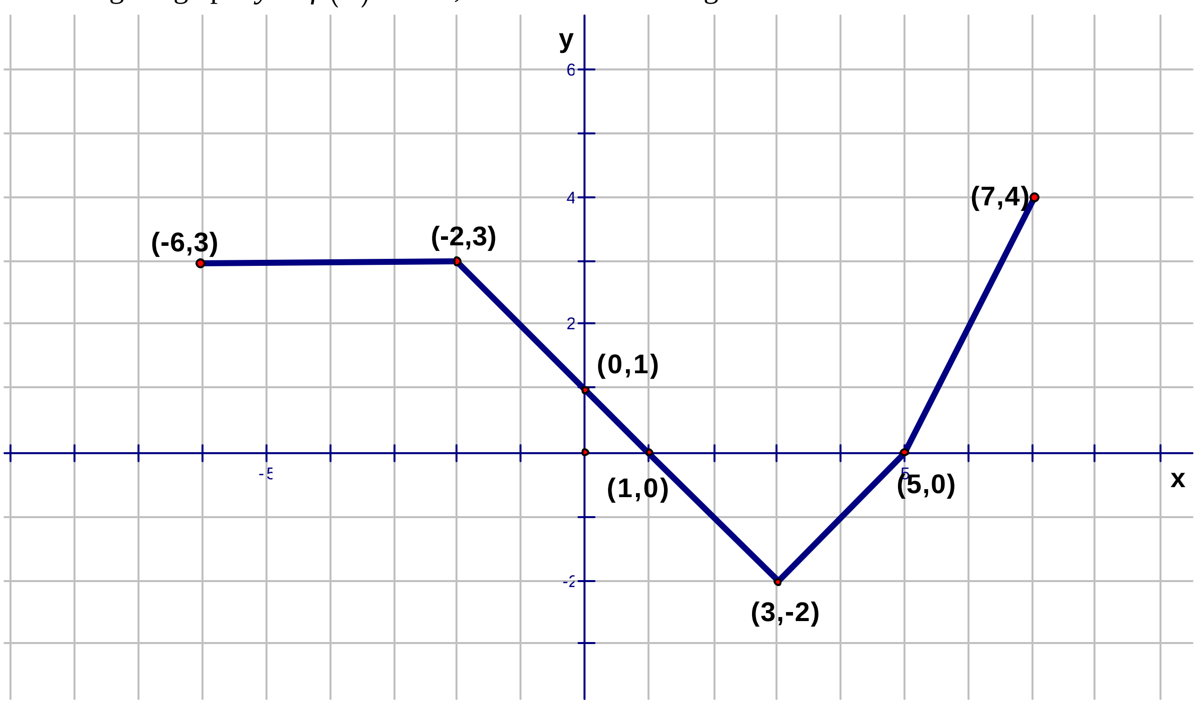
b) Find the slope of the line.

c) Find an equation for the line that is perpendicular to the given line and passes through $(-1,3)$.

6. Graph the following function using transformations. You may use the grid to graph Clearly show all the steps: the equations and their meaning on separate paper. Graph all steps.

$$f(x) = 2\sqrt{x-3} + 1$$

7. Using the graph $y = f(x)$ shown, answer the following:



a) Is y a function of x? Explain.

b) Find the domain and range of f.

c) List the x- and y- intercepts (as ordered pairs).

d) Find $f(3)$.

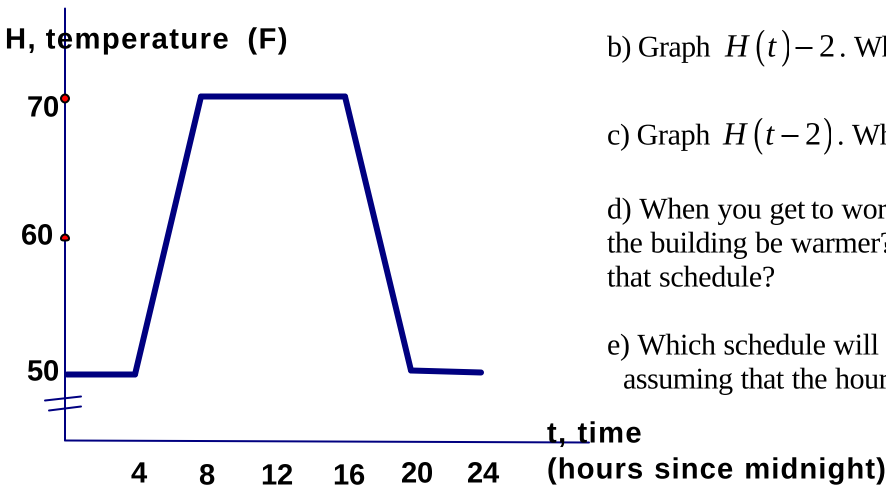
e) For what values of x does $f(x) = 3$

f) Estimate the values for which $f(x) > 1$.

g) Find $(f \circ f)(5)$.

h) Graph $y = f(x+1)$

8. The function $H(t)$ graphed gives the heating schedule of an office building during the winter months. $H(t)$ is the building's temperature in degrees Fahrenheit t hours after midnight.



a) Find the domain and the range . Use correct units.

b) Graph $H(t)-2$. What did the company decide to do?

c) Graph $H(t-2)$. What did the company decide to do?

d) When you get to work at 8 am, under which schedule will the building be warmer? What will the temperature be under that schedule?

e) Which schedule will save the company on heating costs, assuming that the hourly cost of heating depends only on the thermostat setting?

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9. When the Celsius temperature is 0°, the corresponding Fahrenheit temperature is 32°. When the Celsius temperature is 100°, the corresponding Fahrenheit temperature is 212°. Do the following:

a) Express the Fahrenheit temperature as an exact linear function of the Celsius temperature. Use function notation. Clearly define your variables.

b) Graph the equation. Clearly label the axes and the points used.

c) Use function notation to express the corresponding Fahrenheit temperature when the Celsius temperature is 20°. What is that Fahrenheit temperature?

d) Use function notation to express the question, "What is the corresponding Celsius temperature when the Fahrenheit temperature is 100°? What is that Celsius temperature?

e) Is it possible for the Fahrenheit temperature to equal the Celsius temperature? When does that happen?

Math 130

TEST 1 - SOLUTIONS

① ⓐ $f(x)=\begin{cases} 1-x & \text{if } x<-2 \\ 3 & \text{if } -2\le x<1 \\ \sqrt{x} & \text{if } x\geq 1 \end{cases}$

$\boxed{y=1-x}$ if $x<-2$ ①

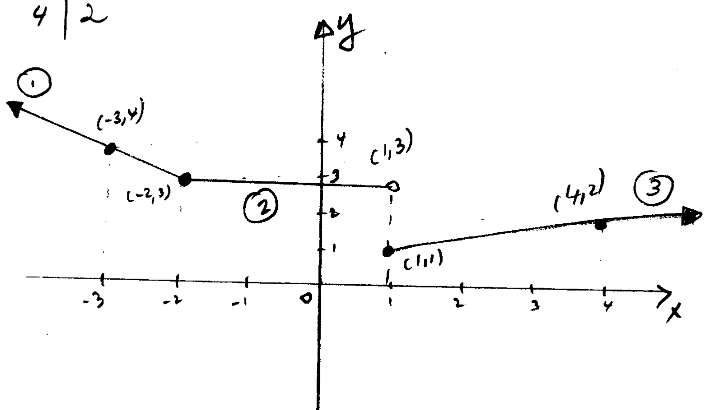
x	y
-2	3
-3	4

$y=1-(-2)=3$
$y=1-(-3)=4$

$\boxed{y=3}$ if $-2\le x<1$ ②
horizontal segment with endpoints $(-2,3)$ and $(1,3)$

$\boxed{y=\sqrt{x}}$ if $x\geq 1$ ③

x	y
1	1
4	2



b) Domain: $x\in\mathbb{R}$
Range: $y\in[1,\infty)$

c) f is increasing on $[1,\infty)$
f is constant on $[-2,1)$
f is decreasing on $(-\infty,-2)$

d) $f(-3)=1-(-3)=4$
$x=-3<-2$

$f(0)=3$
$x=0\in[-2,1)$

$f(3)=\sqrt{3}$
$x=3\geq 1$

e) x-int: none
y-int: $(0,3)$

f) $f(f(1))=f(1)=\sqrt{1}=1$

$f(1)=\sqrt{1}=1$

$(f\circ f)(-3)=f(f(-3))$
$=f(4)$
$=\sqrt{4}=2$

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② a) $A(2,-1)$, $B(-3,1)$
Center of circle $= M(x_M,y_M)$ the midpoint of AB

$$\begin{cases} x_M=\dfrac{x_A+x_B}{2}=\dfrac{2-3}{2}=\dfrac{-1}{2} \\ y_M=\dfrac{y_A+y_B}{2}=\dfrac{-1+1}{2}=0 \end{cases}$$

$M\left(-\frac{1}{2},0\right)$

radius $= r=\frac{1}{2}AB$

$AB^2=(\Delta x)^2+(\Delta y)^2$
$=(2-(-3))^2+(1-(-1))^2$
$=25+4=29$

$r^2=\left(\frac{1}{2}AB\right)^2=\frac{1}{4}AB^2=\frac{29}{4}$

Equation of circle:
$\left(x-\left(-\frac{1}{2}\right)\right)^2+(y-0)^2=\frac{29}{4}$

$$\boxed{\left(x+\frac{1}{2}\right)^2+y^2=\frac{29}{4}}$$

b) No, b/c the circle doesn't pass the vertical line test

c) $\left(x+\frac{1}{2}\right)^2 + y^2 = \frac{29}{4}$

x-int: $y = 0$,

$\left(x+\frac{1}{2}\right)^2 = \frac{29}{4}$

$\sqrt{\left(x+\frac{1}{2}\right)^2} = \sqrt{\frac{29}{4}}$

$x + \frac{1}{2} = \pm \frac{\sqrt{29}}{2}$

$x = \frac{-1}{2} \pm \frac{\sqrt{29}}{2}$

$\boxed{\text{x-int: } \left(\frac{-1}{2} - \frac{\sqrt{29}}{2}, 0\right) \text{ and } \left(\frac{-1}{2} + \frac{\sqrt{29}}{2}, 0\right)}$

y-int: $x = 0$, $\left(\frac{1}{2}\right)^2 + y^2 = \frac{29}{4}$

$y^2 = \frac{29}{4} - \frac{1}{4}$

$y^2 = \frac{28}{4}$

$\sqrt{y^2} = \sqrt{7}$

$y = \pm\sqrt{7}$

$\boxed{\text{y-int: } (0, -\sqrt{7}) \text{ and } (0, \sqrt{7})}$

d) $A = (2, -1)$
$B(-3, 1)$

$m = \frac{\Delta y}{\Delta x} = \frac{1-(-1)}{-3-2} = \frac{2}{-5}$

$y - y_1 = m(x - x_1)$

$y - (-1) = \frac{-2}{5}(x - 2)$

$\boxed{y + 1 = \frac{-2}{5}(x - 2)}$

OR

$y = \frac{-2}{5}x - \frac{1}{5}$

e) yes, a descending line passes the vertical line test.

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③ (a) $\left(2t - \frac{1}{3}\right)^2 - \frac{1}{4} = 0$

$\left(2t - \frac{1}{3}\right)^2 = \frac{1}{4}$

$\sqrt{\left(2t - \frac{1}{3}\right)^2} = \sqrt{\frac{1}{4}}$

$2t - \frac{1}{3} = \pm \frac{1}{2}$

$2t = \frac{1}{3} \pm \frac{1}{2}$

$2t = \frac{1}{3} - \frac{1}{2}$ OR $2t = \frac{1}{3} + \frac{1}{2}$

$2t = \frac{-1}{6}$ $\quad$ $2t = \frac{5}{6}$

$t = \frac{-1}{12}$ $\quad$ $t = \frac{5}{12}$

$\boxed{t \in \left\{\frac{-1}{12}, \frac{5}{12}\right\}}$

(b) $\frac{1}{3}x^2 - 1 = -\frac{1}{2}x$

$\frac{1}{3}x^2 + \frac{1}{2}x - 1 = 0 \quad / \cdot 6$

$2x^2 + 3x - 6 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad$ $a = 2$, $b = 3$, $c = -6$

$x = \frac{-3 \pm \sqrt{9 - 4(2)(-6)}}{2(2)} = \frac{-3 \pm \sqrt{57}}{4}$

$\boxed{x \in \left\{\frac{-3 \pm \sqrt{57}}{4}\right\}}$

(a)

(4) $f(x) = 4x^2 + 2x + 8$

$g(x) = 2x + 5$

$\boxed{D_f = D_g = \mathbb{R}}$ (no restrictions)

$F(x) = \sqrt{4-3x}$

Condition: $4 - 3x \geq 0$

$4 \geq 3x$

$x \leq \frac{4}{3}$

$\boxed{D_F = \left(-\infty, \frac{4}{3}\right]}$

$\ell(x) = \dfrac{2x+1}{3x-1}$

Condition: $3x - 1 \neq 0$

$x \neq \frac{1}{3}$

$\boxed{D_\ell = \mathbb{R} \setminus \left\{\frac{1}{3}\right\}}$

(b) $g(-2x) = 2(-2x) + 5$

$= -4x + 5$

(c) $\dfrac{g(x+h) - g(x)}{h} =$

$= \dfrac{(2(x+h)+5) - (2x+5)}{h}$

$= \dfrac{\cancel{2x} + 2h + \cancel{5} - \cancel{2x} - \cancel{5}}{h} = \dfrac{2h}{h} = 2$

(d) $f(x+1) = 4(x+1)^2 + 2(x+1) + 8$

$= 4(x^2 + 2x + 1) + 2x + 2 + 8$

$= 4x^2 + 8x + 4 + 2x + 10$

$= 4x^2 + 10x + 14$

(5) $\frac{3}{2}x + \frac{1}{5}y = 3$

a)

x	y
0	15
2	0

$(0,15)$ y-∩

$(2,0)$ x-∩

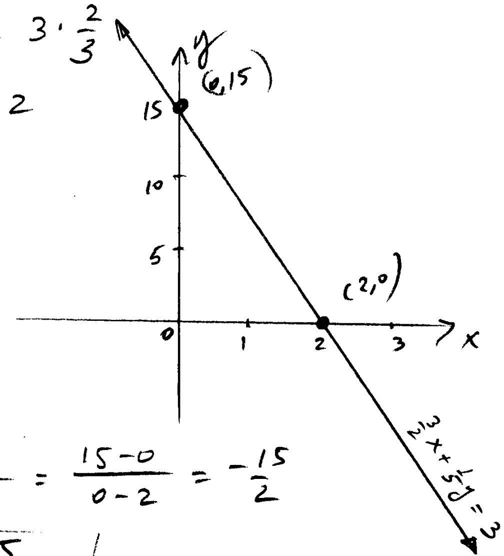
$x = 0, \quad \frac{1}{5}y = 3 \quad / \cdot 5$

$y = 15$

$y = 0 \quad \frac{3}{2}x = 3 \quad / \cdot \frac{2}{3}$

$x = 3 \cdot \frac{2}{3}$

$x = 2$



b) $m = \dfrac{\Delta y}{\Delta x} = \dfrac{15-0}{0-2} = -\dfrac{15}{2}$

$\boxed{m = \dfrac{-15}{2}}$

c) $m_\perp = \dfrac{2}{15}$

Use $m = \frac{2}{15}$ and $(-1, 3)$:

$y - y_1 = m(x - x_1)$

$\boxed{y - 3 = \frac{2}{15}(x+1)}$

OR

$y = \frac{2}{15}x + \frac{47}{15}$

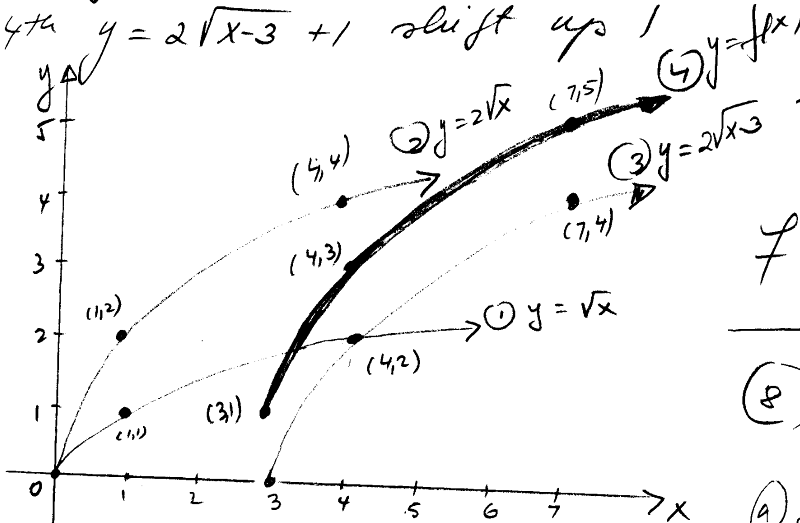
(6) $f(x) = 2\sqrt{x-3} + 1$

1st $y = \sqrt{x}$

2nd $y = 2\sqrt{x}$ vertical stretch by a factor of 2

3rd $y = 2\sqrt{x-3}$ shift right 3

4th $y = 2\sqrt{x-3} + 1$ shift up 1



(7) a) Yes, the graph passes the vertical line test: any vertical line has at most one common point with the graph

b) Domain: $x \in [-6, 7]$
Range: $y \in [-2, 4]$

c) x-∩: $(1,0)$ and $(5,0)$
y-∩: $(0,1)$

d) $f(3) = -2$

e) $f(x) = 3$ iff $x \in [-6, -2]$
OR
$x \approx 6.5$

f) $f(x) > 1$ iff
$x \in [-6, 0) \cup (5.5, 7]$

g) $(f \circ f)(5) = f(f(5))$
$= f(0)$
$= 1$

h) $y = f(x+1)$
shift the graph of $f(x)$ one unit left.

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(8) t = # hours after midnight
$H(t)$ = building's temperature

(a) Domain: $t \in [0h, 24h]$
Range: $H \in [50°F, 70°F]$

(b) $y = H(t) - 2$
shift the given graph 2 units down.
The company decided to lower the temperature by 2° F. The schedule is the same; just the temp. is 2° colder.

(c) $y = H(t-2)$
shift the given graph 2 units to the right.

The company decided to start the heating of the building 2 hours later

(d) The building will be warmer under the original schedule - 70°F at 8am

(e) $y = H(t) - 2$
Lowering the temperature by $2°F$ will save the company on heating costs

(9) Let C = Celsius temperature
F = Fahrenheit temp.

1) F as a linear function of C, thus C = independent
F = dependent

C	F
0	32
100	212

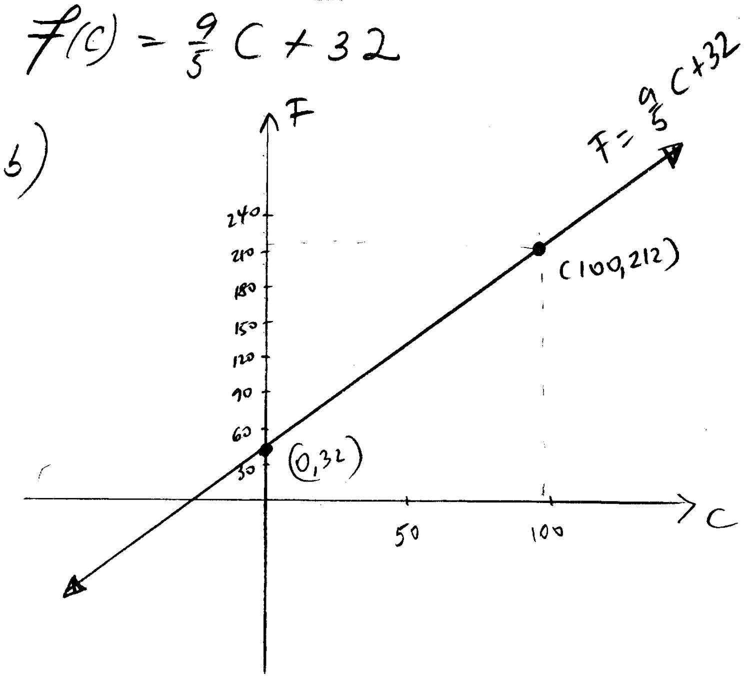
$$m = \frac{\Delta F}{\Delta C} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

$y = mx + b$

$\boxed{F = \frac{9}{5}C + 32}$ Let $F = f(C)$

$$f(C) = \frac{9}{5}C + 32$$

b)



(c) $F = f(C)$

$$f(20) = \frac{9}{5}(20) + 32$$

$f(20) = 77$

when $C = 20°$, $F = 77°$

(d) Solve $f(C) = 100$

$$\frac{9}{5}C + 32 = 100$$

$$\frac{9}{5}C = 68$$

$$C = 68 \cdot \frac{5}{9}$$

$$C = \frac{340}{9}$$

$$C \approx 37.8°$$

if $F = 100°$, $C \approx 37.8°$

(e) $F = C$

$$\frac{9}{5}C + 32 = C$$

$$\frac{9}{5}C - C = -32$$

$$\frac{4}{5}C = -32$$

$$C = -32 \cdot \frac{5}{4}$$

$$C = -40°$$

$F = C = -40°$