Math 130 Spring 08 Name:

QUIZ #2 @ 85 points

Write neatly..Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1) Let $f(x) = -x^2 - 6x - 5$.
Write all the answers and show ALL your work on separate paper.

a) What type of curve is this?
b) What is the y-intercept?
c) What is the vertex ?
d) Find the x- intercept(s) (if any).
e) Sketch its graph. Label the axes, the vertex, and the intercepts.
f) Find the domain and range .

2. Consider the polynomial function

$$f(x) = x^5 + 4x^4 - 3x^3 - 18x^2.$$

Questions $a - g$ below relate to this polynomial function. Write all the answers and show ALL your work on separate paper.

a) Describe the long-term behavior of this function; that is, what happens as $x \to \infty$ and $x \to -\infty$.
b) Use synthetic division to divide $f(x)$ by $x - 3$ and relate dividend, divisor, quotient and remainder in an equation.
c) Using Descartes' rùle of signs, determine the number of positive real zeros and the number of negative real zeros for $f(x)$.
d) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list all possible rational zeros.
e) Find all the real zeros of $f(x)$ and use the zeros to factor f completely.
f) What are the intercepts of the graph of $f(x)$? Write each intercept as an ordered pair.
g) Sketch a graph of $f(x)$ showing how it passes through its intercepts. Clearly label all the points.

3. Find a polynomial function of minimal degree with real coefficients with the following zeros: 2, -3, 1+i.

4. Consider $f(x) = \dfrac{x^2 - 2x - 3}{2x^2 - x - 10}$.
Questions $a - e$ below relate to this polynomial function. Write all the answers and show ALL your work on separate paper.

a) Factor the denominator.
b) What is the domain of the function?
c) What are the vertical asymptotes?
d) What is the horizontal asymptote?
e) What are the intercepts for this function? Write them as ordered pairs.
f) Check if the graph intersects its horizontal asymptote.
g) Plot additional points, as necessary, to get the shape of this function and sketch a graph.

Math 130

Quiz #2 - SOLUTIONS

(1) $f(x) = -x^2 - 6x - 5$

a) The equation represents a parabola opening downward ($a = -1 < 0$).

b) y-int: $x = 0, y = -5$ $\boxed{(0,-5)}$

c) $V(x_v, y_v)$ $x_v = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = -3$

$y_v = -(-3)^2 - 6(-3) - 5 = 4$

$\boxed{V(-3,4)}$

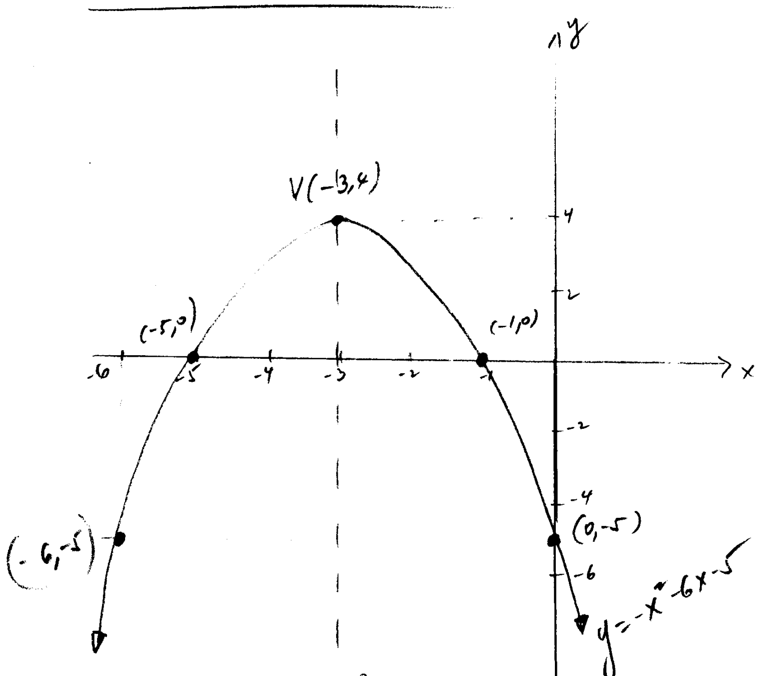
d) x-int: $y = 0$

$-x^2 - 6x - 5 = 0 \quad |(-1)$

$x^2 + 6x + 5 = 0$

$(x+1)(x+5) = 0 \quad \left\langle \begin{array}{l} x = -1 \\ x = -5 \end{array} \right.$

$\boxed{(-1,0) \text{ and } (-5,0)}$



e) Domain: $x \in \mathbb{R}$

Range: $y \in (-\infty, 4]$

(2) $f(x) = x^5 + 4x^4 - 3x^3 - 18x^2$

a) The long-term behavior is given by the leading term x^5

when $x \to \infty$, $y \to \infty$

when $x \to -\infty$, $y \to -\infty$

b)

	1	4	-3	-18	0	0
3	1	7	18	36	108	324

$\boxed{f(x) = (x-3)(x^4 + 7x^3 + 18x^2 + 36x + 108) + 324}$

c) $f(x) = x^5 + 4x^4 - 3x^3 - 18x^2$

Descartes' Rule of Signs can be applied only if the polynomial has a nonzero constant term.

$f(x) = x^2(x^3 + 4x^2 - 3x - 18)$

We'll apply Descartes' Rule of signs for $x^3 + \underbrace{4x^2 - 3x}_{1} - 18$

1 variation in sign $\Rightarrow$

ONE positive real zero

$\underbrace{-x^3 + 4x^2}_{1} + \underbrace{3x - 18}_{2}$

2 variations in sign $\Rightarrow$

Two or zero negative real zeros

d) We will apply the Rational zeros theorem to the polynomial $x^3 + 4x^2 - 3x - 18$

We can apply it because

all coefficients are integers and constant term is nonzero.

$f(x) = x^2\underbrace{(x^3+4x^2-3x-18)}$

$\frac{p}{q} = \frac{\text{factor of } 18}{\text{factor of } 1}$

possible rational zeros

$\frac{p}{q} \in \{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18\}$

(e) Try $x=1$

	1	4	-3	-18
~~1~~	~~1~~	~~5~~	~~2~~	~~-16~~
2	1	6	9	0

Try $x=2$

Therefore,

$f(x) = x^2(x-2)(x^2+6x+9)$

$\boxed{f(x) = x^2(x-2)(x+3)^2}$ the factorization of $f(x)$

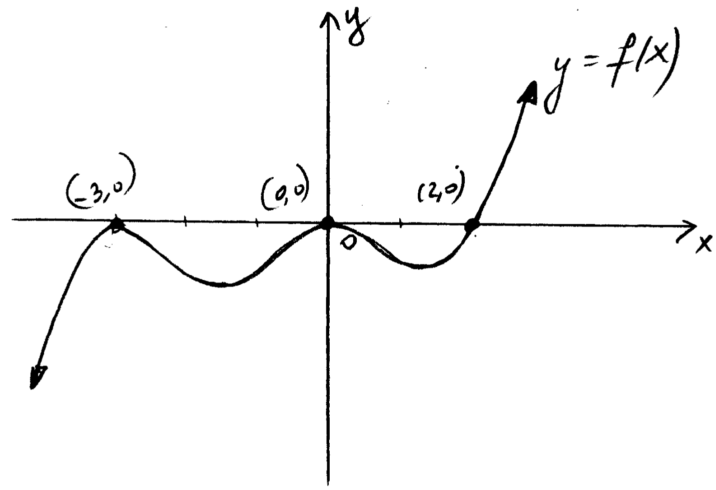
zeros: $\begin{cases} x=0 & m=2 \\ x=2 & m=1 \\ x=-3 & m=2 \end{cases}$

(f) x-n: $(0,0), (2,0), (-3,0)$

y-n: $(0,0)$

(g)

x	$-\infty$	-3	0	2	∞
$f(x)$	$-\infty$	0	0	0	∞
		$m=2$	$m=2$	$m=1$	



③ $\left.\begin{array}{l} x=2 \\ x=-3 \\ x=1+i \end{array}\right\}$ (given)

then $x = 1-i$ is also a zero

$f(x) = (x-2)(x+3)(x-(1+i))(x-(1-i))$

$f(x) = (x-2)(x+3)(x-1-i)(x-1+i)$

$f(x) = (x-2)(x+3)((x-1)^2 - i^2)$

$\boxed{f(x) = (x-2)(x+3)(x^2-2x+2)}$

(4) $f(x) = \frac{x^2-2x-3}{2x^2-x-10}$

(a) $f(x) = \frac{(x-3)(x+1)}{(2x-5)(x+2)}$

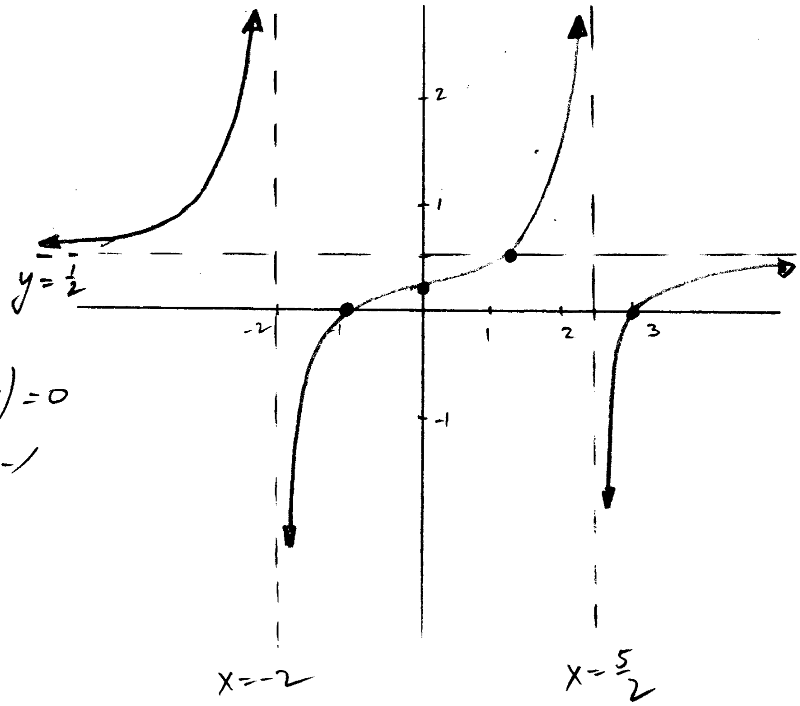
(b) $\begin{cases} 2x-5 \neq 0 & x \neq \frac{5}{2} \\ \text{and} & \\ x+2 \neq 0 & x \neq -2 \end{cases}$

$\boxed{x \in \mathbb{R} \setminus \{\frac{5}{2}, -2\}}$

Domain of $f(x)$

(c) V.A. $\boxed{\begin{matrix} x = \frac{5}{2} \\ x = -2 \end{matrix}}$

(d) H.A $\boxed{y = \frac{1}{2}}$



(e) x-n: $y = 0$ iff $(x-3)(x+1) = 0$
iff $x = 3$ or $x = -1$

x-n: $\boxed{(3,0) \text{ and } (-1,0)}$

y-n: $x = 0$, $y = \frac{-3}{-10} = \frac{3}{10}$

y-n: $\boxed{(0, \frac{3}{10})}$

(f) $f(x) = \frac{1}{2}$

$$\frac{x^2 - 2x - 3}{2x^2 - x - 10} = \frac{1}{2}$$

$$2(x^2 - 2x - 3) = 2x^2 - x - 10$$

$$\cancel{2x^2} - 4x - 6 = \cancel{2x^2} - x - 10$$

$$-6 + 10 = -x + 4x$$

$$3x = 4, \quad x = \frac{4}{3}$$

The graph intersects the line $y = 1$ at $\boxed{(\frac{4}{3}, 1)}$

Note: no test points necessary