Math 71 Spring 07 Name:

TEST 2 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Do the following operations (simplify):

a) $\left(2-\frac{6}{x+1}\right)\left(1+\frac{3}{x-2}\right)$

b) $\frac{2x^2-2x-12}{x^2-49}\cdot\frac{4x^2-1}{2x^2+5x+2}\cdot\frac{2x^2+3x-7}{2x^2-7x+3}$

c) $\frac{x^3-8}{x^2-4}$

d) Divide using long division: $\frac{4a^3+12a^2+7a-1}{2a+3}$

e) $\left(x^{\frac{1}{2}}-3\right)\left(x^{\frac{1}{2}}+5\right)$

f) $\left(\frac{x^{\frac{-5}{4}}y^{\frac{1}{3}}}{x^{-\frac{3}{4}}}\right)^{-6}$ and write the final answer using only positive exponents

g) $\frac{2-3i}{3+i}$

h) $(2-3i)(1-i)-(3-i)(3+i)$

2. Solve the following equations:

a) $\frac{y-1}{y^2-4}+\frac{y}{y^2-y-2}=\frac{2y-1}{y^2+3y+2}$

b) $z=\frac{x-\bar{x}}{s}$ solve for x

3. If $f(x)=x^2-5x+3$, find $\frac{f(a+h)-f(a)}{h}$.

4. If $f(x)=\dfrac{x+2}{x+3}$ and $g(x)=\dfrac{x+1}{x^2+2x-3}$, find all the values of a for which $f(a)=g(a)+1$.

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5. If $f(x)=2x^2-3x+1$, find $f(i)$.

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6. If $g(x)=x^2-6x-4$, find $g\left(\sqrt{a+1}-\sqrt{a-1}\right)$.

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7. If $h(x)=x+\sqrt{x+5}$, find x such that $h(x)=7$.

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8. if $f(x)=\dfrac{x^2+19}{2-x}$, find $f(3i)$.

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9. Let $f(x)=\sqrt{x-2}$.

a) What is the domain of this function?

b) Sketch the graph of the function by plotting points.

c) What is the range of this function?

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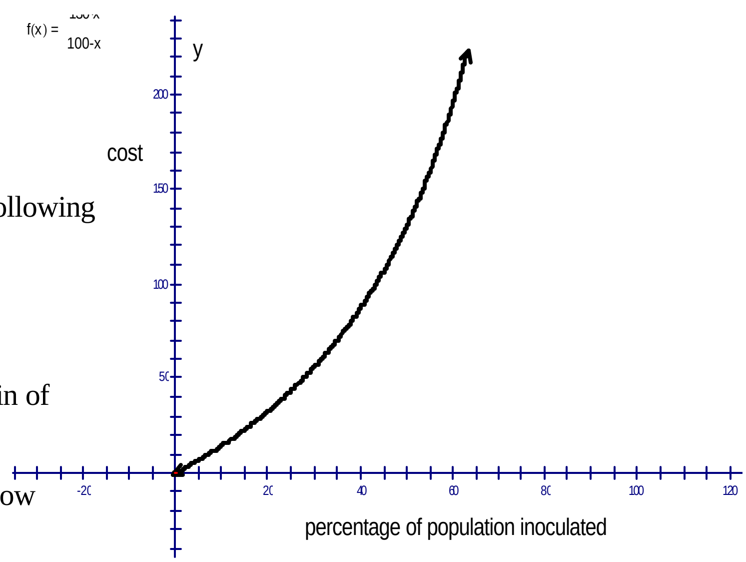
10. The rational function

$$f(x)=\frac{130x}{100-x}$$

models the cost, f(x), in millions of dollars, to inoculate x% of the population against a particular strain of flu. The graph of the rational function is shown. Answer the following questions:



a) Find and interpret $f(60)$.

b) What value of x must be excluded from the domain of of the function?

c) What happens to the cost as x approaches 100? How is this shown by the graph?

10. Police use the function $f(x)=\sqrt{20x}$ to estimate the speed of a car, f(x), in miles per hour, based on the length, x, in feet, of its skid marks upon sudden braking on a dry asphalt road. A motorist is involved in an accident. A police officer measures the car's skid mark to be 45 feet long. Estimate the speed at which the motorist was traveling before braking. If the posted speed limit is 35 miles per hour and the motorist tells the officer she was not speeding, should the officer believe her? Explain.

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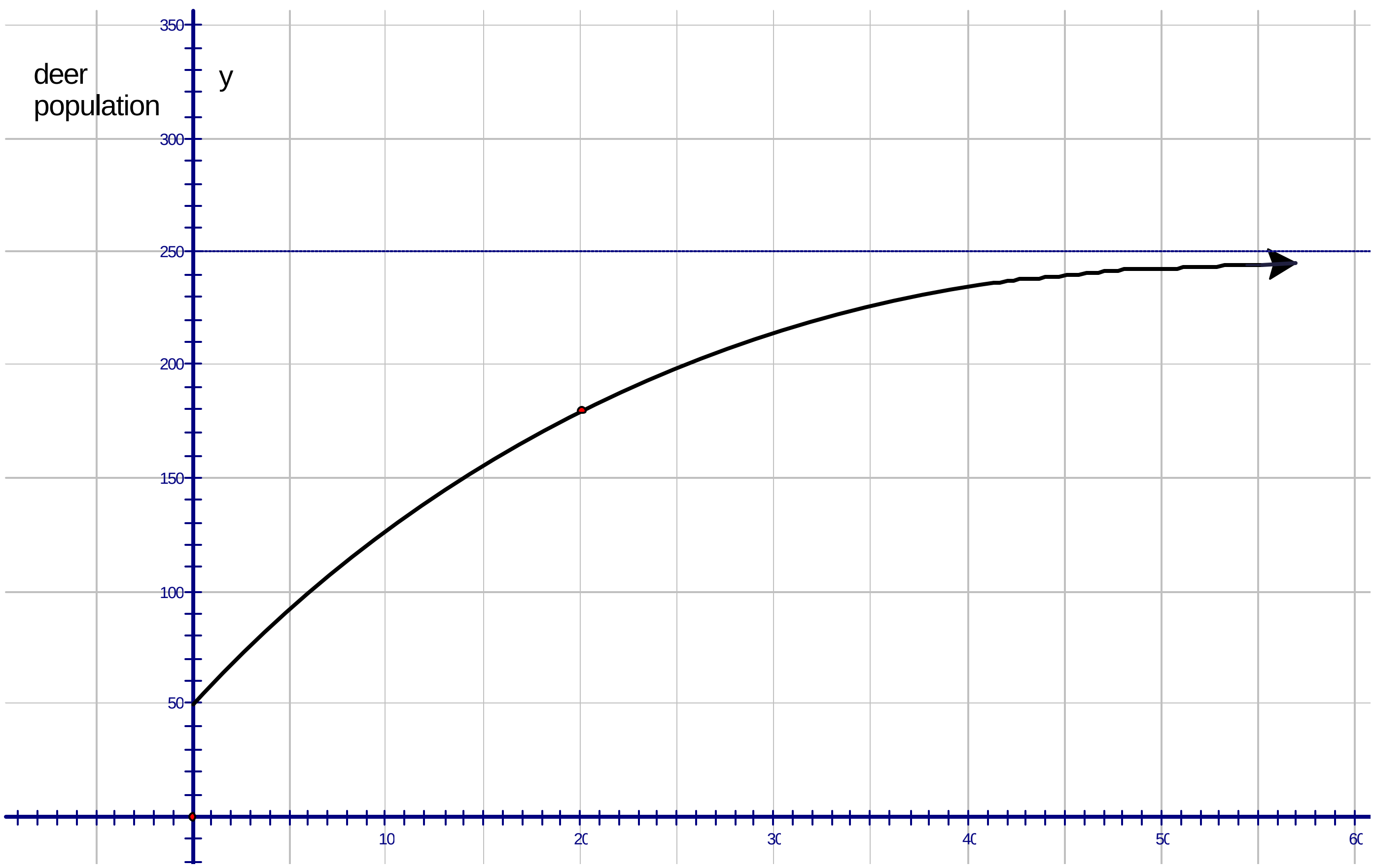
12. Deer are placed into a newly acquired habitat. The deer population over time is modeled by a rational function whose graph is shown in the figure. Use the graph to answer each of the following questions:

a) How many deer were introduced into the habitat?

b) What is the population after 25 years?

c) What is the equation of the horizontal asymptote shown in the figure? What does this mean in terms of the deer population?



SOLUTIONS

(1a) $\left(2-\frac{6}{x+1}\right)\left(1+\frac{3}{x-2}\right) =$

$= \frac{2(x+1)-6}{x+1}\cdot\frac{x-2+3}{x-2}$

$= \frac{2x-4}{\cancel{x+1}}\cdot\frac{\cancel{x+1}}{x-2} = \frac{2(x-2)}{x-2} = \boxed{2}$

(b) $\frac{2x^2-2x-12}{x^2-49}\cdot\frac{4x^2-1}{2x^2+5x+2}\cdot\frac{2x^2+13x-7}{2x^2-7x+3}=$

$=\frac{2(x^2-x-6)}{(x-7)\cancel{(x+7)}}\cdot\frac{(2x-1)\cancel{(2x+1)}}{\cancel{(2x+1)}(x+2)}\cdot\frac{\cancel{(2x-1)}\cancel{(x+7)}}{\cancel{(2x-1)}(x-3)}$

$=\frac{2\cancel{(x-3)}(x+2)}{(x-7)}\cdot\frac{2x-1}{\cancel{x+2}}\cdot\frac{1}{\cancel{x-3}}$

$=\boxed{\frac{2(2x-1)}{x-7}}$

(c) $\frac{x^3-8}{x^2-4} = \frac{x^3-2^3}{x^2-2^2}$

$=\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$

$=\boxed{\frac{x^2+2x+4}{x+2}}$

(d) $\frac{4a^3+12a^2+7a-1}{2a+3} = \boxed{2a^2+3a-1+\frac{2}{2a+3}}$

$$\begin{array}{r|l} & 2a^2+3a-1 \\ \hline 2a+3 & 4a^3+12a^2+7a-1 \\ & -4a^3-6a^2 \\ \hline & \quad 6a^2+7a-1 \\ & -6a^2-9a \\ \hline & \qquad -2a-1 \\ & \qquad +2a+3 \\ \hline & \qquad\quad 2 \ R \end{array}$$

(e) $(x^{\frac12}-3)(x^{\frac12}+5) =$

$=(x^{\frac12})^2+5x^{\frac12}-3x^{\frac12}-15$

$=x+2x^{\frac12}-15$

$=\boxed{x+2\sqrt{x}-15}$

(f) $\left(\frac{x^{-\frac54}y^{\frac13}}{x^{-\frac34}}\right)^{-6} = \left(x^{-\frac54-(-\frac34)}y^{\frac13}\right)^{-6}$

$=\left(x^{-\frac12}y^{\frac13}\right)^{-6}$

$=\left(x^{-\frac12}\right)^{-6}\left(y^{\frac13}\right)^{-6}$

$=x^3y^{-2}$

$=\boxed{\frac{x^3}{y^2}}$

(g) $\frac{2-3i}{3+i} = \frac{(2-3i)(3-i)}{(3+i)(3-i)}$

$=\frac{6-2i-9i+3i^2}{3^2-i^2}$

$=\frac{6-11i+3(-1)}{9-(-1)}$

$=\boxed{\frac{3-11i}{10}}$

(h) $(2-3i)(1-i)-(3-i)(3+i)=$

$=(2-2i-3i+3i^2)-(3^2-i^2)$

$=2-5i-3-9+i^2$

$=-10-5i-1$

$=\boxed{-11-5i}$

(2) (a)

$$\frac{y-1}{y^2-4}+\frac{y}{y^2-y-2}=\frac{2y-1}{y^2+3y+2}$$

$$\frac{^{y+1}\,y-1}{(y-2)(y+2)}+\frac{^{y+2}\,y}{(y-2)(y+1)}=\frac{^{y-2}\,2y-1}{(y+1)(y+2)}$$

Conditions: $\begin{cases} y \neq 2 \\ y \neq -2 \\ y \neq -1 \end{cases}$

$LCD = (y-2)(y+2)(y+1)$

$(y+1)(y-1)+y(y+2)=(y-2)(2y-1)$

$\cancel{y^2}-1+\cancel{y^2}+2y = \cancel{2y^2}-5y+2$

$-1+2y = -5y+2$

$2y+5y = 2+1$

$7y=3 \Rightarrow \boxed{y=\frac{3}{7}}$

(b) $z=\frac{x-\bar{x}}{s}$ solve for x

$zs = x-\bar{x}$

$\boxed{x = zs+\bar{x}}$

(3) $f(x) = x^2-5x+3$

$\frac{f(a+h)-f(a)}{h} =$

$= \frac{\left((a+h)^2-5(a+h)+3\right)-(a^2-5a+3)}{h}$

$= \frac{\cancel{a^2}+2ah+h^2\cancel{-5a}-5h\cancel{+3}\cancel{-a^2}\cancel{+5a}\cancel{-3}}{h}$

$= \frac{2ah+h^2-5h}{h} = \frac{\cancel{h}(2a+h-5)}{\cancel{h}}$

$= \boxed{2a+h-5}$

(4) $f(x) = \frac{x+2}{x+3}$

$g(x) = \frac{x+1}{x^2+2x-3}$

Find a such that

$f(a) = g(a)+1$

Solution

$f(a) = g(a)+1$

$\frac{a+2}{a+3} = \frac{a+1}{a^2+2a-3}+1$

$\frac{^{a-1}\,a+2}{a+3} = \frac{a+1}{(a+3)(a-1)} + \frac{(a+3)(a-1)}{1}$

Conditions: $\begin{cases} a \neq -3 \\ a \neq 1 \end{cases}$

$LCD = (a+3)(a-1)$

$(a-1)(a+2) = a+1+a^2+2a-3$

$\cancel{a^2}+a-2 = \cancel{a^2}+3a-2$

$a=3a$

$2a=0 \Rightarrow \boxed{a=0}$

(5) $f(x) = 2x^2-3x+1$

$f(i) = 2(i)^2-3i+1$

$= 2(-1)-3i+1$

$\boxed{f(i) = -1-3i}$

(6) $g(x) = x^2 - 6x - 4$

$g(\sqrt{a+1} - \sqrt{a-1}) =$

$= (\sqrt{a+1} - \sqrt{a-1})^2 - 6(\sqrt{a+1} - \sqrt{a-1}) - 4$

$= a + \cancel{1} - 2\sqrt{a+1}\sqrt{a-1} + a - \cancel{1}$

$-6\sqrt{a+1} + 6\sqrt{a-1} - 4$

$= \boxed{2a - 2\sqrt{a^2-1} - 6\sqrt{a+1} + 6\sqrt{a-1}}$ /

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(7) $h(x) = x + \sqrt{x+5}$

Find x : $h(x) = 7$

Solution

$h(x) = 7$

$x + \sqrt{x+5} = 7$

$\sqrt{x+5} = 7 - x \quad /^2$

$(\sqrt{x+5})^2 = (7-x)^2$

$x + 5 = 49 - 14x + x^2$

$x^2 - 15x + 44 = 0$

$(x-4)(x-11) = 0$ — $x = 4$ OR $\cancel{x = 11}$

Check $x = 4$:

$4 + \sqrt{4+5} \overset{?}{=} 7$

$4 + 3 = 7$ true

Check $x = 11$

$11 + \sqrt{11+5} \overset{?}{=} 7$ false

Solution set is $\{4\}$

(8) $f(x) = \dfrac{x^2+19}{2-x}$

$f(3i) = \dfrac{(3i)^2+19}{2-(3i)}$

$= \dfrac{9i^2+19}{2-3i} = \dfrac{9(-1)+19}{2-3i}$

$= \dfrac{10}{2-3i}$

$= \dfrac{10(2+3i)}{(2-3i)(2+3i)}$

$= \dfrac{10(2+3i)}{2^2-(3i)^2} = \dfrac{10(2+3i)}{4-9i^2}$

$= \dfrac{10(2+3i)}{4-9(-1)} = \dfrac{10(2+3i)}{13}$

$\boxed{f(3i) = \dfrac{10(2+3i)}{13}}$ /

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(9) $f(x) = \sqrt{x-2}$

a) Domain = ?

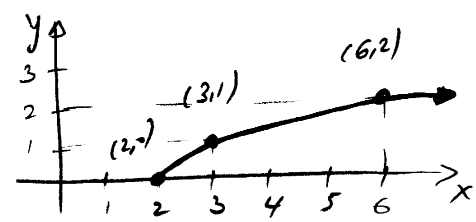
Condition: $x - 2 \geq 0$

$x \geq 2$

$\boxed{\text{Domain: } x \in [2, \infty)}$ /

b)

x	y
2	0
3	1
6	2



c) $\boxed{\text{Range: } y \in [0, \infty)}$ /

(10) $f(x) = \frac{130x}{100-x}$

a) $f(60) = \frac{130 \cdot 60}{100-60} = \frac{130 \cdot 6\cancel{0}}{4\cancel{0}}$

$\boxed{f(60) = 195 \text{ million } \$}$

The cost to innoculate 60% of the population is \$195 million

b) $\boxed{x \neq 100}$

c) $x \to 100$, $f(x) \to \infty$
The graph has a vertical asymptote at $x = 100$
As $x \to 100$, the cost is increasing.

(11) $f(x) = \sqrt{20x}$
x = length (in ft) of the skid marks
$f(x)$ = speed of car (mi/hr)
if $x = 45$ ft, find $f(x)$
$f(45) = \sqrt{20(45)}$
$= \sqrt{900}$
$f(45) = 30$ mi/hr
The motorist was traveling at 30 mi/hr before braking
The officer should believe the motorist.

(12) Let D = number of deer
(a) when $t=0$, $D = 50$
See point $(0,50)$ on the graph.
(b) when $t=25$, $D=200$
See point $(25,200)$ on the graph.

(c) $y = 250$ horizontal asymptote

The population will approach 250 deer over time.