Review Test #2
Chapter 3 (3.1 – 3.5) and Chapter 4 (4.2 – 4.6)

To prepare for the test, you may study:

- All examples and exercises done in class
- Quiz #2
- Handout Section 3.1 – Quadratic Functions – Exercises # 1, 2, 3, 5, 7 (see website for handout and solutions)
- Handout Sections 3.2 & 3.3 – Synthetic Division; Zeros of Polynomials – ll exercises (see website for handout and solutions)
- Handout Section 3.5 – Graphs of Rational Functions - all exercises (see website for handout and solutions)
- Handout Section 4.2 – Exponential Functions - Exercises # 2, 3, 7, 8, 9, 10 (see website for handout and solutions)
- All homework problems

More practice

Chapter 3

1) Consider the following polynomial function $f(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$.
Questions a-g below relate to this polynomial function.

a) Use the leading term to describe the long-term behavior of this function; that is, what happens as $x \to \pm\infty$.

b) Use synthetic division to divide $f(x)$ by $x-1$ and relate dividend, divisor, quotient and remainder in an equation.

c) Compute and compare the values of $f(1)$ and $f(2)$. What can you conclude using the intermediate value theorem?

d) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list all possible rational zeros for $f(x)$.

e) Find all the zeros of the polynomial.

f) Factor $f(x)$ completely.

g) What are the x- and y-intercepts of the graph?

h) Sketch a graph of $f(x)$ showing how it passes through its intercepts.

2) $f(x) = 2x^4 - 19x^3 + 57x^2 - 64x + 20$.

Questions a – g below relate to this polynomial function.

a) Describe the long-term behavior of this function; that is, what happens as $|x| \to \infty$.

b) Compute and compare the values of $f(0)$ and $f(1)$. What can you conclude using the Intermediate value theorem?

c) Using Descartes' rule of signs, determine the number of positive real zeros and the number of negative real zeros for $f(x)$.

d) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list possible rational zeros.

e) Find all the real zeros of $f(x)$.

f) What are the intercepts of the graph of $f(x)$? Write each intercept as an ordered pair.

g) Sketch a graph of $f(x)$ showing how it passes through its intercepts. Plot additional points, as necessary, to get the shape of the graph. Clearly label all the points.

3) Let $g(x)=\dfrac{1}{x+1}$

a) Sketch a graph of the function (using transformations or by finding asymptotes and plotting points).
b) What are the asymptotes for the graph?
c) State its domain and range.
d) Find the intercepts.
e) Calculate $g(-2)$.
f) Solve $g(x)=-2$.
g) Find points that correspond to parts (d) and (e) on the graph of the function.

4) Let $f(t)=1+\ln t$.

a) Graph the function.
b) State the domain, range, and vertical asymptote.
c) Find the exact *x*- and *y*-intercepts (if any).
d) Does the function have an inverse> Explain. Find $f^{-1}(x)$.
e) Graph the inverse $f^{-1}(x)$ showing the symmetry through $y=x$.
f) State the domain, range, and horizontal asymptote for the inverse function $f^{-1}(x)$.
g) Find the exact *x*- and *y*-intercepts of the inverse function $f^{-1}(x)$ (if any).

5) Let $f(x)=3^{x-1}-2$.

a) Graph the function.
b) State the domain, range, and horizontal asymptote.
c) Find the exact *x*- and *y*-intercepts (if any).
d) Does the function have an inverse? Explain. Find the inverse function $f^{-1}(x)$.
e) Graph the inverse function showing how it can be obtained from the graph of *f*.
f) Find the exact *x*- and *y*-intercepts for $f^{-1}(x)$ (if any).

Chapter 4

Solve for x in Problems 1 – 9.

1) $10^{x+3} = 5e^{7-x}$ 2) $2e^{3x} = 4e^{5x}$ 3) $2x-1 = e^{\ln x^2}$ 4) $9^x = 2e^{x^2}$

5) $5^x = 3^{2x-1}$ 6) $3^{x^2-4} = 27$ 7) $\log_8(x+5) - \log_8 2 = 1$

8) $10^{2x} + 3(10^x) - 10 = 0$ 9) $\log_2(\log_3 x) = -1$ 10) $e^x - e^{-x} = 1$

11) $\ln(-x) + \ln 3 = \ln(2x-15)$ 12) $\ln 5x - \ln(2x-1) = \ln 4$

13) $\log x = \sqrt{\log x}$

For problems 11 – 12, solve for t. Assume a and b are positive constants and k is nonzero.

14) $P = P_0 e^{kt}$ 15) $ae^{kt} = e^{bt}$, where $k \neq b$. 16) $r = p - k\ln t$

17) $I = \dfrac{E}{R}\left(1 - e^{-\frac{Rt}{2}}\right)$

Simplify the expressions in 13 – 16 completely.

18) $5e^{\ln(A^2)}$ 19) $\ln\left(e^{2ab}\right)$ 20) $\ln\left(\dfrac{1}{e}\right) + \ln AB$ 21) $2\ln\left(e^A\right) + 3\ln B^e$

29) If the size of a bacteria colony doubles in 5 hours, how long will it take for the number of bacteria to triple?

30) Suppose a certain radioactive substance has a half-life of 5 years. An object starts with 20 kg of the radioactive material.

a) How much of the radioactive material is left after 10 years?

b) The object can be moved safely when the quantity of the radioactive material is 0.1 kg or less. How much time must pass before the object can be moved?

32) You place $800 in an account that earns 4% annual interest, compounded annually. How long will it be until you have $2000

31) The number of bacteria present in a culture after t hours is given by the formula $N = 1000e^{0.69t}$.

a) How many bacteria will be there after ½ hour?

b) How long will it be before there are 1,000,000 bacteria?

c) What is the doubling time?

33) (Textbook 4.5 # 66) At the World Championship races held at Rome's Olympic Stadium in 1987, American sprinter Carl Lewis ran the 100-m race in 9.86 sec. His speed in meters per second after t seconds is closely modeled by the function defined by $f(t) = 11.65\left(1 - e^{-\frac{t}{1.27}}\right)$.

a) How fast was he running as he crossed the finish line?

b) After how many seconds was he running at the rate of 10 m per sec?

35) (Textbook 4.6 #21) Find the doubling time of an investment earning 2.5% interest if interest is compounded continuously.

36) (Textbook 4.6 # 25) In 2000 India's population reached 1 billion, and in 2025 it is projected to be 1.4 billion.

a) Find values for P_0 and a so that $f(x) = P_0 a^{x-2000}$ models the population of India in year x.

b) Estimate India's population in 2010.

c) When will India's population might reach 1.5 billion?

Answers Chapter 4

1) 0.515
2) -0.347
3) x=1
4) 1.81, 0.38
5) $\frac{\log_5 3}{2\log_5 3 - 1}$
6) $\pm\sqrt{7}$
7) 11
8) $\log 2$
9) $\sqrt{3}$
10) $\ln\frac{1+\sqrt{5}}{2}$
11) $\varnothing$
12) 4/3
13) 1,10
14) $\ln(P/P_0)/k$
15) $\frac{\ln a}{b-k}$
16) $e^{\frac{p-r}{k}}$
17) $-\frac{2}{R}\ln\left(1-\frac{RI}{E}\right)$
18) $5A^2$
19) 2ab
20) -1+lnA+lnB sec.
21) $2A + 3e\ln B$

29) 7.925 hours;

30) a) 5 kg; b) 38.2 years;
31) a) 1412 bacteria; b) 10 hours; c) 1 hour;
32) 23.4 years

33) a) 11.6451 m per sec; b) 2.4823 sec

35) about 27.73 years
36) a) 1 and a=1.01355; b) about 7.2 yr. ; c) 2030.