Math 130 Spring 2008 Name:

QUIZ #3 @ 85 points

Write neatly. Show all work. **Write all responses on separate paper.**

1. Write the 1[st], 2[nd], and 20[th] term of the following sequence: $a_n = \dfrac{(-1)^n}{2n}$, $n \geq 1$

2. Find the first four terms of the sequence given by the following recursive formula:

$$\begin{cases} a_1 = 1 \\ a_2 = 3 \\ a_n = a_{n-1} + a_{n-2}, \quad if \quad n \geq 3 \end{cases}$$

3. Expand and evaluate: $\displaystyle\sum_{k=1}^{6} \frac{k}{2k-1}$

4. Use summation notation to rewrite the sequence:

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots - \frac{1}{128}$$

5. Let $3, -2, -7, -12, \ldots$ be a sequence.

a) Is this an arithmetic sequence or a geometric sequence?
b) Find the common difference .
c) Find a formula for the nth term.
d) Find the sum of the first 30 terms of the sequence.

6. Give an example of a geometric sequence. Write the formula for the nth term and for the sum of the first n terms. Then find the sum of the first 20 terms of your geometric sequence.

7. Solve the following systems using matrices.

a) $\begin{cases} x + y - z = 6 \\ 2x - y + z = -9 \\ x - 2y + 3z = 1 \end{cases}$

b) $\begin{cases} x - y + 2z + w = 4 \\ y + z = 3 \\ z - w = 2 \\ x - y = 0 \end{cases}$

8. Let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 6 & -7 \\ -2 & 1 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix}$$

Do the following operations. If not defined, say so and explain why.

a) $A - C$
b) $3A$
d) AC
e) AD

(6) In general,
a geometric sequence is:
$a, ar, ar^2, \ldots$
We start with a first term $a \neq 0$
and we multiply by a constant ratio $r \neq 0$
over and over

$$a_n = ar^{n-1}$$
$$S_n = \frac{a(1-r^n)}{1-r}$$

(7a)
$$\begin{cases} x+y-z=6 \\ 2x-y+z=-9 \\ x-2y+3z=1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ \boxed{2} & -1 & 1 & -9 \\ \boxed{1} & -2 & 3 & 1 \end{array}\right) \xrightarrow[R_3 \to -R_1+R_3]{R_2 \to -2R_1+R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -3 & 3 & -21 \\ 0 & \boxed{-3} & 4 & -5 \end{array}\right) \xrightarrow[R_3 \to -R_2+R_3]{R_2 \to \frac{1}{3}R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -1 & 1 & -7 \\ 0 & 0 & 1 & 16 \end{array}\right)$$

3rd row: $z = 16$
2nd row: $-y+z=-7 \Rightarrow -y+16=-7$
$y = 23$
1st row: $x+y-z=6$
$\Rightarrow x+23-16=6 \Rightarrow x=-1$
The solution is $(-1, 23, 16)$.

(7b)
$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 4 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 & 2 \\ \boxed{1} & -1 & 0 & 0 & 0 \end{array}\right) \xrightarrow[R_4 \to -R_1+R_4]{}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 4 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & \boxed{-2} & -1 & -4 \end{array}\right) \xrightarrow[R_4 \to 2R_3+R_4]{}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 4 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & -3 & 0 \end{array}\right)$$

4th row: $-3w=0 \Rightarrow w=0$
3rd row: $z-w=2 \Rightarrow z=2$
2nd row: $y+z=3 \Rightarrow y=1$
1st row: $x-y+2z+w=4$
$x-1+4=4 \Rightarrow x=1$
The solution is $(1, 1, 2, 0)$.

(8a) $A-C=$

$$= \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-1 & 1-1 & 0-3 \\ -1+2 & 2-3 & -3+5 \\ 0-1 & 5-0 & -1+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & -3 \\ 1 & -1 & 2 \\ -1 & 5 & 0 \end{pmatrix}$$

b) $3A = 3\begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix}$

$= \begin{pmatrix} 6 & 3 & 0 \\ -3 & 6 & -9 \\ 0 & 15 & -3 \end{pmatrix}$

d) $AC = ?$

$\dim A = 3 \times 3$

$\dim C = 3 \times 3$

so $\dim AC = 3 \times 3$

$AC = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix} =$

$= \begin{pmatrix} 0 & 5 & 1 \\ -8 & 5 & -10 \\ -11 & 15 & -24 \end{pmatrix}$

$a_{11} = 2 \cdot 1 + 1(-2) + 0(1) = 0$

$a_{12} = 2 \cdot 1 + 1 \cdot 3 + 0 \cdot 0 = 5$

$a_{13} = 2 \cdot 3 + 1(-5) + 0(-1) = 1$

$a_{21} = -1(1) + 2(-2) + (-3)1 = -8$

$a_{22} = (-1)1 + 2 \cdot 3 + (-3)0 = 5$

$a_{23} = (-1)3 + 2(-5) + (-3)(-1) = -10$

$a_{31} = 0(1) + 5(-2) + (-1)1 = -11$

$a_{32} = 0(1) + 5(3) + (-1)0 = 15$

$a_{33} = 0(3) + 5(-5) + (-1)(-1) = -24$

e) $AD = ?$

$\dim A = 3 \times 3$

$\dim D = 3 \times 2$

so $\dim AD = 3 \times 2$

$AD = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 3 & 2 \\ -13 & -7 \\ -8 & -2 \end{pmatrix}$.

Quiz #3 – Solutions

(1) $a_n = \frac{(-1)^n}{2n}$

$a_1 = \frac{(-1)^1}{2(1)} = \frac{-1}{2}$

$a_2 = \frac{(-1)^2}{2(2)} = \frac{1}{4}$

$a_{20} = \frac{(-1)^{20}}{2(20)} = \frac{1}{40}$

(2) $\begin{cases} a_1 = 1 \\ a_2 = 3 \\ a_n = a_{n-1} + a_{n-2}, & n \geq 3 \end{cases}$

$a_1 = 1$, $a_2 = 3$ (given)

$a_3 = a_2 + a_1$
$= 3 + 1$
$= 4$ so $a_3 = 4$

$a_4 = a_3 + a_2$
$= 4 + 3$
$= 7$ so $a_4 = 7$

(3) $\sum_{k=1}^{6} \frac{k}{2k-1} = \frac{1}{2(1)-1} + \frac{2}{2(2)-1} + \frac{3}{2(3)-1} + \frac{4}{2(4)-1} + \frac{5}{2(5)-1} + \frac{6}{2(6)-1}$

$= \frac{1}{1} + \frac{2}{3} + \frac{3}{5} + \frac{4}{7} + \frac{5}{9} + \frac{6}{11}$

(4) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{128} = ?$

$a_1 = 1$

$a_2 = \frac{-1}{2}$

$a_3 = \frac{1}{4} = \frac{1}{2^2}$

$a_4 = \frac{-1}{8} = \frac{-1}{2^3}$

$a_n = (-1)^{n-1} \frac{1}{2^{n-1}}$

$\boxed{a_n = \frac{1}{(-2)^{n-1}}}$

if $n =$ odd, $a_n > 0$
$n =$ even, $a_n < 0$

so $1 - \frac{1}{2} + \frac{1}{4} - \cdots - \frac{1}{128} = \sum_{n=1}^{8} \frac{1}{(-2)^{n-1}}$

check: if $n=1$, $\frac{1}{(-2)^{1-1}} = \frac{1}{1} = 1$

$n = 2$, $\frac{1}{(-2)^{2-1}} = \frac{-1}{2}$

$n = 3$, $\frac{1}{(-2)^{3-1}} = \frac{1}{4}$

$n = 8$, $\frac{1}{(-2)^{8-1}} = \frac{-1}{128}$

(5) $3, -2, -7, -12, \ldots$

(a) This is an arithmetic sequence with $a_1 = 3$ and d

(b) $d = a_2 - a_1 = -2 - 3 = -5$
$= a_3 - a_2 = -7 - (-2) = -5$

$d = -5$

(c) $a_n = a_1 + (n-1)d$
$a_n = 3 + (n-1)(-5)$
$a_n = 3 - 5n + 5$
$a_n = 8 - 5n$

(d) $S_n = \frac{(a_1 + a_n)n}{2}$

$S_{30} = \frac{(a_1 + a_{30})30}{2}$

$a_1 = 3$
$a_{30} = 8 - 5(30)$
$= 8 - 150$
$= -142$

$S_{30} = \frac{(3 - 142)30}{2}$

$S_{30} = -2085$